EFFECTS OF SOIL PROPERTIES ON TIME OF SATURATION IN EARTH DAMS +

Ihssan A. Abdul Hussain * Mona A. Hamid ** Ahmed A. Diwan***

Abstract:

In the present work a numerical analysis of the time required for saturation of an earth dam is investigated using variably saturated flow model. The sensitivity analysis to the soil properties such as hydraulic conductivity, soil retention parameters (n and α), drainable porosity and initial pressure head are studied for different degree of hydraulic anisotropy. The progress of saturation is observed and inflow and outflow rates are computed for assigned times and then employed to indicate the time required for saturation (T_S) of earth dam. The results are compared with the available analytical solution which ignored most of the soil properties. The results for different values of degree of hydraulic anisotropy indicate that the hydraulic conductivity, soil retention parameters, drainable porosity and initial pressure head have significant effect on T_S. On the other hand, the results indicate that T_S increases considerably with an increase in the degree of hydraulic anisotropy.

Key words: Earth dams, Time of saturation, Variably saturated zone, Anisotropy, Soil properties.

المستخلص:

يتضمن ألعمل ألحالي تحليل عددي للزمن اللازم لوصول سد ترابي متجانس إلى حالة التشبع باستخدام نموذج جريان متغير التشبع. إن تحليل حساسية خصائص التربة مثل الاتصالية الهيدروليكية ومعاملات احتجاز التربة والمسامية المرشحة و ضغط عمود الماء الأولي قد درست لدرجات مختلفة من عدم توحيد الخواص الهيدروليكية. لوحظ أن هناك تقدم في التشبع ومعدلات الجريان الداخلة والخارجة قد حسبت لأوقات محددة ثم وظفت لكي تشير إلى الوقت المطلوب لتشبع السد الأرضي. قورنت النتائج مع الحلول التحليلية المتوفرة لحالات مبسطة أهملت تأثير معظم خواص التربة أعلاه. دلت النتائج لقيم مختلفة من درجات عدم توحيد الخواص الهيدروليكية على أن قابلية الاتصالية الهيدروليكية ومعاملات احتجاز التربة والمسامية المرشحة وضغط عمود الماء الأولي لها تأثير مهم في حساب الزمن اللازم لوصول سد ارضي لحالة التشبع. من ناحية أخرى، أشارت النتائج إلى أن الزمن اللازم لوصول سد ارضي لحالة التشبع يزداد إلى حد بعيد مع زيادة درجة عدم توحيد الخواص.

+ Received on 15/10/2006 , Accepted on 24/5/2007

* Lecturer / Civil Department /Technical Institute of Basrah
** Assist. Lecturer / Department of Surveying / Technical Institute of Basrah
*** Assist. Lecturer, , Civil Department , Technical Institute of Basrah

Introduction:

Numerical models are often employed to predict the seepage flow development behavior in earthen dams. The quality of any particular model depends upon the accuracy and completeness of the physical understanding upon which the model is built.

Many models, both analytical and numerical in nature, have been developed for analysis of seepage of earthen dams. Most of the analytical models for seepage analysis make use of the Dupuit's assumptions with the tail water at the downstream being zero [1], [2] and [3]. The limitations of analytical solutions are overcome by the development of numerical models. Numerical models for seepage analysis can be broadly divided into two categories; (i) saturated flow models and (ii) variably saturated flow models. In the classical saturated flow approach, the free surface is taken as the upper boundary of the flow domain [4], [5], [6] and [7]. Common failings of fully saturated flow models for describing seepage problems include the observation that the fully saturated flow models predict slower responses in seepage line position than actually occur. Therefore, the incorrect assumption of treating the free surface as a stream line in saturated flow approach can be avoided by modelling seepage through an earth dam using variably saturated flow approach [8], [9], [10], and [11].

The author in reference [12] has developed some graphs to calculate the approximate time of saturation of earth dam. To use this graphs the upstream water depth and the base width of the dam and its hydraulic conductivity must be known. The graphs neglects the effect of capillarity, initial condition and degree of hydraulic anisotropy.

It is evident from literature that, often researchers have analyzed the seepage through an earth dam without giving any significant to the soil properties and their effects on the progress of saturation and the time required for saturation of the earth dam. The objective of the present work is to develop a seepage analysis of an earth dam and perform the sensitivity analysis of soil properties such as hydraulic conductivity, soil retention parameters, drainable porosity and initial pressure head to the time required for saturation of an earth dam for different degree of hydraulic anisotropy.

Modeling of Seepage Development flow:

Variably saturated unsteady water flow through the earth dam system can be described by Richard's equation [13]. The equation is as follows:

$$\left(C+\frac{\theta}{\phi}S_o\right)\frac{\partial H}{\partial T}=\frac{\partial}{\partial X}\left(K_X(\theta)\frac{\partial H}{\partial X}\right)+\frac{\partial}{\partial Z}\left(K_Z(\theta)\frac{\partial H}{\partial Z}+K_Z(\theta)\right) \quad \text{------------ (1)}$$

Where X and Z are the horizontal and vertical (+ve upwards) coordinates respectively, T is the time, $H(X,Z,T)$ is the pressure head, θ is the volumetric water content, $C = d\theta / dH$ is soil moisture capacity, ϕ is the porosity, S_o is the specific storage; $K_X (= K_{XS}.K_{xr})$ and $K_Z (= K_{ZS}.K_{zr})$ are capillary conductivities in X and Z directions respectively, K_{XS} and K_{ZS} are the saturated hydraulic conductivities in X and Z directions respectively, $K_{xr}(\theta)$ and $K_{zr}(\theta)$ are the relative hydraulic conductivities in X and Z-directions respectively.

In order to solve equation (1), the soil hydraulic functions (θ *vs.* H *and* K *vs.* H) have to be specified. In the present study, the soil hydraulic functions are described by a set of equations given by [14].

i. H (θ) **Relationship**:

$$S_e(H) = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left[\frac{1}{1 + (\alpha|H|)^n}\right]^m \quad for\ H < 0 \qquad \text{-------------} \ (2)$$
$$S_e(H) = 1 \quad for\ H \geq 0$$

Where S_e is the effective saturation, θ_s and θ_r are saturated and residual moisture contents, α and n are unsaturated soil parameters as described in [14].

$$m = 1 - 1/n, \quad n > 1 \qquad \text{----------------------} \ (3)$$

ii. K (θ) **Relationship**

$$K_{XR} = K_{ZR} = S_e^{1/2}\left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \quad for\ H < 0 \qquad \text{---------------} \ (4)$$
$$K_{XR} = K_{ZR} = 1 \quad for\ H \geq 0$$

Note that the relative hydraulic conductivity $K(\theta)$ does not depend upon the value of parameter α .

In most variably saturated flow situations S_o does not play an important role except for short period of time near saturation [15] and [16]. The inflow and outflow rate through the boundary, Γ, of the domain is given by:

$$Q = \int_\Gamma K_X \frac{\partial H}{\partial X} n_x \, d\Gamma + \int_\Gamma K_Z \left(\frac{\partial H}{\partial Z} + 1\right) n_z \, d\Gamma \qquad \text{--------------------} \ (5)$$

Where n_x *and* n_z are direction cosines of the normal to line Γ.

Steady State Seepage Development under Reservoir Full Condition

The initial and boundary conditions for reservoir development steady state condition are as follow (Refer Fig. 1)

Initial Condition:

$$H(X, Z, 0) = H_i \qquad \text{-------------------------}(6a)$$

Where, H_i is uniform initial pressure head across the dam section.

Boundary Conditions:

$$H(X, Z, T) = H_o - Z, \ (along\ AB) \qquad \text{------------------------} \ (6b)$$

Where H_o is the upstream water level in the dam section.

$$\frac{\partial H}{\partial X} = 0, \quad \frac{\partial H}{\partial Z} = -1, \ (along\ BC\ and\ DE) \qquad \text{------------------------} \ (6c)$$

$$\frac{\partial H}{\partial Z} = 0, \ (along\ AF) \qquad \text{-------------------------}(6d)$$

$$\frac{\partial H}{\partial Z} = -1, (along\ \ CD\, and\ AF) \qquad \text{---------------------------(6e)}$$

$$\frac{\partial H}{\partial X} = 0, \quad \frac{\partial H}{\partial Z} = -1, (along\, EF,\, if\ unsaturated) \qquad \text{------------------------(6f)}$$

$$H = 0, (along\, EF,\, if\ saturated) \qquad \text{--------------------- (6g)}$$

Where E is the point of separation between the unsaturated zone (DE) and the saturated zone (EF) along the downstream slope. EF is termed as the seepage face, E is not known a priori. Its position is determined iteratively during modeling.

A numerical model incorporating a finite difference solution of equation (1) has been developed. The solution is based upon the Iterative Alternating Direction Implicit Explicit Scheme [17]. The solution provides spatial distribution of the pressure head (H) at pre-stipulated discrete times. At each discrete time the contour of zero pressure head is determined, which represents the free surface up to the downstream slope. Along the downstream slope this is deemed as the seepage face. Further, the flow rate toward the dam section (inflow) and the flow rate pass through the downstream (outflow) are computed using equation (5) at the end of each assigned time.

Problem Description:

Fig. 1 shows the schematic diagram of an earth dam section on an impermeable foundation. The earth dam is assumed to be homogeneous and anisotropic. The progress of saturation through an earth dam is observed to study the sensitivity and effect of the soil properties on the required for saturation for different degree of hydraulic anisotropy η. Here, the hydraulic anisotropy (η) is defined as the ratio of saturated horizontal hydraulic conductivity and the saturated vertical hydraulic conductivity (K_{XS} / K_{ZS}). The geometric details of an earth dam and the relevant soil properties along with their range used in the analysis are presented in Table 1. The geometric of earth dam in the present problem is selected such away to represents the average dimensions and slopes of the small and moderate homogeneous dams. In the base case, the values of soil parameters used are: α=2.0 m^{-1}, n=3.0 and θ_s=0.35 whereas the value of saturated hydraulic conductivity and initial pressure head are 0.3 m/day and -1.5 m respectively.

The seepage in the dam section is modeled to arrive at the steady state distribution of the pressure head under the reservoir full condition by assigning boundary conditions described by equations (6a-g). A uniform pressure head is assigned as the initial pressure head condition. On the upstream a constant water level of 12.5m is maintained while a constant water level of zero meter is maintained on the downstream side. Commencing from this initial and boundary conditions, the model is run until a steady state condition is reached (saturation is achieved). $\mathbf{T_S}$ is achieved when the maximum difference in computed pressure head between two successive time steps reaches to 10^{-3} m. Pressure head distributions are determined numerically and the corresponding upstream flow rate (inflow) and downstream flow rate (outflow) are estimated employing equation (5). The inflow , outflow are computed and the progress of the free surface position is estimated at different assigned times (1, 5, 10, 20, 40, 80, 160, 320, 500 and 800 days) reaching to steady state condition. The stabilization of inflow and outflow at equal value is another indication of saturation and reaching steady state condition.

Table (1): Soil Properties and Geometric Details

Soil Properties	
Soil Parameter	Value
$\alpha\left(m^{-1}\right)$	0.5, 1.0, 2.0, 4.0
n	1.5, 3.0, 4.5
$n_e=(\theta_s-\theta_r)$	0.25, 0.35, 0.45
θ_r	0.01
η	1, 10, 20
$K_S\ (m/day)$	0.03, 0.15, 0.3, 1.5, 3.0
Geometric Details	
Height of the Dam (m)	15
Top width (m)	5
Upstream Slope	1V : 2.75H
Downstream Slope	1V : 2.5H
Upstream Water Level (m)	12.5
Downstream Water Level (m)	0

Fig. 2 shows the progress of the free surface and the seepage face for different time till reaching steady state for base case. Obviously, it can be seen that as the time increases, the position of the free surface advanced to the downstream face till the steady state and saturation achieved. It is clear from Fig. 2 that the steady state is achieved after 159 days, when the position of the free surface at the time 159 days is coincide with those of further times.

Fig. 3 shows the variation of model computed inflow and outflow with different time for base case. It is clear from Fig. 3 that as the time advances the inflow decreases till outflow start seep through the downstream face reaching to steady state and saturation when the inflow being approximately equal outflow after 159 days.

Model Validation:

The model is validated with the available analytical solution performed by [13]. The solution is based on an drainable porosity of 0.25 and h=1 ft. To estimate the time of saturation of an actual dam, the time determined from the analytical solution must be multiplied by the actual water depth in feet. The time of saturation for the base data (α =2 m^{-1}, n=3 and $\eta=1$) and different values of hydraulic conductivity and initial head are calculated using the variably saturated flow model. In contrast, the time of saturation is calculated for the same values of hydraulic conductivity above using analytical solution and for h=12.5m(41ft). Fig. 4 shows the comparison between $\mathbf{T_S}$ computed by the numerical model and analytical solution. The results indicated that the curves have same shape and the curve computed by numerical model of hi=-1.5 m is the closer and approximately coincide with that one of analytical solution.

Sensitivity Analysis:

Effect of Initial Condition:

The seepage in the dam section is modeled under reservoir full condition by assigning the initial and boundary conditions described in equations (6) and criteria described in the previous paragraph to compute the time required for saturation $\mathbf{T_S}$.

The time required for saturation $\mathbf{T_S}$ under reservoir full condition are studied for different values of initial pressure head and degree of hydraulic anisotropy. At each degree of hydraulic anisotropy, a uniform pressure head of different values (-0.5, -1.5 and -3.0m) is assigned as the initial pressure head condition to study the effect of this parameter on $\mathbf{T_S}$. Fig. 5 shows the variation of $\mathbf{T_S}$ with initial condition for different degree of hydraulic anisotropy. It can be observed that as the initial pressure head increases $\mathbf{T_S}$ decreases. This is due to the fact that with an increase in initial pressure head, the volumetric water content increases, hence the pores have less resistance to flow transmission and then the seepage develops faster leading to decrease in $\mathbf{T_S}$. On the other hand, it is observed that as the value of uniform initial pressure head decreases the rate of increment of $\mathbf{T_S}$ is increased. For example, $\mathbf{T_S}$ are determined for η=10, α=2 m^{-1}, n=3 and K_s = 0.3 m/day and different values of h_i. These values for h_i=-0.5, -1.5 and -3.0 m are 104, 255 and 489 days respectively.

Effect of Soil Properties:

The variation of $\mathbf{T_S}$ with different combination of soil properties are computed for different degree of hydraulic anisotropy as shown in figures 6-9. It is clear from Fig. 6 that as K_S decreases $\mathbf{T_S}$ increases significantly. This is due to the fact that the decrease in the saturated hydraulic conductivity values leads to reduction in flow rates and hence slow development in the seepage inside the earth dam. Also, it is observed that as K_S decreases the rate of increment in $\mathbf{T_S}$ increases considerably. For example, $\mathbf{T_S}$ are determined for η=10, h_i=-1.5 m, α = 2 m^{-1}, n=3 and different values of K_S. These values for K_S=3, 1.5, 0.3, 0.15 and 0.03 m/day are 42, 72, 255, 428 and 2155 days respectively. Then, one can recommend to use soil of K_S value less than 0.3 m/day in construction of homogeneous earth dams.

Fig. 7 shows the variation of $\mathbf{T_S}$ with different values of α . It is observed that as α increases $\mathbf{T_S}$ increases significantly. As α increases, the height of the capillary fringe decreases and the soil has tendency to drain considerable amount of moisture from the vadose zone. Since at large values of α small amount of moisture is retained at tension , preventing the easy movement of the surface of atmospheric pressure well below the region of significant moisture. Thus, it need more time to convey the soil under consideration to the saturation reaching to the steady state condition. Also, it is observed that as α increases the rate of increment in $\mathbf{T_S}$ decreases particularly at low and moderate degree of hydraulic anisotropy. For example, $\mathbf{T_S}$ are determined for η=10, h_i=-3 m, K_s = 0.3 m/day and n=3 and different values of α . These values for α=0.5, 1, 2 and 4 m^{-1} are 64, 172, 255 and 320 days respectively. Then, one can recommend to use soil of α value ranges from 1 to 2 in construction of homogeneous earth dams.

Fig. 8 shows the variation of $\mathbf{T_S}$ with different values of n . It is observed that as n increases $\mathbf{T_S}$ decreases. The value of n is the measure of the uniformity of the soil, as n increases the uniformity and the number of the large pores in comparison to the mean pores increase in porous media. Hence, the flow rate faces low resistance passing through the large pores due to the increase in the relative hydraulic conductivity and then leading to decrease in $\mathbf{T_S}$. Despite the increase in n leads to an decrease in $\mathbf{T_S}$ **,** the rate of decrease reduces.. For example, $\mathbf{T_S}$ are determined for η=10, h_i=-1.5 m, K_s = 0.3 m/day and α=2 m^{-1} and different values of n. The value of $\mathbf{T_S}$ for n=1.5, 3.0 and 4.5 are 286, 255 and 241 days respectively. These results also indicate that at high values of n its effect being less significant on $\mathbf{T_S}$. Then, one can concluded that n value has less significant effect in construction of homogeneous earth dams.

Fig. 9 shows the variation of $\mathbf{T_S}$ with drainable porosity (n_e) for different degree of hydraulic anisotropic. It is observed that as (n_e) increases $\mathbf{T_S}$ increases. This can explained by equations (2-4) in which the hydraulic conductivity is a function of pressure head. From this equations, it is clear that, at any given pressure, the relative hydraulic conductivity increases as the value of n_e decreases. As a result, the progress of saturation of a dam will be in a faster rate and then $\mathbf{T_s}$ decreases. Also, it is observed that as (n_e) increases, the rate of increment in $\mathbf{T_S}$ increases. For example, $\mathbf{T_S}$ are determined for η=10, h_i=-1.5 m, K_s = 0.3 m/day and α =2 m^{-1} and different values of $(\theta_S - \theta_r)$. The value of $\mathbf{T_S}$ for (n_e) =.25, 0.35, and 0.45 are 202, 255 and 324 days respectively.

Effect of Degree of hydraulic Anisotropy:

Upon scrutiny of the curves in Figs. 5-9, it is possible to evaluate the effect of degree of hydraulic anisotropy on $\mathbf{T_S}$. It is seen that as the degree of hydraulic anisotropy increases, $\mathbf{T_S}$ increases. The decrease in vertical hydraulic conductivity (associated with larger degree of hydraulic anisotropy) makes the media have great resistance to progress of saturation and then increase the $\mathbf{T_S}$. Also, it is observed that as the degree of hydraulic anisotropy increases the rate of increment decreases. For example, $\mathbf{T_S}$ are determined for h_i=-1.5 m, K_s = 0.3 m/day, n=3, α =2 m^{-1} and (n_e)=0.35. The value of $\mathbf{T_S}$ for η=1, 10, and 20 are 159, 255 and 280 days respectively. Thus, one can recommend not to use soil of high degree of hydraulic anisotropy and then not exaggerated in compaction process.

The dam is filled up to the maximum level is not actually fulfilled in practice. In actual practice the dam is filled for three months and is emptied for three months. Hence, in addition to the recommendations above, it is recommended to avoid any combination of soil properties and degree of hydraulic anisotropy give $\mathbf{T_S}$ greater than three mounts (90) days.

Conclusions:

The present study discusses the effect of initial condition, the soil properties and hydraulic anisotropy on the progress of saturation and time required for saturation (steady state) of an earthen dam. For this purpose, a variably saturated numerical model is developed for the analysis of seepage through an earthen dam. The time required for saturation $\mathbf{T_S}$ is computed for each combination of soil properties and degree of hydraulic anisotropy. The free surface position, inflow and outflow are computed for assigned times and then employed to indicate the correct time of saturation. The results are compared well with the available analytical solution performed by Cedergreen. The following conclusions are drawn from the study.

(i) $\mathbf{T_S}$ increases as the initial pressure head is decreased.

(ii) The decreasing of saturated hydraulic conductivity K_S and soil retention parameter n and increasing of drainable porosity n_e and soil retention parameter α leads to an increase of $\mathbf{T_S}$. However, K_S, n_e and α have significant effect on $\mathbf{T_S}$ whereas at high values of the soil retention parameters n its effect being less significant on $\mathbf{T_S}$.

(iii) $\mathbf{T_S}$ increases with an increase in the degree of hydraulic anisotropy.

(iv) It is recommended to use soil of α value range from 1 to 2 m^{-1} in construction of an earth dam.

(v) It is recommended to use soil of moderate degree of hydraulic anisotropy and $\eta \leq 10$ is convenient.

Notation:

The following symbols are used in this paper:

C =Soil moisture capacity;
H =Pressure head;
K =Capillary conductivities;
K_r =Relative hydraulic conductivity;
K_s =Saturated hydraulic conductivity;
m , n =Retention parameters;
n_x *and* n_z =Direction cosines
Q =Flow rate;
S_e =Effective water saturation;
$S_\circ$ =Specific storage;
T =Time;
T_S =Time needed to reach steady state;
X =Horizontal coordinate;
Z =Vertical coordinate;
θ =Volumetric water content;
θ_r =Residual water content;
θ_s =Saturated water content;
ϕ =Porosity;
η =Degree of hydraulic anisotropy;
α = Retention parameter

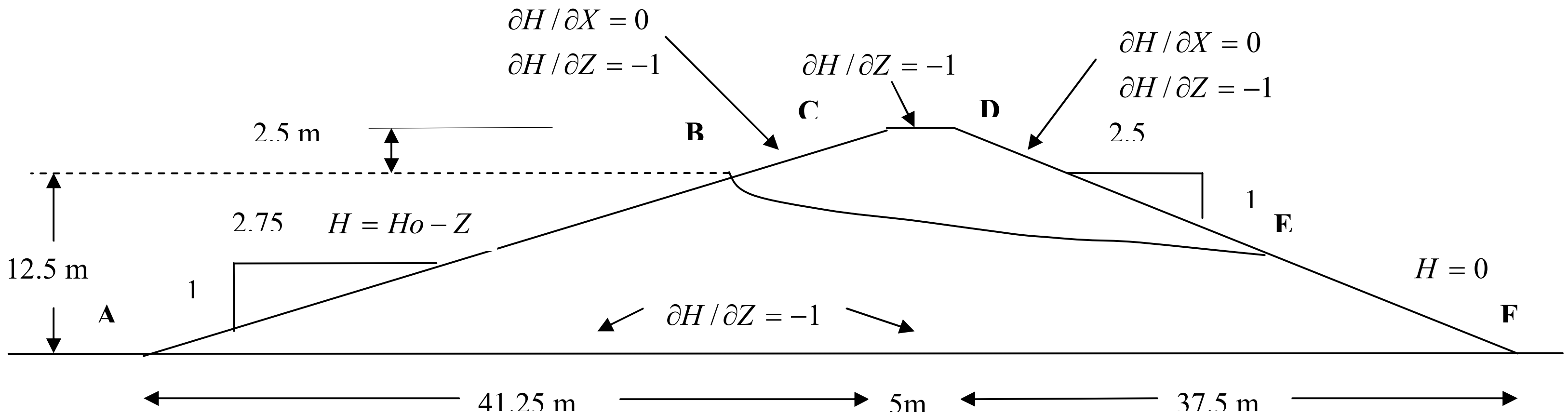

Fig.(1):Dam Geometry and Boundary Conditions

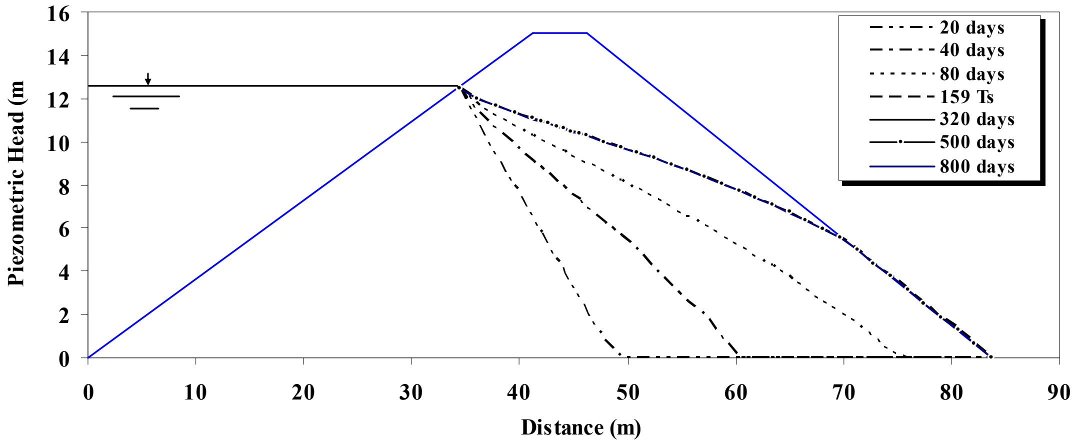

Fig. (2): Progress of Free Surface with Time for Base Case
(h_i=-1.5 m, n=3.0 and α =2 m^{-1}, n_e=0.35 and K_s=0.3 m/day,)

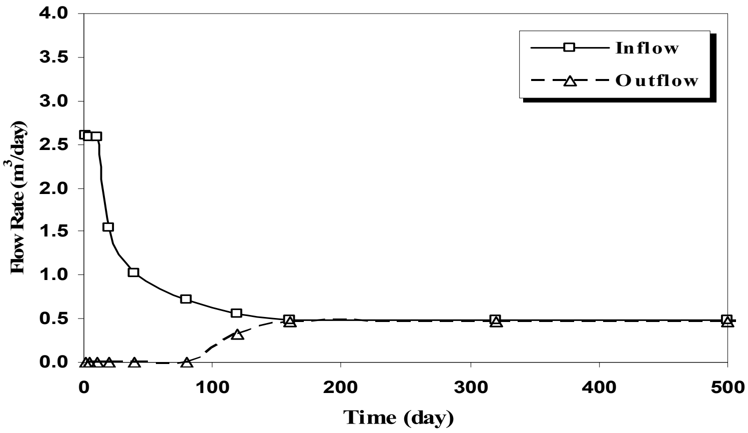

Fig. (3): Variation of Inflow and outflow of the Base Case

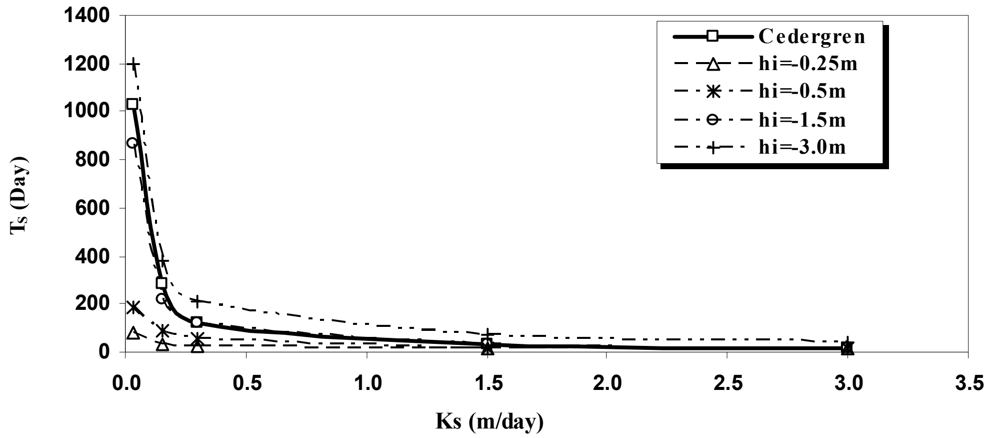

Fig. (4): Model Validation

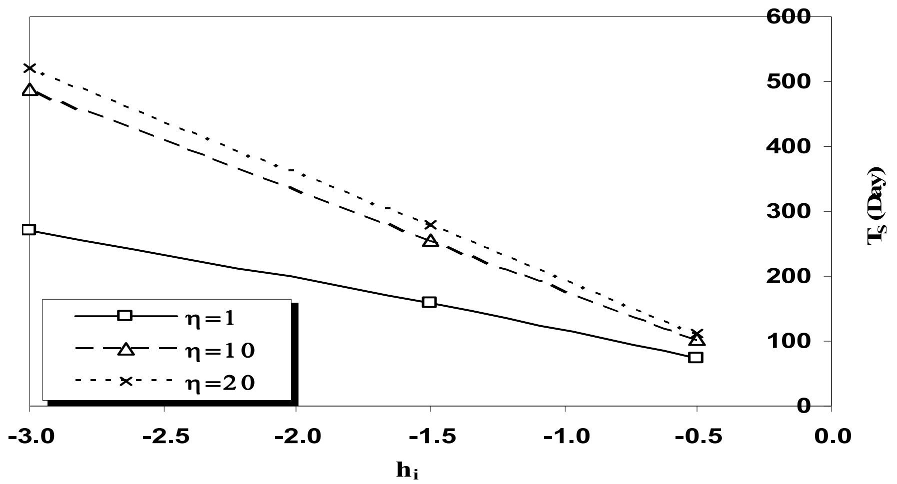

Fig. (5): Effect of Initial condition on T_S of the Base Case (α =2.0 m^{-1}, n =3.0 and n_e=0.35 and K_s=0.3 m/day)

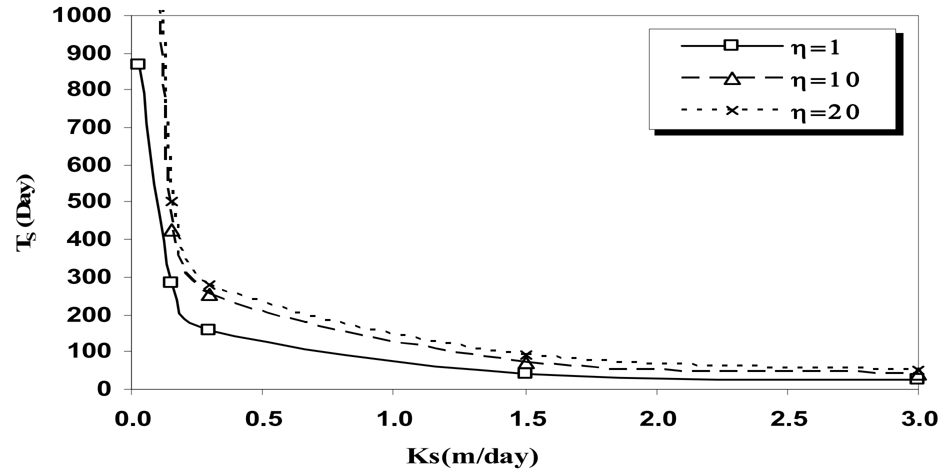

Fig. (6): Effect of Hydraulic conductivity on T_S of the Base Case (α =2.0 m^{-1}, n =3.0 n_e=0.35 and h_i=-1.5 m) for Different values of Hydraulic Anisotropy

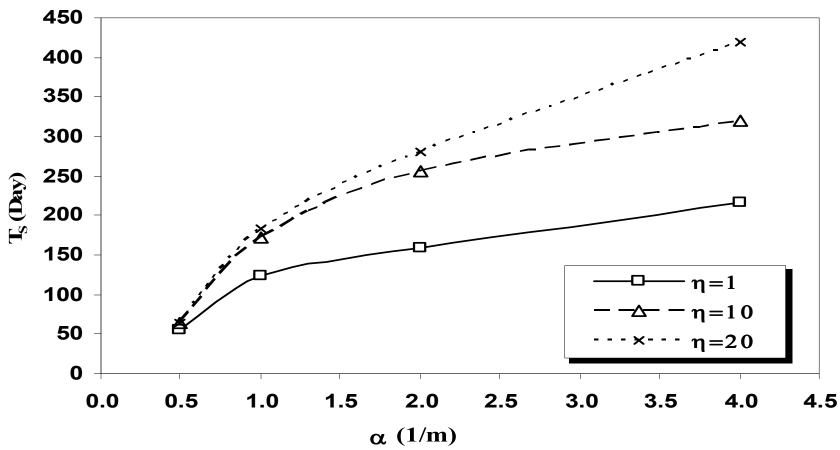

Fig. (7): Effect of Soil Retention Parameter α on T_S of the Base Case for Different values of Hydraulic Anisotropy

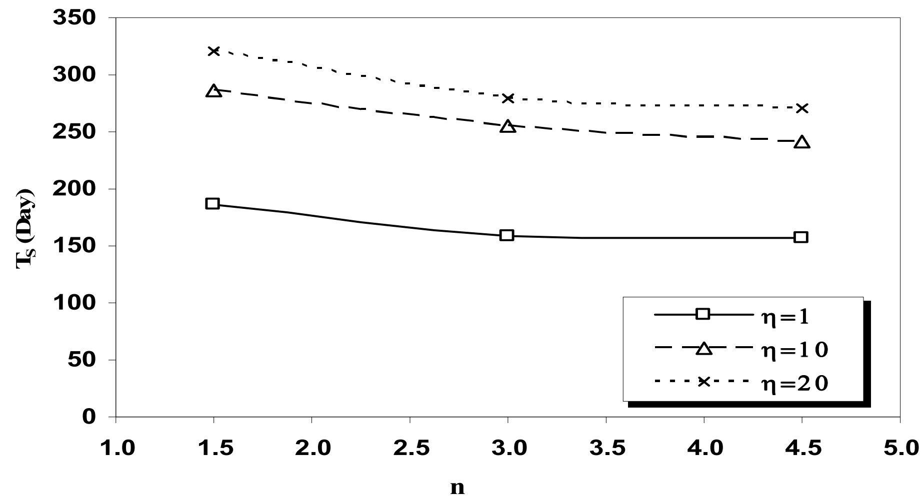

Fig. (8): Effect of Soil Retention Parameter n on T_S of the Base Case(α =2.0 m^{-1}, h_i=-1.5 m and K_s=0.3 m/day)
for Different values of Hydraulic Anisotropy

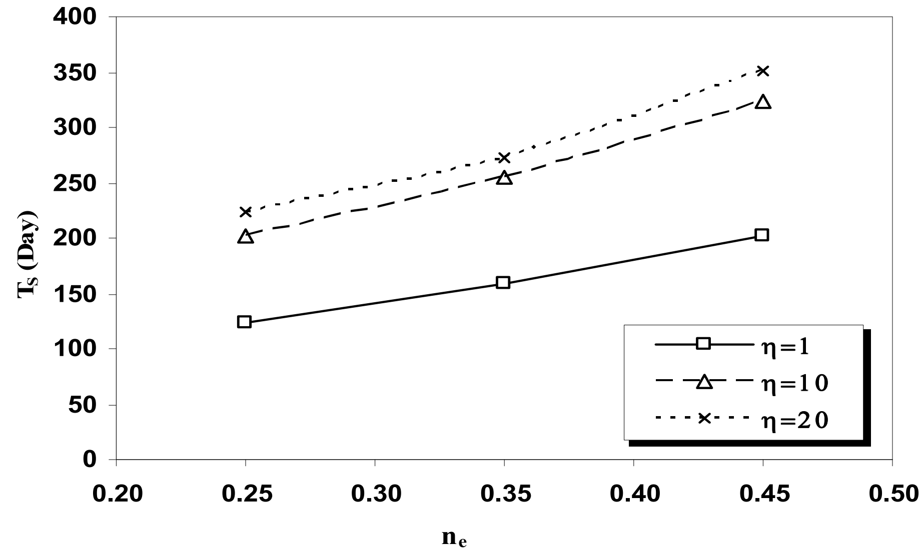

Fig. (9): Effect of Soil Retention Parameter n_e on T_S of the Base Case (α =2.0 m^{-1}, h_i=-1.5 m and K_s=0.3 m/day)
for Different values of Hydraulic Anisotropy

<u>References:</u>

1. Pavlovsky, N. N. "*Seepage through earth dam.*" Instit. Gidrotekhniki i Melioratsii,Leningrad. Translated by U. S. Corps of Engineers, 1931.
2. Casagrande, L. "*Naeherungsmethoden zur bestimmung von art und menge der sickerung durch geschuettete daemme.*" Thesis, Technische Hochschule, Vienna. Translated by U. S. Corps of Engineers, Waterways Exp. Sta., Vicksburg, Miss, 1932.
3. Casagrande, A. "*Seepage through dams.*" in 'Contributions to Soil Mechanics

1925-1940,' Boston Society of Civil Engineers, Boston, 1940.

4. Shaw, F. S., and R. V. Southwell. "*Relaxation methods applied to engineering problems, 7, Problems relating to the percolation of fluids through porous material*." *Proc. Roy. Soc.* London, A, 178, 1, 1941.
5. Neuman, S.P., and P. A. Witherspoon "*Analysis of non-steady flow with a free surface using the finite element method*." *Water Resour. Res*., 7(3), 611-623, 1971.
6. Liggett, J. A., and P. L. F. Liu "*The boundary integral equation method for porous media flow*." *George Allen and Unwin.* London, 225 pp., 1983.
7. Rao, H. S., and S. Sankaranarayanan "*Location of water table by FEM and verification by electrical analogy experiments*" *Ground Water,* 33(3), 372-377, 1995.
8. Freeze, R. A. "*Three dimensional, transient saturated-unsaturated flow in a ground water basin*." *Water Resou. Res.* 7(4), 347-366, 1971a.
9. Neuman, S.P. "*Saturated-unsaturated seepage by finite element*" *J. Hydr. Div., ASCE,* 99(12),2233-2250, 1973.
10. Thieu, N. T. M., Fredlund, D. G., and Hung, V. Q. "*General partial differential equation solvers for saturated-unsaturated seepage." Unsaturated Soils for Asia,* Rotterdam, 201-206, 2000.
11. Giu, S., Zhang, R., Turner, J.P., and Xue, X. "Probabilistic slope stability with stochastic soil hydraulic conductivity." *J. Geot. and Geoenvi. Engrg.,* 126(1), 1-9*,* 2000.
12. Cedergren, H. R. *Seepage, drainage and flow nets.* John Wiley & Sons, Inc., New york, N.Y., 1967.
13. Ahmad, S., Kashyap, D. and Mathur, B. S. "Mathematical modelling of saturated –unsaturated flow to drain." *J. of Irrigation and Drainage Engrg., ASCE,* 119(1), 18-33, 1993.
14. Van Genuchten, M. T. "A closed form equation for predicting the hydraulic conductivity of unsaturated soil." *Soil Sci. Soc. Am. Proc.,* 44, 892-898, 1980.
15. Freeze, R. A. "Role of subsurface flow in generating surface runoff 1: Base flow contribution to channael flow." *Water Resour. Res.,* 8, 609-623, 1972.
16. Wise, W. R, Clement, T. P and Molz, J. F " Variably saturated modeling of transient drainage: sensitivity to soil properties" *J. Hydro.,* The Netherlands, 161, 91-108, 1994.
17. Remson, I., Hornberger, G. M, and Molz, F. D. *Numerical methods in subsurfacehydrology.* John Wiley, New York, 1971.