



Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: <http://tjps.tu.edu.iq/index.php/j>



On Micro – $\dot{b}$ – open sets and Micro – $\dot{b}$ – continuity

Taha H. Jasim , Marwah D. Sultan

Department of Mathematics , College of Computer Science and , Mathematics , Tikrit University , Tikrit , Iraq

DOI: <http://dx.doi.org/10.25130/tjps.25.2020.099>

ARTICLE INFO.

Article history:

-Received: 2 / 2 / 2020

-Accepted: 30 / 7 / 2020

-Available online: / / 2020

Keywords: micro – $\dot{b}$ – open sets, micro continuous, micro – $\dot{b}$ – continuity, micro – $\dot{b}$ – irresolute.

Corresponding Author:

Name: Marwah D. Sultan

E-mail:

marwahmm29@gmail.com

Tel:

ABSTRACT

The purpose of this research is generalize the concepts of micro – $\dot{b}$ – openset, so we study the relation this new concept open set micro-pre-open set, micro-semi-open set, micro- α – open set, and micro – β – openset and micro – $\dot{b}$ – open continuity and micro – $\dot{b}$ – irresolute continuity in micro topological space and study the relationship among them, many theorems were proved as characteristic of these type of continuity and some examples were introduced.

1 Introduction

In 2013 Thivagar [1] introduced nano topology; based on the concept of lower approximation, upper approximation and boundary region. Nano topology have maximum five nano open sets and minimum three nano open sets including $U, \emptyset$ and " on nano – open sets " introduced in 2015 by Revalthy .A and Ganambal llango [4]

In 1996 Andrijevic [6] introduced a study about " b – open sets " and "on nano b – open sets in nano topological spaces" introduced in 2016 by M.parimala, C.indirani and S.jafari [7]. Andrijevic. D.[5] in 1987 introduced research " on the topology generated by pre – open set " and " on semi pre – open sets in nano topological spaces " introduced by Mary .D .A and Arokia Rani. A .[8] in 2014 .

Suppose we want add some more open sets ,For that in 2019 S. Chandrasekar [3] introduced " on Micro topological spaces " which is extension concept of nano topological space such that we have some more open sets and every nano topological space is Micro topological space.

Levine [9] introduced semi – opensets and semi continuity in topological spaces in 1963. Mashhour [10] introduced a study about on pre – continuous mappings and weak pre continuous mappings in 1982 . Power .P.L and Rajak . K [12] introduced " Fine irresolute mappings " in year 2012 and " A new of

contra contiuity in Bi – supra topological space " introduced by Taha. H. J. and others [11] in 2018 .

In this paper we define micro – $\dot{b}$ – open set, and study the relationships with micro – pre – open set micro – semi – open set, micro – α – openset and micro – open set .

At last micro – $\dot{b}$ – continuity and micro – $\dot{b}$ – irresolute were studied and investigated some characterization.

The important of this paper to generalize some class of open sets and contioutiuy in order to extension this subject to a new spaces .

2 Preliminary

Now recall some definition and examples which is useful in our work.

Definition2.1. [1]

Let U be an non- empty finite set of objects named the universe and R be an equivalence relationship on U called the indiscernibility relation. Elements of the same category of equality are said to be indiscernible from each other. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$, Then :

1. The lower approximation of X relative to R is a set of all objects, which can be classified as X relative to R and is defined by $L_R(X) = \bigcup_{x \in U} \{ R(x) : R(x) \subseteq X \}$, where $R(x)$ denotes the equivalence class specified by x

2. The upper approximation of X relative to R is a set of all objects, which can be possibly classified as X relative to R and is defined by $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

3. The boundary area of X relative to R is the set of all objects, which can be classified neither as X nor as not X relative to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [1]

Let U be the universe, R be an equivalence relation on

U and $T_R(X) =$

$\{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $T_R(X)$ satisfies the following axioms:

1. U and $\emptyset \in T_R(X)$.
2. The union of elements of any subset of $T_R(X)$ is in $T_R(X)$.
3. The intersection of elements of any finite subset of $T_R(X)$ is in $T_R(X)$.

That is $T_R(X)$ forms a topology on U is named the nano topology on U with respect to X . We called $(U, T_R(X))$ the nano topological space. The element's of $T_R(X)$ is said to be nano open sets.

Definition 2.3 [3]

The pair $(U, T_R(X))$ is nano topological space here

$\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in$

$T_R(X)$ and called it micro topology of $T_R(X)$ by μ where $\mu \in T_R(X)$.

Definition 2.4 [3]

The micro topology $\mu_R(X)$ satisfy the following axioms :

1. $U, \emptyset \in \mu_R(X)$
2. The union of elements of any sub collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is named the micro topology on U relative to X . The triple $(U, T_R(X), \mu_R(X))$ is said to be micro topological spaces and the elements of $\mu_R(X)$ are named micro open sets and the complement of micro open sets is said an micro closed set.

Definition 2.5.[3]

let $(U, T_R(X), \mu_R(X))$ be a micro topological space and $A \subset U$. then A is called to be micro- pre - open set if $A \subseteq \text{Mic} - \text{int}(\text{Mic} - \text{cl}(A))$ and micro - pre - closed set if $\text{Mic} - \text{cl}(\text{Mic} - \text{int}(A)) \subseteq A$. A complement of micro - pre - open set of U is said to be micro - pre - closed set.

Definition 2.6 [3]

Let $(U, T_R(X), \mu_R(X))$ be an micro topological space and $A \subset U$, then A is said to be micro - semi - open set if $A \subseteq \text{Mic} - \text{cl}(\text{Mic} - \text{int}(A))$ and micro - semi_closed set if $\text{Mic} - \text{int}(\text{Mic} - \text{cl}(A)) \subseteq A$. A complement of micro - semi - open set of a space U is said micro - semi - closed set.

3 Micro - $\dot{b}$ - open sets

In this section we defined and study 'micro - $\dot{b}$ - open sets' some of their properties are analogous for

open sets .and the relation it by micro-pre -open, micro -semi-open and micro- α -open sets, and micro - β - open sets .

Definition 3.1:

suppose $(U, T_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$, Then A is said to be micro- $\dot{b}$ -open if $A \subseteq \text{Mic} - \text{int}(\text{Mic} - \text{cl}(A)) \cup \text{Mic} - \text{cl}(\text{Mic} - \text{int}(A))$, We denoted by $\text{Mic} - \dot{b}$ - open .

Definition 3.2 :

Let $(U, T_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$, then A be called micro - $\dot{b}$ - closed if its complement is $\text{Mic} - \dot{b}$ - open set and denoted by $(\text{Mic} - \dot{b} - \text{closed})$.

Example 3.3:

Let $U = \{p, q, r, s, t\}$

$U/R = \{\{p\}, \{q, r, s\}, \{t\}\}$

And let $X = \{p, q\} \subseteq U$

Then $T_R(X) =$

$\{U, \emptyset, \{p\}, \{t\}, \{p, t\}, \{p, q, r, s\}, \{q, r, s\}\}$

If $\mu = \{t\}$

$\text{Micro} - \emptyset = \mu_R(X) =$

$\{U, \emptyset, \{p\}, \{t\}, \{p, t\}, \{p, q, r, s\}, \{q, r, s\}, \{q, r, s, t\}\}$

micro closed sets are

$U, \emptyset, \{p\}, \{p, t\}, \{t\}, \{q, r, s\}, \{p, q, r, s\}$ and $\{q, r, s, t\}$

$A \subseteq \{q, r, s\}$ then A is $\text{Mic} - \dot{b}$ - open

Theorem 3.4:

Every $\text{Mic} - \text{open}$ set is $\text{Mic} - \dot{b}$ - open set .

Proof:

Let A be $\text{Mic} - \text{open}$, then $A = \text{Mic} - \text{int}(A) \subseteq \text{Mic} - \text{cl}(\text{Mic} - \text{int}(A))$

But; $A \subseteq \text{Mic} - \text{cl}(A)$, since $A = \text{Mic} - \text{int}(A)$

Then $A \subseteq \text{Mic} - \text{cl}(\text{Mic} - \text{int}(A))$

Hence $A \subseteq \text{Mic} - \text{cl}(\text{Mic} - \text{int}(A)) \cup \text{Mic} - \text{int}(\text{Mic} - \text{cl}(A))$ $\square$

The opposite of the upside theorem is not right, can be see in the next example .

Example 3.4.

Let $U = \{1, 2, 3, 4, 5\}$ and $U/R =$

$\{\{1\}, \{2, 3, 4\}, \{5\}\}$

supoos $X = \{2, 3\} \subseteq U$

hence $T_R(X) = \{U, \emptyset, \{2, 3, 4\}\}$

Then $\mu = \{1\}$ then $\text{Mic} - \emptyset = \mu_R(X) =$

$\{U, \emptyset, \{1\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$

Clearly $A = \{1, 2, 3, 4\}$ is $\text{Mic} - \dot{b}$ - open but not $\text{Mic} - \text{open}$.

Theorem 3.5:

Every $\text{Mic} - \text{pre} - \text{open}$ set is $\text{Mic} - \dot{b}$ - open set .

Proof:

Let A be a mic - pre - open set in $(U, T_R(X), \mu_R(X))$, then $A \subseteq \text{Mic} - \text{Int}(\text{Mic} - \text{cl}(A))$

And since $\text{Mic} - \text{Int}(\text{Mic} - \text{cl}(A)) \subseteq$

$\text{Mic} - \text{Int}(\text{Mic} - \text{cl}(A)) \cup$

$\text{Mic} - \text{cl}(\text{Mic} - \text{Int}(A))$, then $A \subseteq$

$Mic - Int (Mic - Cl(A)) \cup$
 $Mic - cl (Mic - Int(A))$, Hence A is $Mic - \dot{b} -$
 open set in $(U, T_R(X), \mu_R(X))$ $\square$
 The opposite of the upside theory is not right as
 shown in the following example.

Example 3.6:

let $U = \{1, 2, 3, 4, 5\}$, $U/R = \{\{1\}, \{2, 3, 4\}, \{5\}\}$
 and let $X = \{2, 3\}$ then $T_R(X) =$
 $\{U, \emptyset, \{2, 3, 4\}\}$, then $\mu = \{1\}$,
 then $Mic - O = \mu_R(X) =$
 $\{U, \emptyset, \{1\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$,
 Here $A = \{1, 2, 3, 5\}$ is $Mic - b -$
 open set but not $Mic - pre - open set$.

Theorem 3.7:

Every $Mic - semi - open set$ is $Mic - \dot{b} - open set$.

Proof:

suppose A be a $Mic - semi - open set$ in $(U, T_R(X), \mu_R(X))$,
 Then $A \subseteq Mic - cl (Mic - Int (A))$.

And since $Mic - cl (Mic - Int (A)) \subseteq$

$$Mic - cl (Mic - Int (A)) \cup$$

$$Mic - Int (Mic - cl (A))$$

$$\text{Then } A \subseteq Mic - cl (Mic - Int (A)) \cup$$

$$Mic - Int (Mic - cl (A)),$$

Hence A is $Mic - \dot{b} - open set$ in

$$(U, T_R(X), \mu_R(X)) \quad \square$$

The opposite of the upside theorem need not to be
 right as shown by the following example.

Example 3.8:

Let $U = \{a, b, c, d\}$ with $U/R =$
 $\{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\} \subseteq U$
 Then $T_R(X) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$, then $\mu =$
 $\{d\}$,
 then $Mic - O = \mu_R(X) =$
 $\{U, \emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$,
 Here $A =$
 $\{a, c, d\}$ is $Mic - b - open set$ but it is not $Mic - semi -$

Definition 3.9:

Suppose $(U, T_R(X), \mu_R(X))$ be a micro topological
 space and $A \subset U$, then A is said to be $micro - \alpha -$
 open set briefly ($Mic - \alpha - open$) if $A \subseteq Mic -$
 $int (Mic - cl (Mic - int(A)))$.

Definition 3.10 :

Let $(U, T_R(X), \mu_R(X))$ be an micro topological space
 a subset A of U be called $micro - \alpha - closed set$ if its
 complement is $Micro - \alpha - open set$

Briefly ($Mic - \alpha - closed set$).

Theorem 3.11:

Every $Mic - \alpha - open set$ is $Mic - \dot{b} - open set$.

Proof:

Let A be a $Mic - \alpha - open set$ in $(U, T_R(X), \mu_R(X))$,
 then

$$A \subseteq$$

$$Mic - int (Mic - cl (Mic -$$

$int(A))$, and since $Mic - int (Mic - cl (Mic -$
 $int (A)) \subseteq$

$$Mic - cl (Mic -$$

$$int (A)) \cup Mic - cl (Mic - int (A))$$

$$\text{Then } A \subseteq Mic - cl (Mic - int (A)) \cup$$

$$Mic - int (Mic - cl(A)),$$

Hence A is $Mic - \dot{b} -$

$$open set in (U, T_R(X), \mu_R(X)) \quad \square$$

The opposite of the upside theorem needed not to be
 right as shown by the following example.

Example 3.12:

In the Example(3.6), a set A is $mic - \dot{b} -$
 open set but not $Mic - \alpha - open set$.

Definition 3.13:

Let $(U, T_R(X), \mu_R(X))$ be a micro topological space
 and $A \subset U$, then A is said to be $micro - \beta - open$
 set, briefly ($Mic - \beta - open set$) if $A \subseteq Mic -$
 $cl (Mic - int (Mic - cl(A)))$.

Definition 3.14 :

Let $(U, T_R(X), \mu_R(X))$ be an micro topological space
 and $A \subset U$, then A is said to be $micro - \beta -$
 closed set briefly ($Mic - \dot{b} - closed set$) if its
 complement is $Mic - \beta - open set$.

Theorem 3.15 :

Every $Mic - \dot{b} - open set$ is $Mic - \beta - open set$.

Proof:

Let A be $Mic - \dot{b} - open set$ in $(U, T_R(X), \mu_R(X))$.

$$\text{Then } A \subseteq Mic - cl (Mic - int(A)) \cup Mic -$$

$$int (Mic - cl(A)) \subseteq Mic - cl (Mic -$$

$$int (Mic - cl(A)) \dots \dots (1)$$

$$\text{Since } A \subseteq Mic - cl(A) \text{ then } Mic - int(A) \subseteq$$

$$Mic - int (Mic - cl(A)), \text{ thus } Mic - cl (Mic -$$

$$int(A)) \subseteq Mic - cl (Mic - int (Mic -$$

$$cl(A)) \dots \dots (2)$$

From (1) and (2) we get A is $Mic - \beta - open set$
 $\square$

The opposite of the upside theorem is not right shown
 by the following example

Example 3.16:

In Example (3.6), a set $A = \{3, 4, 5\}$ is $Mic - \beta -$
 open set but not $Mic - b - open set$.

Remark 3.17 :

Intersection of two $Mic - \dot{b} - open sets$ needed not to
 be $Mic - \dot{b} - open set$.

Example 3.18:

Let $U = \{1, 2, 3, 4\}$ with $U/R =$

$$\{\{1\}, \{2, 3\}, \{4\}\}, \text{ and } X = \{1, 3\} \subseteq U$$

$$T_R(X) = \{U, \emptyset, \{1, 2, 3\}, \{2, 3\}, \{1\}\} \text{ then } \mu =$$

$$\{3\}$$

$$Micro - O = \mu_R(X) =$$

$$\{U, \emptyset, \{1\}, \{2, 3\}, \{1, 2, 3\}, \{3\}, \{1, 3\}\}$$

$$\text{Then } \{1, 4\} \text{ and } \{3, 4\} \text{ are } Mic - \dot{b} -$$

$$open sets, \text{ but } \{4\} \text{ is not } Mic - \dot{b} - open set.$$

Definition 3.19:

Let $(U, T_R(X), \mu_R(X))$ be a micro topological space an element $x \in A$ is called $Mic - \dot{b}$ - interior point of A if there exist a micro $-\dot{b}$ - open set H such that $x \in H \subseteq U$

Definition 3.20:

Let $(U, T_R(X), \mu_R(X))$ be micro topological space and $A \subseteq U$, then the union of all $Mic - \dot{b}$ - open sets in U which contained in A be named a micro $-\dot{b}$ - interior of A and denoted by $(Mic - \dot{b} - interior)$.

Definition 3.21:

Let $(U, T_R(X), \mu_R(X))$ be micro topological space and $A \subseteq U$, then the intersection of all $Mic - \dot{b}$ - closde sets including A be called aMicro $-\dot{b}$ - closure of A and denoted by $Mic - Cl_b(A)$.

Remark 3.22:

It is clear that $Mic - int_b(A)$ is $Mic - \dot{b}$ - open set and $Mic - cl_b(A)$ is $Mic - \dot{b}$ - closed set.

Theorem 3.23:

1. $A \subseteq Mic - cl_b(A)$ and $A = Mic - cl_b(A)$ if and only if A is $Mic - \dot{b}$ - closed set.
2. $Mic - int_b(A) \subseteq A$ and $A = Mic - int_b(A)$ if and only if A is $Mic - \dot{b}$ - open set.

Theorem 3.24:

1. Arbitrary union of $Mic - \dot{b}$ - open sets is $Mic - \dot{b}$ - open set.
2. Arbitrary intersection of $Mic - \dot{b}$ - closed sets is $Mic - \dot{b}$ - closed set.

Proof:

1. Let $\{A_\alpha \mid \alpha \in I\}, I = 1, 2, \dots, n$

The family of $Mic -$

$\dot{b}$ - open sets in U . By definition (3.20)

This implies that $A_\alpha \subseteq [(Mic - int(Mic - cl(A_\alpha))) \cup (Mic - Cl(Mic - int(A_\alpha)))] \forall \alpha \in I$

Then $\cup A_\alpha \subseteq \cup [(Mic - int(Mic - cl(A_\alpha))) \cup (Mic - Cl(Mic - int(A_\alpha)))] \forall \alpha \in I$.

Since $\cup (Mic - int(Mic - cl(A_\alpha))) \subseteq Mic - int(\cup Mic - cl(A_\alpha))$ and

$Mic - int(\cup Mic - cl(A_\alpha)) \subseteq (Mic - int(Mic - cl(\cup A_\alpha)))$ and by the same way we get $\cup (Mic - Cl(Mic - int(A_\alpha))) \subseteq (Mic - Cl(Mic - int(\cup A_\alpha)))$

This implies that :

$\cup A_\alpha \subseteq Mic - int(Mic - cl(\cup A_\alpha)) \cup Mic - int(Mic - cl(\cup A_\alpha))$

Hence $\cup A_\alpha$ is $Mic - \dot{b}$ - open

□

2. Let $\{B_\alpha \mid \alpha \in I\}, I =$

$0, 1, 2, \dots, n$ be a family of $Mic - \dot{b}$ - closed sets in U .

suppose $A_\alpha = B_\alpha^c, \forall \alpha \in I$, then $\{A_\alpha \mid \alpha \in I\}$ is a family of $Mic - \dot{b}$ - sets, then by (1)

$\cup A_\alpha = \cup B_\alpha^c$ is $Mic - \dot{b}$ - open,

Hence $(\cap B_\alpha)$ is $Mic - \dot{b}$ - closed set □

Theorem 3.25:

Let $A \subset U (U, T_R(X), \mu_R(X))$ then $Mic - \dot{b} - int(A)$ is equal to the union of all $Mic - \dot{b}$ - open sets contained in A .

Proof:

We need to prove that $Mic - \dot{b} - int(A) = \cup \{B \mid B \subset A, B \text{ is } Mic - \dot{b} - \text{open set}\}$.

Let $x \in Mic - \dot{b} - int(A)$, then there exist $Mic - \dot{b} - \text{open set } B_0$ in U such that $x \in B_0 \subseteq Mic - \dot{b} - int(A)$ by definition (3.20)

Conversely, suppose $x \in \cup \{B \mid B \subset A, B \text{ is } Mic - \dot{b} - \text{open set}\}$ then there exist a set B_0 is $Mic - \dot{b} - \text{open set}$, such that $x \in B_0 \subseteq A$.

Hence $x \in Mic - \dot{b} - int(A)$

And $\cup \{B \mid B \subset A, B \text{ is } Mic - \dot{b} - \text{open set}\} \subset A$

So $Mic - \dot{b} - int(A) = \cup \{B \mid B \subset A, B \text{ is } Mic - \dot{b} - \text{open set}\}$ □

Theorem 3.26:

1. $Mic - int_b(A \cup B) \supseteq Mic - int_b(A) \cup Mic - int_b(B)$.
2. $Mic - int_b(A \cap B) = Mic - int_b(A) \cap Mic - int_b(B)$.

Proof:

1. since $A \subset A \cup B$, then $Mic - int_b(A) \subseteq Mic - int_b(A \cup B) \dots \dots (1)$

and $B \subset A \cup B$, then $Mic - int_b(B) \subseteq Mic - int_b(A \cup B) \dots \dots (2)$

from (1) and (2) we get $Mic - int_b(A) \cup Mic - int_b(B) \subseteq Mic - int_b(A \cup B)$.

2. since $A \cap B \subseteq A$ then $Mic - int_b(A \cap B) \subseteq Mic - int_b(A)$ and $A \cap B \subseteq B$ then $Mic - int_b(A \cap B) \subseteq Mic - int_b(B)$.

That is $Mic - int_b(A \cap B) \subseteq Mic - int_b(A) \cap Mic - int_b(B) \dots \dots (1)$

Now $Mic - int_b(A) \subseteq A$ and $Mic - int_b(B) \subseteq B$ therefore $Mic - int_b(A) \cap Mic - int_b(B) \subseteq A \cap B$.

That is $Mic - int_b[Mic - int_b(Mic - int_b(A) \cap Mic - int_b(B))] \subseteq Mic - int_b(A \cap B)$.

Since $Mic - int_b[Mic - int_b(A) \cap Mic - int_b(B)] = [Mic - int_b(A) \cap Mic - int_b(B)]$.

Therefore $[Mic - int_b(A) \cap Mic - int_b(B)] \subseteq Mic - int_b(A \cap B) \dots \dots (2)$

From (1) and (2) we get $Mic - int_b(A) \cap Mic - int_b(B) = Mic - int_b(A \cap B)$

By reverse the steps we get to the converse □

Remark 3.27:

1. Intersection of $Mic - \alpha$ - open set and $mic - \dot{b}$ - open set is $Mic - \dot{b}$ - open set.
2. Union of $Mic -$ semi open and $Mic -$ pre - open sets is $Mic - \dot{b}$ - open set.
3. Union of $Mic -$ pre - open and $Mic - \dot{b}$ - open is $Mic - \dot{b}$ - open set.
4. Union of $Mic -$ semi - open and $Mic - \dot{b}$ - open is $Mic - \dot{b}$ - open set.

4 Micro $-\dot{b}$ - continuity

In this section we introduced a new class of functions, namely micro $\dot{b}$ -continuous briefly ($Mic - \dot{b} - continuous$) function in micro topological spaces and study some characterization of it and so introduced micro $\dot{b}$ -irresolute continuous function briefly ($Mic - \dot{b} - i - continuous$) functions and its decomposition with $Mic - \dot{b} - continuous$ function.

Definition 4.1:

Let $(U, T_R(X), \mu_R(X))$ and $(V, \dot{T}_R(X), \dot{\mu}_R(X))$ be two Micro_topological spaces, then $f: (U, T_R(X), \mu_R(X)) \rightarrow (V, \dot{T}_R(X), \dot{\mu}_R(X))$ called $Mic - \dot{b} - continuous$ function if $f^{-1}(v)$ is $Mic - \dot{b} - open$ in U when ever v is $Mic - open$ in V .

Example 4.2:

$U = \{p, q, r, s, t\}, U \setminus R = \{\{p\}, \{q\}, \{r, s, t\}\}$ and $X = \{p, t\}$
Then $T_R(X) = \{U, \emptyset, \{p\}, \{p, r, s, t\}, \{r, s, t\}\}$ then $\mu = \{t\}$
Then $Micro - O = \mu_R(X) = \{U, \emptyset, \{p\}, \{t\}, \{p, t\}, \{p, r, s, t\}, \{r, s, t\}\}$.
 $V = \{1, 2, 3, 4, 5\}$ and $V \setminus R = \{\{1\}, \{2, 3, 4\}, \{5\}\}$
 $X = \{1, 2\} \subseteq U$, then $T_R(X) = \{U, \emptyset, \{1\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$
If $\mu = \{4\}$
Then $\mu_R(X) = \{U, \emptyset, \{1\}, \{4\}, \{1, 4\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$.
And the function is defined by $f(p) = 1, f(q) = 5, f(r) = 2, f(s) = 3$ then f is $Mic - \dot{b} - continuous$ since $H = \{2, 3, 4\}$ is $Mic - open$ set in V and $f^{-1}(H) = \{r, s, t\}$ is $Mic - \dot{b} - open$ set in U .

Theorem 4.3:

Every $Mic - continuous$ function is $Mic - \dot{b} - continuous$ function.

Proof:

Let $f: U \rightarrow V$ be $Mic - continuous$ function that is $f^{-1}(H)$ is $Mic - open$ in U , when ever H is $Mic - open$ in V , Then by (theorem 3.3) and hence $f^{-1}(V)$ is $Mic - open$ in V

Hence $f: U \rightarrow V$ be $Mic - \dot{b} - continuous$ function
□

The following theorem we establish a characterization of $Mic - \dot{b} - continuous$ functions in terms of $Mic - \dot{b} - closure$.

Theorem 4.4:

A function $f: (U, T_R(X), \mu_R(X)) \rightarrow (V, \dot{T}_R(X), \dot{\mu}_R(X))$ is $Mic - \dot{b} - continuous$ if and only if $f(Mic - cl_b(A)) \subseteq Mic - cl_b(f(A))$ for every subset of U .

Proof:

Let f be $Mic - \dot{b} - continuous$ and $A \subseteq U$. Then $f(A) \subseteq V, Mic - cl_b(f(A))$ is $Mic - \dot{b} - closed$ in V . since f is $Mic - \dot{b} - continuous$, $f^{-1}(Mic - cl_b(f(A)))$ is micro closed in U . Since $f(A) \subseteq Mic - cl_b(f(A))$,

$A \subseteq f^{-1}(Mic - cl_b(f(A)))$. Thus $f^{-1}(Mic - cl_b(f(A)))$ is $Mic - \dot{b} - closed$ set containing A . Therefore $Mic - cl_b(A) \subseteq f^{-1}(Mic - cl_b(f(A)))$, That is $f(Mic - cl_b(A)) \subseteq Mic - cl_b(f(A))$.

Conversely, let $f(Mic - cl_b(A)) \subseteq Mic - cl_b(f(A))$ for every subset A of U . If F is $Mic - \dot{b} - closed$ in V , since $f^{-1}(F) \subseteq U, f(Mic - cl_b(f^{-1}(F))) \subseteq Mic - cl_b(f(f^{-1}(F))) \subseteq Mic - cl_b(F)$. that is, $Mic - cl_b(f^{-1}(F)) \subseteq f^{-1}(Mic - cl_b(F))$. Therefore, $Mic - cl_b(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is micro closed in U for every $Mic - \dot{b} - closed$ set F in V . That is f is $Mic - \dot{b} - continuous$ function □

Now we characterize $Mic - \dot{b} - continuous$ functions in terms of inverse image of micro closure.

Theorem 4.5:

A function $f: (U, T_R(X), \mu_R(X)) \rightarrow (V, \dot{T}_R(X), \dot{\mu}_R(X))$ is $Mic - \dot{b} - continuous$ if and only if $Mic - cl_b(f^{-1}(B)) \subseteq f^{-1}(Mic - cl_b(B))$ for every subset B of V .

Proof :

Let f be $Mic - \dot{b} - continuous$ and $B \in V$, $Mic - cl_b(B)$ is $Mic - \dot{b} - closed$ in V and hence $f^{-1}(Mic - cl_b(B))$ is a micro closed in U . Therefore,

$Mic [cl_b(f^{-1}(Mic - cl_b(B)))] = f^{-1}(Mic - cl_b(B))$. Since $B \subseteq Mic - cl_b(B), f^{-1}(B) \subseteq f^{-1}(Mic - cl_b(B))$. Therefore, $Mic - cl_b(f^{-1}(B)) \subseteq Mic - cl_b(f^{-1}(Mic - cl_b(B))) = f^{-1}(Mic - cl_b(B))$. That is, $Mic - cl_b(f^{-1}(B)) \subseteq f^{-1}(Mic - cl_b(B))$ for every $B \in V$. Let B $Mic - \dot{b} - closed$ in V . Then $Mic - cl_b(B) = B$.

By assumption, $Mic - cl_b f^{-1}(B) \subseteq f^{-1}(Mic - cl_b(B)) = f^{-1}(B)$. Thus, $Mic - cl_b(f^{-1}(B)) \subseteq f^{-1}(B)$. That is, $f^{-1}(B)$ is a micro closed set in U for every $Mic - \dot{b} - closed$ set B in V . Therefore, f is $Mic - \dot{b} - continuous$ on U □

Definition 4.6:

Let $(U, T_R(X), \mu_R(X))$ and $(V, \dot{T}_R(X), \dot{\mu}_R(X))$ be two Micro topological spaces, then $f: (U, T_R(X), \mu_R(X)) \rightarrow (V, \dot{T}_R(X), \dot{\mu}_R(X))$ is Micro $\dot{b}$ -irresolute continuous function briefly ($Mic - \dot{b} - i - continuous$) if $f^{-1}(v)$ is $Mic - \dot{b} - open$ set in U whenever v is $Mic - \dot{b} - open$ set in V .

Theorem 4.7:

Every $Mic - continuous$ function is $Mic - \dot{b} - i - continuous$ function.

Proof:

Let $f: U \rightarrow V$ be Mic – continuous. Let $H \subset V$ and h be a micro open set in V by (theorem 3.3) and since (f is Mic- continuous). $f^{-1}(H)$ is Mic – $\dot{b}$ – openset, then f is Mic – $\dot{b}$ – i – continuous

□

Theorem 4.8:

Let $(U, T_R(X), \mu_R(X))$ and $(V, \dot{T}_R(X), \dot{\mu}_R(X))$ and $(W, \ddot{T}_R(X), \ddot{\mu}_R(X))$ be three Micro topological spaces if $f: U \rightarrow V$ is Micro – $\dot{b}$ – continuous, and $g: V \rightarrow W$ is Micro – $\dot{b}$ – i – continuous then $gof: U \rightarrow W$ is Mic – $\dot{b}$ – continuous.

References

- [1] Thivagar .M.L and Richard .C, (2013), On nano forms of weakly open sets, International Journal of Mathematics and Statistic Invention, **1/1**, 31 – 37 .
- [2] Bhuvaneswari. M, (2017). A study on nano topology, Coimatore 641 105, India department of mathematics Nehru arts and science college , **1(2017)**, 52 – 54 .
- [3] Chandrasekar. S, (2019) On Micro topological spaces, Journal of New theory, PG and research department of mathematics, Arignar anna government arts college, Namakkal (DT), Tamil Nadu, India, **2149 – 1402**, 23 – 31 . [http:// www. Newtheory .org](http://www.Newtheory.org)
- [4] Revalthy. A and Ganambal llango,(2015) On Nano –open sets, Int. Jr. of Engineering, Contemporary Mathematics and Scinces, **1(2)**, 2250-3099
- [5] Andrijevic. D.(1987). On the topology generated by pre – open set, Mat. Vesnik, **39**, 367 – 376.
- [6] Andrijevic. D, (1996), b open sets, Math . Vesnik, **48(1)**, 59 – 64 .

Proof:

Let G be Mic – $\dot{b}$ – open set in W , since g is Micro – $\dot{b}$ – i – continuous ,then $g^{-1}(G)$ is Mic – $\dot{b}$ – openset in V .

Now $(gof)^{-1}(G) = (f^{-1}og^{-1})(G) = f^{-1}o(g^{-1}(G))$, Take $g^{-1}(G) = H$ which is Mic – $\dot{b}$ – open in V , Since f is Mic – $\dot{b}$ – continuous then $f^{-1}(H)$ is Mic – open in U , Hence $gof: U \rightarrow W$ is Micro – $\dot{b}$ – continuous function □

- [7] Primalia. M, and others ,(2016) On nano b – open sets in Nano – topological spaces , Journal of mathematics and statistics (JJMS) **9(3)**, 173 – 184 .
- [8] Mary. D.A and Arokia Rani. A, (2014) On semi pre – open sets in nano topological spaces, Math. Sci. Int. Res. Jnl, **3**, 771 – 773.
- [9] Levine.N. (1963). Semi open sets and semi continuouty in Topological spaces . Amer. Math . Monthly, **70**, 36 – 41 .
- [10] Mashhour. A.S, Abd El – Monsef. M.E and El – Deeb. S.N,(1982), On pre – continuous mappings and weak pre continuous mappings. Proc. Math. and Phys. Soc. Egypt. **53**, 47 – 53.
- [11] Taha. H. J. and others, (2018). A new Types of Contra Continuity in Bi-supra Topological Space ,Tikrit Journal of pure Science, **20(4)**, 170-176.
- [12] Power. P. L and Rajak. K. (2012). Fine irresolute mappings, Journal of Advanced Studies in Topology, **3/4**, 125 -139.

حول المجموعات المايكروية المفتوحة من النمط $\dot{b}$ و الاستمرارية المايكروية من النمط $\dot{b}$

ظه حميد جاسم ، مروه ضامن سلطان

قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

الغرض من هذا البحث هو تعميم لمفهوم المجموعات المايكروية المفتوحة من النمط $\dot{b}$ وكذلك درسنا علاقتها مع بعض المجموعات المفتوحة الأخرى مثل المجموعة المفتوحة القبلية و المجموعة شبه المفتوحة والمجموعة المفتوحة من النمط ألفا والمجموعة المفتوحة من النمط بيتا كما قمنا بتعميم الاستمرارية المايكروية من النمط $\dot{b}$ والاستمرارية المايكروية المترددة من النمط $\dot{b}$ و دراسة العلاقة فيما بينهما و لقد قدمنا الكثير من المبرهنات و الخصائص لهذه الأنواع من الاستمرارية مع بعض الأمثلة .