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## Fekete -Szego Problem for Certain Subclass of $q$ -Difference Operator Using Quasi-Subordination

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### ABSTRACT

In this paper , a new certain class of  $q$ -Difference Operator with quasi-subordination is defined and Fekete-Szego problems for functions belongs to the class are derived . the results are presented here equip extensions of these given in some previous works

### 1. Introduction

Let  $\mathcal{U}(p)$  denote the class of functions in open unit disk

$$\mathcal{D} = \{ z \in \mathbb{C} : |z| < 1 \}.$$

The multivalent function  $\mathcal{F}$  in  $\mathcal{D}$  of the form  $\mathcal{F}(z) = z^p + \sum_{t=1}^{\infty} \alpha_{t+p} z^{p+t}$ ,  $p \in \mathbb{N} = \{ 1, 2, 3, \dots \}$ ,  $z \in \mathcal{D}$ . (1)

Now, if  $p=1$  then  $\mathcal{U}(1) = \mathcal{U}$ , denote by  $\mathcal{F} * \mathcal{h}$  Hadamard product of the functions  $\mathcal{F}$  and  $\mathcal{h}$  are analytic in  $\mathcal{D}$ .  $\mathcal{F}$  form (1) and

$$\mathcal{h}(z) = z^p + \sum_{t=1}^{\infty} \beta_{t+p} z^{t+p}$$

then

$$(\mathcal{h} * \mathcal{F})(z) = z^p + \sum_{t=1}^{\infty} \alpha_{t+p} \beta_{t+p} z^{t+p}$$

in [3], [4] Jackson defined and for  $(0 < q < 1)$ , the operator  $\mathcal{D}_q$  is called  $q$ -derivative of the function  $\mathcal{F}$  by

$$\mathcal{D}_q \mathcal{F}(z) = \begin{cases} \frac{\mathcal{F}(z) - \mathcal{F}(qz)}{(1-q)z} & \text{if } z \neq 0, \\ \mathcal{F}'(0) & \text{if } z = 0 \end{cases} \quad (2)$$

By the equation (2) it follows that if  $\mathcal{F} \in \mathcal{U}(p)$  form the equation (1), then

$$\mathcal{D}_q \mathcal{F}(z) = [p]_q z^{p-1} + \sum_{t=1}^{\infty} [t+p]_q \alpha_{t+p} z^{t+p-1}$$

Where  $[t]_q = \frac{1-q^t}{(1-q)}$ ,  $\lim_{q \rightarrow 1} [t]_q = t$ .

By using  $q$ -derivative Jackson well-defined is the operator  $\delta_{p,q}^s : \mathcal{U}(p) \rightarrow \mathcal{U}(p)$ ,  $s \in \mathbb{N}$ .

If  $\mathcal{F} \in \mathcal{U}(p)$  by the equation (1) it follows that

$$\delta_{p,q}^s \mathcal{F}(z) = (\mathcal{F} * H_{p,q}^s)(z), \quad p \in \mathbb{N}_0, z \in \mathcal{D}$$

Where

$$H_{p,q}^s(z) = z^p + \sum_{t=1}^{\infty} ([t+p]_q)^s z^{t+p}$$

Moreover

$$\delta_{p,q}^s \mathcal{F}(z) = z^p + \sum_{t=1}^{\infty} ([t+p]_q)^s \alpha_{t+p} z^{t+p}$$

$$\lim_{q \rightarrow 1^-} \delta_{p,q}^s \mathcal{F}(z) = z^p + \sum_{t=1}^{\infty} (t+p)^s \alpha_{t+p} z^{t+p}$$

For  $\lambda \geq 0$ , with the assist of the operator  $\delta_{p,q}^s$  well-define  $q$ -derivative the new operator  $\mathcal{M}_{\lambda,p,q}^{s,m} : \mathcal{U}(p) \rightarrow \mathcal{U}(p)$  by

$$\mathcal{M}_{\lambda,p,q}^{s,m} \mathcal{F}(z) = (1 - \lambda) \mathcal{M}_{\lambda,p,q}^{s,m-1} \mathcal{F}(z) + \lambda \frac{z}{p} (\mathcal{M}_{\lambda,p,q}^{s,m} \mathcal{F}(z))'$$

$$\mathcal{M}_{\lambda,p,q}^{s,0} \mathcal{F}(z) = \delta_{p,q}^s \mathcal{F}(z),$$

From the definition we can see that if  $\mathcal{F} \in \mathcal{U}(p)$  of the form the equation (1), then

$$\mathcal{M}_{\lambda,p,q}^{s,m} \mathcal{F}(z) = z^p + \sum_{t=1}^{\infty} ([t+p]_q)^s \left( \frac{p+\lambda t}{p} \right)^m \alpha_{t+p} z^{t+p} \quad +$$

$$\sum_{t=1}^{\infty} ([t+p]_q)^s \left( \frac{p+\lambda t}{p} \right)^m \alpha_{t+p} z^{t+p}$$

For real complex numbers  $a, b, c$ , the Gaussian-hypergeometric function is formed by

$${}_2F_1(a, b, c; z) = \sum_{t=1}^{\infty} \frac{(a)_t (b)_t}{(c)_t} \frac{z^{t+p}}{t!},$$

the function  $\theta_p(a, b, c; z)$  defined by

$$\theta_p(a, b, c; z) = z^p + \sum_{t=1}^{\infty} \frac{(a)_t (b)_t}{(c)_t} \frac{z^{p+t}}{t!}$$

The well known Gaussian-hypergeometric function conformable to the function

$$\theta_p(a, b, c; z), \text{ using the convolution for } \mathcal{M}_{\lambda, p, q}^{s, m} \mathcal{F}(z), \text{ define the operator } \Psi_{\lambda, p, q}^{s, m} \text{ by}$$

$$\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z) = \theta_p(a, b, c; z) * \mathcal{M}_{\lambda, p, q}^{s, m} \mathcal{F}(z)$$

$$\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z) = z^p + \sum_{t=1}^{\infty} ([t+p]_q)^s \left(\frac{p+\lambda t}{p}\right)^m \frac{(a)_t (b)_t}{(c)_t} \alpha_{t+p} \frac{z^{t+p}}{t!} \quad (3)$$

Let

$$H_t = ([t+p]_q)^s \left(\frac{p+\lambda t}{p}\right)^m \frac{(a)_t (b)_t}{t! (c)_t}$$

$$\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z) = z^p + \sum_{t=1}^{\infty} H_t \alpha_{t+p} z^{t+p}$$

For analytic functions  $\mathcal{F}, h$ , the function  $\mathcal{F}$  is subordinate to  $h$  in  $\mathcal{D}$  (see[5]).

Can be written

$$\mathcal{F}(z) < h(z). \quad (4)$$

If there exists an analytic function  $\omega$ , with  $|\omega(z)| < 1$  and  $\omega(0) = 0$  such that

$$\mathcal{F}(z) = h(\omega(z)). \quad (5)$$

If  $h$  is the univalent function in  $\mathcal{D}$ , then  $\mathcal{F}(z) < h(z)$  is equipollent to  $\mathcal{F}(0) = h(0)$

,  $\mathcal{F}(\mathcal{D}) = h(\mathcal{D})$ .

Minda and Ma [6] introduced and studied the classes  $(\Omega)$  and  $\mathcal{S}^*(\Omega)$  as below

$$(\Omega) = \{ \mathcal{F}(z) \in \mathcal{U} : 1 + \frac{z \mathcal{F}''(z)}{\mathcal{F}'(z)} < \Omega(z) \} \quad (6)$$

$$\mathcal{S}^*(\Omega) = \{ \mathcal{F}(z) \in \mathcal{U} : \frac{z \mathcal{F}'(z)}{\mathcal{F}(z)} < \Omega(z) \}, \quad (7)$$

Where the analytic function is  $\Omega(z)$  with part real positive in  $\mathcal{D}$ ,  $\Omega(\mathcal{D})$  is like with respect to real axis and the starlike with respect to  $\Omega(0) > 0$  and  $\Omega(0) = 1$ . the class  $\mathcal{C}(\Omega)$  and  $\mathcal{S}^*(\Omega)$  contain several wellknown subclasses of convex and starlike function is specific case.

In [5],  $\mathcal{F}$  is the function quasi-subordinate to  $h$  in  $\mathcal{D}$ , written as next

$$\mathcal{F}(z) <_q h(z) \quad (8)$$

If there exist  $\omega$  and  $\mathcal{B}$  analytic functions, with  $|\mathcal{B}(z)| \leq 1$ , and  $|\omega(z)| < 1$  and  $\omega(0) = 0$  then

$$\mathcal{F}(z) = \mathcal{B}(z) h(\omega(z)). \quad (9)$$

"When  $\mathcal{B}(z) = 1$ , then  $\mathcal{F}(z) = h(\omega(z))$ , that  $\mathcal{F}(z) < h(z)$  in  $\mathcal{D}$ . that if  $\omega(z) = z$ , then  $\mathcal{F}(z) = \mathcal{B}(z) h(z)$ , it is called that  $\mathcal{F}$  is majorized by  $h$  and written  $\mathcal{F}(z) \ll h(z)$  in  $\mathcal{D}$ . it is obvious that quasi-subordination is a generalization of subordination as well as majorization". see [10].

Darus and Mohd [1] introduced the classes  $\mathcal{C}_q(\Omega)$  and  $\mathcal{S}_q^*(\Omega)$  as below

$$\mathcal{C}_q(\Omega) = \{ \mathcal{F}(z) \in \mathcal{U} : \frac{z \mathcal{F}''(z)}{\mathcal{F}'(z)} <_q \Omega(z) - 1 \} \quad (10)$$

$$\mathcal{S}_q^*(\Omega) = \{ \mathcal{F}(z) \in \mathcal{U} : \frac{z \mathcal{F}'(z)}{\mathcal{F}(z)} - 1 <_q \Omega(z) - 1 \}. \quad (11)$$

**Definition 1.1** let the class  $\chi_{s, q}^p(\nu, b; \Omega)$  consists of function  $\mathcal{F}(z) \in \mathcal{U}(p)$  satisfy

The quasi-subordination

$$\frac{1}{b} \left( \frac{z (\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z))'}{(1-\nu) \Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z) + \nu z (\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z))'} - p \right) <_q \Omega(z) - 1 \quad (12)$$

( $z \in \mathcal{D}$ ;  $p \in \mathbb{N}$ ,  $p > s$ ;  $s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ ;  $b \in \mathbb{C}^*$ ;  $0 \leq \nu \leq 1$ ).

Clearly, we have the next relationship:  $\chi_{0, q}^1(0, 1; \Omega) = \mathcal{S}_q^*(\Omega)$ ;  $\chi_{0, q}^1(1, 1; \Omega) = \mathcal{C}_q(\Omega)$

It is well known this n-th coefficient of the univalent function  $\mathcal{F}(z) \in \mathcal{U}$  bounded by n

(see[3]). bounds for coefficient giving datum about various geometric properties of the function. there are many authors having inspected the bounds for coefficient Fekete-Szegő for various class [7, 9]. in particular, authors start to study the problem of Fekete-Szegő for various class use quasi-subordination [7, 11].

**Lemma 1.2** if  $\omega \in \Pi$ ,  $\Pi$  is the class of analytic functions  $\omega(z)$ , then

$$|\omega_2 - j\omega_1^2| \leq \begin{cases} -j, & \text{if } j \leq -1 \\ 1, & \text{if } -1 \leq j \leq 1 \\ j, & \text{if } j \geq 1 \end{cases}$$

## 2. Main results.

let  $\mathcal{F}(z) = z + a_{1+p} z^{1+p} + a_{2+p} z^{2+p} + \dots$ ,  $\Omega(z) = 1 + B_1 z + B_2 z^2 + \dots$ ,  $\emptyset(z) = C_0 + C_1 z + C_2 z^2 + \dots$ ,  $\omega(z) = \omega_1 z + \omega_2 z^2 + \dots$ ,  $B_1 > 0$ .

**Theorem 2.1** if  $\mathcal{F}(z) \in \mathcal{U}(p)$  belongs to  $\chi_{s, q}^p(\nu, b; \Omega)$ , then a

$$|a_{p+1}| \leq \frac{|b| |C_0| B_1 \sqrt{2B_1}}{\sqrt{|b| C_0 B_1^2 [( \nu + 2) H_1^2 ( \nu - 1) + 2 H_2 ( \nu + 2)] - 2(B_2 - B_1)( \nu + 1)^2 H_1^2}} \quad (13)$$

$$|a_{p+2}| \leq \frac{|b| |C_0| B_1}{( \nu + 2) H_2} + \frac{|b| |C_1| B_1}{( \nu + 2) H_2} + \left( \frac{|b| |C_0| B_1}{( \nu + 1) H_1} \right)^2 \quad (14)$$

and, For each real number  $m$ ,

$$|a_{p+2} - m a_{p+1}^2| \leq \frac{|b| |C_1| |C_0| B_1 \sqrt{2B_1} (|b| |C_0| B_1)^2}{(( \nu + 2) H_2)^2 (( \nu + 1) H_1)^2} \quad (15)$$

**Proof.**

If  $\mathcal{F}(z) \in \chi_{s, q}^p(\nu, b; \Omega)$ , then there exist analytic function  $\omega(z)$  and  $\emptyset(z)$  with

$|\omega(z)| \leq 1$ ,  $\emptyset(z) < 1$  and  $\emptyset(0) = 0$  such that

$$\frac{1}{b} \left( \frac{z (\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z))'}{(1-\nu) \Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z) + \nu z (\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z))'} - p \right) =$$

$$\emptyset(z) (\omega(\omega(z)) - 1). \quad (16)$$

Since

$$\frac{z (\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z))'}{(1-\nu) \Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z) + \nu z (\Psi_{\lambda, p, q}^{s, m} \mathcal{F}(z))'} - p =$$

$$\frac{\sqrt{|b| C_0 B_1^2 [H_1^2 ( \nu - 1) ( \nu + 2) + 2 H_2 ( \nu + 2)] - 2(B_2 - B_1) ( \nu + 1)^2 H_1^2}}{\alpha_{1+p} z^{1+p}} + [2( \nu + 2) \omega_2 \alpha_{2+p} - 2( \nu + 2) \omega_2 \alpha_{1+p}^2] z^2 + \dots,$$

$$\emptyset(z) (\omega(\omega(z)) - 1) = B_1 C_0 \omega_1 z + [B_1 C_1 \omega_1 + C_0 (B_1 \omega_2 + B_2 \omega_1^2)] z^2 + \dots,$$

From the equation (16) it follows that

$$a_{p+1} =$$

$$\sqrt{\frac{\omega_1 b C_0 B_1 \sqrt{2B_1}}{|b C_0 B_1^2[(v+2)H_1^2(v-1)+2H_2(v+2)]-2(B_2-B_1)(v+1)^2H_1^2|}} \quad (17)$$

$$a_{p+2} = \left( \frac{2}{(v+2)H_2} + \left( \frac{1}{(v+1)H_1} \right)^2 \right) (B_1 C_1 \omega_1 + B_1 C_0 \omega_2 + C_0(b B_1^2 C_0 + B_2) \omega_1^2) .$$

Further ,

$$a_{p+2} - m a_{p+1} = \frac{b}{((v+2)H_2)^2 ((v+1)H_1)^2} [C_1 \omega_1 +$$

$$C_0(\omega_2 - j\omega_1)] , \quad (18)$$

Since  $\phi(z)$  is bounded and analytic in  $\mathcal{D}$  , in [8] we have

$$|C_n| \leq 1 - |C_0|^2 \leq 1 .$$

Using by these fact and the wellknown this inequality  $|\omega_1| \leq 1$  in (17) and (18) , we get

$$|a_{p+1}| \leq \sqrt{\frac{|b| |C_0| B_1 \sqrt{2B_1}}{|b C_0 B_1^2[(v-1)(v+2)H_1^2 + 2(v+2)H_2] - 2(B_2-B_1)(v+1)^2H_1^2|}} \quad (19)$$

And

$$|a_{p+2} - m a_{p+1}| \leq \frac{|b| |C_1| |C_0| B_1 \sqrt{2B_1} (|b| |C_0| B_1)^2}{((v+2)H_2)^2 ((v+1)H_1)^2}$$

(19)

Applying Lemma1.2 and the tringle inequality to (19) , obtain (15) . the result is severe for the function

$$1/b \left( \frac{z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))'}{(1-v)\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z) + v z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))} - p \right) = (1+z)(\omega(z^2) - 1)$$

Or

$$1/b \left( \frac{z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))'}{(1-v)\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z) + v z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))} - p \right) = (1+z)(\omega(z) - 1) .$$

For  $m = 0$  , in (15) , have (14) . the proof of theorem2.1 is complete .

**Corollary2.2** if  $\mathcal{F}(z) \in \mathcal{U}$  belongs to  $\mathcal{S}_q^*(\Omega)$  , then

$$|a_2| \leq \frac{|b| |C_0| B_1 \sqrt{B_1}}{\sqrt{|b C_0 B_1^2 - (B_2 - B_1)|}} ,$$

$$|a_3| \leq \frac{|b| |C_0| B_1}{2} + \frac{|b| |C_1| B_1}{2} + (|b| |C_0| B_1)^2 .$$

$$|a_{p+2} - m a_{p+1}| \leq \frac{|b| |C_1| m |C_0| B_1 \sqrt{B_1} (|b| |C_0| B_1)^2}{2} .$$

**Corollary2.3** if  $\mathcal{F}(z) \in \mathcal{U}$  belongs to  $\mathcal{C}_q(\Omega)$  , then

$$|a_2| \leq \frac{|b| |C_0| B_1 \sqrt{B_1}}{\sqrt{|3b C_0 B_1^2 - 4(B_2 - B_1)|}}$$

$$|a_3| \leq \frac{|b| |C_0| B_1}{3} + \frac{|b| |C_1| B_1}{3} + \left( \frac{|b| |C_0| B_1}{2} \right)^2$$

$$|a_{2+p} - m a_{1+p}| \leq \frac{m |b| |C_1| |C_0| B_1 \sqrt{B_1} (|b| |C_0| B_1)^2}{3} .$$

**Theorem2.4**  $\mathcal{F}(z) \in \mathcal{U}(p)$  satisfies

$$1/b \left( \frac{z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))'}{(1-v)\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z) + v z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))} - p \right) \ll \Omega(z) - 1 ,$$

Then the following inequalities are hold

$$|a_{p+1}| \leq \sqrt{\frac{|b| |C_0| B_1 \sqrt{2B_1}}{|b C_0 B_1^2[(v-1)H_1^2(v+2)+2H_2(v+2)]-2(B_2-B_1)(v+1)^2H_1^2|}} \quad (20)$$

$$|a_{p+2}| \leq \frac{|b| |C_0| B_1}{(v+2)H_2} + \frac{|b| |C_1| B_1}{(v+2)H_2} + \left( \frac{|b| |C_0| B_1}{(v+1)H_1} \right)^2 .$$

and , For each real number  $m$  ,

$$|a_{p+2} - m a_{p+1}| \leq \frac{|b| |C_1| |C_0| B_1 \sqrt{2B_1} (|b| |C_0| B_1)^2}{((v+2)H_2)^2 ((v+1)H_1)^2}$$

**Proof.** the result follows by taking  $\omega(z) = z$  in the proof of Theorem2.1 .

**Theorem2.5**  $\mathcal{F}(z) \in \mathcal{U}(p)$  belongs to  $\chi_{s,q}^p(v, b; \Omega)$  , then for each real number  $m$  ,  $b > 0$  , and  $\tau_1, \tau_2, \tau_3$  , are the real numbers .

$$|a_{p+2} - m a_{p+1}| \leq \begin{cases} bv(1-j) & , m \leq \tau_1 , \\ 2bv , & \tau_1 \leq m \leq \tau_2 \\ bv(1+j) & , m \geq \tau_2 . \end{cases}$$

(20)

Further , if  $\tau_1 \leq m \leq \tau_3$  , then

$$|a_{p+2} - m a_{p+1}| + T_1 |a_{p+1}|^2 \leq 2bv . \quad (21)$$

If  $\tau_3 \leq m \leq \tau_2$  , then

$$|a_{p+2} - m a_{p+1}| + T_2 |a_{p+1}|^2 \leq 2bv . \quad (22)$$

For each real number  $m$  and  $b > 0$  ,

$$|a_{p+2} - m a_{p+1}| \leq \begin{cases} -bv(1-j) & , m \leq \tau_2 , \\ -2bv , & \tau_2 \leq m \leq \tau_1 \\ -bv(1+j) & , m \geq \tau_1 . \end{cases}$$

(23)

Further , if  $\tau_2 \leq m \leq \tau_3$  , then

$$|a_{p+2} - m a_{p+1}| + T_2 |a_{p+1}|^2 \leq -2bv . \quad (24)$$

If  $\tau_3 \leq m \leq \tau_1$  , then

$$|a_{p+2} - m a_{p+1}| + T_1 |a_{p+1}|^2 \leq -2bv . \quad (25)$$

Where

$$v = \frac{2B_1}{H_2(v+2)} + \left( \frac{B_1}{H_1(v+1)} \right)^2 ,$$

$$\tau_1 = \frac{b |C_0| B_1^2 + B_1 - B_2}{(H_1(v+1))^2 (H_2(v+2))^2 b |C_0| B_1^2} ,$$

$$\tau_2 = \frac{b |C_0| B_1^2 + B_2 + B_1}{(H_1(v+1))^2 (H_2(v+2))^2 b |C_0| B_1^2} ,$$

$$\tau_3 = \frac{b |C_0| B_1^2 + B_2}{(H_1(v+1))^2 (H_2(v+2))^2 b |C_0| B_1^2} ,$$

**Proof.** we assume that  $b > 0$  . from (15) , have

$$|a_{2+p} - m a_{1+p}| \leq bv[\max\{1, |j|\}] .$$

If  $m \leq \tau_1$  , then  $j \leq -1$  . so , by applying Lemma1.2 , we have the first inequality in (20) .

If  $m \leq \tau_2$  , then  $j \geq 1$  . applying Lemma1.2 , we get the last inequality in (20) .

When  $\tau_1 \leq m \leq \tau_2$  , then  $|j| \leq 1$  . so applying Lemma1.2 , we obtain the middle inequality in (20) .

If  $m \leq \tau_1$  or  $m \leq \tau_2$  , the result is severe for the function

$$1/b \left( \frac{z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))'}{(1-v)\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z) + v z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))} - p \right) = \phi(z)(\Omega(z^2) - 1) .$$

If  $\tau_1 \leq m \leq \tau_2$  , the result is severe for the function

$$1/b \left( \frac{z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))'}{(1-v)\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z) + v z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))} - p \right) = \phi(z)(\Omega(z^2) - 1) .$$

If  $m = \tau_1$  , the result is severe for the function

$$\frac{1}{v} \left( \frac{D^l \psi_{\lambda,p,q}^{s,m} \mathcal{F}(z)}{D^l \psi_{\lambda,p,q}^{s,m} \mathcal{F}(z)} - p^{l-1} \right) = \phi(z)(\Omega \left( \frac{z(z+v)}{1+vz} \right) - 1) , \quad 0 \leq v \leq 1 .$$

If  $m = \tau_2$  , the result is sharp for the function

$$1/b \left( \frac{z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))'}{(1-v)\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z) + v z(\psi_{\lambda,p,q}^{s,m} \mathcal{F}(z))} - p \right) =$$

$$\phi(z)(\Omega \left( -\frac{z(z+v)}{1+vz} \right) - 1) .$$

Moreover, (21) and (22) are established by an application of Lemma1.2.

Applying Lemma1.2, we can prove (23) and (25) for  $b > 0$ . the proof of theorem2.5 is complete.

**Corollary2.7**  $\mathcal{F}(z) \in \mathcal{U}$  belong to  $\mathcal{S}_q^*(\Omega)$ , then for each number  $m$

$$|a_3 - m a_2^2| \leq \begin{cases} \frac{|C_0|B_1\sqrt{B_1} [1-2m-(|b| |C_0|B_1)^2]}{\sqrt{|b C_0 B_1^2-(B_2-B_1)|}}, & m \leq \tau_1, \\ \frac{|C_0|B_1\sqrt{B_1}}{\sqrt{|C_0 B_1^2-(B_2-B_1)|}}, & \tau_1 \leq m \leq \tau_2 \\ \frac{|C_0|B_1\sqrt{B_1} [1+2m-(|C_0|B_1)^2]}{\sqrt{|C_0 B_1^2-(B_2-B_1)|}}, & m \geq \tau_2. \end{cases}$$

If  $\tau_1 \leq m \leq \tau_3$ , then

$$|a_3 - m a_2^2| + \frac{(2m-1)C_0 B_1^2-(B_2-B_1)}{2 C_0^2 B_1^2} |\alpha_2|^2 \leq$$

$$\frac{|C_0|B_1\sqrt{B_1}}{\sqrt{|C_0 B_1^2-(B_2-B_1)|}},$$

If  $\tau_3 \leq m \leq \tau_2$ , then

$$|a_3 - m a_2^2| + \frac{(2m+1)C_0 B_1^2-(B_2-B_1)}{2 C_0^2 B_1^2} |\alpha_2|^2 \leq$$

$$\frac{|C_0|B_1\sqrt{B_1}}{\sqrt{|C_0 B_1^2-(B_2-B_1)|}}.$$

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## مشكلة فيكت سيكو لفئة فرعية معينة من المؤثر q-Difference باستخدام شبه التابعية

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## المخلص

في هذا البحث ، تم تحديد فئة جديدة معينة من المؤثر q-Difference مع شبه التابعية و مشاكل فيكت سيكو للدوال التي تنتمي إلى الفئة. النتائج المعروضة هنا لتجهيز هذه الملحقات في بعض الأعمال السابقة.