On Fuzzy V-B-ideals

Shaymaa Abed Yasseen
University of Kufa
Faculty of Mathematics of computer science
Department of Mathematics
Husseinh.abbas@uokufa.edu.iq

Layth Salah Mahdi
Ministry of Education/ The General
Directorate For Al-najaf Alashraf Breeding
Al-forat Preparatory School
layths.mahdi@uokufa.edu.iq

Abstract--- *In this paper, we present the idea in fuzzy V-B-ideal in V-B-ḂH algebra, we state and gives some propositions and examples which explain the relationships between these notions and the other types of fuzzy ideal s in ḂH algebra.*

Keyword: V-B-ḂH algebra, V-B-ideal, fuzzy subst.

INTRODUCTION

Y.Imaį and Ķ.Isekį formulated the idea of a ḂCK-algebra and ḂCI-algebras a present of the concept of set-theoretic difference and propositional calculus in 1966[6]. In 1998 ,Y. Ḃ. Jŭn, E. 'H. Roh and 'H .S. Kįm presented the notion of ḂH-algebra and the notions of ideal [9].In 2002 , j.neggers and 'Hee Sįk Kįm introduced the notions of B-algebra[5].In 2015,H.H. Abass and L. S. Mahdi introduced the notions in V-B-BH-algebra and the notions of V-B-ideal of V-B-ḂH-algebra[4].

On the other hand, we shall mention the development in fuzzy sense.

The notions of fuzzy subset presented by L.A. Zadeḩ In 1965as a method for representing uncertainty in real physical world [7]. In 2006,C.'H. Parkstudied the interval-valued Fuzzy ideal in BH-algebras[1].

In this paper, we present the notions in fuzzy V-B-ideal in V-B-BH algebra.

I. PRELIMINARIES

In this section, we present some concepts about a B-algebra, such as BCK-algebra, BCI-algebra, ḂH-algebra and V-B-ḂH algebra (BC A-part, homomorphism, ideal ,V-B-ideal) in BH-algebra with some theorems, propositions and examples. Also, we review some fuzzy preliminaries about fuzzy set, level set fuzzy set, fuzzy singleton, fuzzy ideal ,(image and preimage) of fuzzy set under function , and some other concepts that we need in our work.

Definition (1.1):[5]

A **B-algebra** is a nonempty set ψ with a constant ọ and a binary operation "⍟" satisfying the following axioms:

i. $ă ⍟ ă = ọ$
ii. $ă ⍟ ọ = ă$
iii. $(ă ⍟ č) ⍟ ĝ = ă ⍟ \big(ĝ ⍟ (ọ ⍟ č)\big)$, $for\ every\ ă, č, ĝ \in \psi$.

Definition (1.2) :[6]

A BCI-algebra is an algebra (ψ,⍟,ẹ) , where ψ is a nonempty set, "⍟" is a binary operation and ọ is a constant , satisfying the following axioms:

i. $\big((ă ⍟ č) ⍟ (ă ⍟ ĝ)\big) ⍟ (ĝ ⍟ č) = ẹ, for\ every\ ă, č, ĝ \in \psi$,
$ii.\ \big(ă ⍟ (ă ⍟ č)\big) ⍟ č = ọ,\ for\ every\ ă, č \in \psi$.
$iii.\ ă ⍟ ă = ọ, for\ every\ ă \in \psi$.
$iv.\ ă ⍟ č = ọ\ and\ č ⍟ ă = ọ\ imply\ ă = č, for\ every\ ă, č \in \psi$.

Definition (1.3) :[6]

A *BCK-algebra* is a BCI-algebra satisfying the axiom: $ọ ⍟ ă = ọ\ for\ all\ ă \in \psi$.

Definition (1.4) :[9]

A ***BH-algebra is*** a nonempty set ψ with a constant ọ and a binary operation"⍟" satisfying the following conditions:

i. $ă ⍟ ă = ọ, for\ every\ ă \in \psi$.
ii. $ă ⍟ č = ọ\ and\ č ⍟ ă = ọ\ imply\ ă = č, for\ every\ ă, č \in \psi$.
iii. $ă ⍟ ọ = č, for\ every\ ă \in \psi$.

Remark (1.5):[8]
Let X and Y be $BH-algebras$. A mapping $\Omega : X \to Y$ is called a *homomorphism* if $\Omega(\varkappa \circledast \gamma) = \Omega(\varkappa) \circledast \Omega(\gamma)$ for all $\varkappa, \gamma \in X$. A *homomorphism* Ω is called a *epimorphism* if it is *injective*. The set $\{\varkappa \in X: \Omega(\varkappa) = 0'\}$ is called the *kernel* of Ω, denoted by $Ker(\Omega)$, and the set $\{\Omega(\varkappa): \varkappa \in X\}$ is called *image* of Ω, denoted by $Im(\Omega)$. Notice that $\Omega(0) = 0'$, for all *homomorphism* Ω, and $\Omega^{-1}(Y) = \{\varkappa \in X: \Omega(\varkappa) = \gamma, \text{for some } \gamma \in Y\}$

Definition (1.6) :[9]
Let I be a nonempty subset of a BH-algebra ψ. Then I is said to be ***ideal*** of ψ if it satisfies:
i. $o \in I$.
ii. $ă \circledast č \in I$ and $č \in I$ imply $ă \in I$.

Definition (1.7): [4]
A ***V-B-BH-algebra*** is defined to be a BH-algebra ψ in which there exists a proper subset **V** of ψ, such that:
i. $0 \in \mathbf{V}$, $|\mathbf{V}| \geq 2$.
ii. **V** is a B-algebra.

Definition (1.8): [4]
A nonempty subset I of a V-B-BH algebra ψ is said to be ***V-B-ideal*** of ψ related to V if it satisfies:
i. $o \in I$.
ii. $ă \circledast č \in I$ and $č \in I \Rightarrow ă \in I$, for every $ă \in V$.

Definition (1.9) [7]:
Let ψ be a non-empty set. A fuzzy set Ά in ψ (a ***fuzzy subset*** of ψ) is a function from ψ into the real interval [0 ,1] .

Definition (1.10) :[7]
Let Ά and ฿ be two fuzzy sets in ψ, then:
i. Ά = ฿ *if and only if* Ά (ă)=฿(ă), $\forall$ ă $\in \psi$.
ii. Ά $\subseteq$ ฿ *if and only if* Ά(ă) $\leq$ ฿ (ă), $\forall$ ă $\in \psi$.
iii. Ά $\subset$ ฿ *if and only if* Ά(ă) $<$ ฿ (ă), *for every* ă $\in \psi$ where Ά is called a proper fuzzy subset of ฿

Definition (1.11) :[2]
Let Ά and ฿ be two fuzzy sets in ψ, then
i. (Ά ∩ ฿)(x)=min{Ά (x), ฿(x)}, *for every* $x \in \psi$.
ii. (Ά ∪ ฿)(x)=max{Ά (x), ฿(x)}, *for every* $x \in \psi$.

Ά ∩ ฿ *and* Ά ∪ ฿

are fuzzy sets in ψ.

In general, if $\{Ά_\epsilon , \epsilon \in \gamma\}$ is a family of fuzzy sets in ψ, then :

$\underset{\epsilon \in \gamma}{\cap} Ά_\epsilon(ă) = \inf\{Ά_\epsilon(ă), \epsilon \in \gamma\}$,

for every ă $\in \psi$ and

$\underset{\epsilon \in \gamma}{\cup} Ά_\epsilon(ă) = \sup\{Ά_\epsilon(ă), \epsilon \in \gamma\}$,

for every ă $\in \psi$

which are also fuzzy sets in ψ.

Definition (1.12) :[7]
Let Ά be a fuzzy set in ψ. For $t \in [0,1]$. The set $Ά_t$={ă $\in \psi$, Ά (ă) $\geq t$} is called a ***level subset of*** Ά.

Definition (1.13) :[1]
A fuzzy subset Ά of a BH-algebra ψ is said to be a ***fuzzy ideal*** if and only if:
i. For any ă $\in \psi$, Ά (o) $\geq$ Ά (ă).
ii. For any ă, č $\in \psi$, Ά (ă) $\geq$ *min*{Ά (ă $\circledast$ č) , Ά (č)}.

Definition (1.14):[3]
Let Ά and ฿ be any two sets, $A^{\circledast}$ be any fuzzy set in Ά and
Ω : Ά$\to$ ฿ be any function. If Ω^{-1}(č) = {ă∈Ά|Ω(ă) =č} for č∈B, then the fuzzy set $B^{\circledast}$ in B which is defined by
$B^{\circledast}$(č)=

$$\begin{cases} \sup\{A^{\circledast}(ă) \mid x \in \Omega^{-1}(č)\} & \text{if } \Omega^{-1}(y) \neq \emptyset \\ o & \text{otherwise} \end{cases}$$

for every č∈฿, is said to be ***image*** of $A^{\circledast}$ under Ω and denoted by $\Omega(A^{\circledast})$.

Definition (1.15):[3]
Let $\mathcal{A}$ and B be any two sets, Ω: $\mathcal{A} \to$ ฿ be any function and $B^{\circledast}$ be any fuzzy set in $\Omega(\mathcal{A})$. The fuzzy set $A^{\circledast}$ in A defined by $A^{\circledast}$(ǎ) = $B^{\circledast}$(Ω(ǎ)) for every ǎ∈ψ is called the ***preimage*** of $B^{\circledast}$ under Ω and is denoted by $\Omega^{-1}(B^{\circledast})$.

II. THE MAIN FUZZY RESULTS
In this section, we present the concept of a fuzzy V-B-ideal of a V-B-BH algebra. For our discussion, we relate this idea with other species of fuzzy ideals of a BH-algebra. Also, we stated and proved some theorems and examples about these concepts.

Definition (2.1):
A fuzzy subset Ά of a V-B-BH-algebra ψ is said to be a $\boldsymbol{fuzzy\ V-B-ideal}$ if and only if:

i. Ά(ọ) ≥ Ά(ǎ), $for\ every$ ǎ $\in \psi$.
ii. Ά(ǎ) ≥ min{Ά(ǎ⍟č), Ά(č)}, $for\ every$ ǎ $\in V$, č $\in \psi$.

Example(2.2) :
Consider the V-B-BH-algebra ψ={ọ,1,2,3} with binary operation "⍟" defined as follows.

⍟	ọ	1	2	3
ọ	ọ	2	1	3
1	1	ọ	2	1
2	2	1	ọ	2
3	3	2	1	ọ

where V={ọ,1,2}. The fuzzy subset Ά of ψ which is defined by:

Ά(ǎ)= $\begin{cases} 0.5 & ǎ = ọ \\ 0.4 & ǎ = 1,2 \\ 0.3 & ǎ = 3 \end{cases}$

is a fuzzy V-B-ideal of ψ.

Proposition (2.3):
Let ψ be a V-B-BH-algebra. Then every fuzzy ideal of ψ is a fuzzy V-B-ideal of ψ.

Proof:
Let A be a fuzzy ideal of ψ. To prove that Ά is a fuzzy V-B-ideal of ψ.

⇒i. Ά(ọ)≥Ά(ǎ), for every ǎ∈ψ [Since Ά is a fuzzy ideal of ψ. By definition(1.13) (i)]

ii. Let ǎ∈ V, č∈ψ⇒ǎ∈ψ [Since V ⊆ψ]

⇒ ǎ, č ∈ ψ ⇒ Ά (ǎ) ≥ min{ Ά(ǎ ⍟ č), Ά(č)} [Since Ά is a fuzzy ideal. By definition(1.13)(ii)]

Therefore, Ά is a fuzzy V-B-ideal of ψ.

Remark (2.4):
The converse of the proposition (2.3) may not be true in general as the following example shows.

Example (2.5) :
The fuzzy V-B-ideal Ά of ψ in example(2.2) is not a fuzzy ideal of ψ, since Ά (3) = ọ.3 < min {Ά (3⍟1), Ά (1)} = Ά (1) = ọ.4

Proposition (2.6) :
Let ψ be a V-B-BH-algebra and let Ά be a fuzzy V-B-ideal of ψ such that Ά (ǎ ⍟ č) ≤ min{ Ά (ǎ) , Ά (č)} for every ǎ $\notin V$, č $\in \psi$. Then Ά is a fuzzy ideal of ψ.

Proof :

Let Ά $be\ a\ fuzzy$ V − B − ideal $of\ \psi$.

To prove that Ά is a fuzzy ideal of ψ.

i. Let ǎ∈ψ⇒Ά(ọ)≥Ά(ǎ), for every ǎ∈ψ. [By definition (2.1)(i)]

ii. Let ǎ, č∈ψ.

Then we have two cases.

$Case1$: If ǎ $\in V$ ⇒Ά (ǎ)
≥ min{ Ά (ǎ ⍟ č), Ά (č)}
[$By\ definition$ (2.1)(ii)]

$Case2$: If ǎ $\notin V$ ⇒Ά (ǎ ⍟ č)
≤ min{A(ǎ) , A(č)}
⇒ A(ǎ ⍟č) ≤ A(ǎ) and A(ǎ ⍟ č)
≤ Ά (č) ⇒Ά (ǎ ⍟ č)
= min{ Ά (ǎ ⍟ č) , Ά (č)}
⇒Ά (ǎ) ≥ min{ Ά (ǎ ⍟ č) , Ά (č)}
⇒Ά $is\ a\ fuzzy\ ideal\ of\ \psi$.

Theorem (2.7):

Let ψ be a V-B-BH-algebra. Then Ά is a fuzzy V-B -ideal of ψ if and only if $Ά_t$ is a V-B-ideal of ψ, for every t∈[ọ, $\sup_{\varepsilon\in\psi}$Ά (ǎ)], ǎ ∈ ψ.

Proof:

Let A be a fuzzy V-B-ideal of ψ and t∈[ọ, $\sup_{\varepsilon\in\psi}$Ά (ǎ)]

To prove that $Ά_t$ is a V-B-ideal of ψ.

i. Since Ά(ọ) ≥Ά(ǎ), for every ǎ∈ψ⇒Ά(ọ) ≥ t ⇒ọ∈$Ά_t$.

ii. Let ǎ∈ V and ǎ ⊛č∈$Ά_t$, č∈$Ά_t$.

⇒Ά (ǎ ⊛ č)≥ t, Ά (č)≥ t

⇒ min{Ά (ǎ ⊛ č), Ά (č)}≥ t .

But Ά (ǎ)≥min{Ά (ǎ ⊛ č), Ά (č)} [Since A is a fuzzy V-B-ideal]

⇒Ά(ǎ) ≥t ⇒ǎ∈$Ά_t$⇒$Ά_t$ is a V-B-ideal of ψ.

Conversely,

To prove that Ά is a fuzzy V-B-ideal of ψ

i. Let t∈[ọ, $\sup_{\varepsilon\in\psi}$Ά (ǎ)]

⇒$Ά_t$ is a V-B-ideal of ψ

⇒ọ∈$Ά_t$⇒Ά(ọ) ≥t ⇒Ά(ọ) ≥Ά(ǎ),for every ǎ∈ψ.

[Since t∈[ọ, $\sup_{\varepsilon\in\psi}$Ά (ǎ)]

ii. Let ǎ∈V, č∈$\psi$and t= min{Ά(ǎ⊛č),Ά(č)}

⇒A(ǎ ⊛ č)≥ t and Ά (č)≥ t

⇒ǎ ⊛ č ∈ $Ά_t$ and y ∈ $Ά_t$⇒ǎ ∈ $Ά_t$[Since $Ά_t$ is a V-B-ideal of ψ]

⇒Ά(ǎ) ≥t ⇒Ά(ǎ) ≥min{Ά(ǎ ⊛č), Ά(č)}

⇒Ά is a fuzzy V-B-ideal of ψ.

Proposition (2.8):

Let{$Ά_\epsilon$(ǎ), ε ∈ γ}be a family of fuzzy V-B-ideals of a V-B-BH-algebra ψ. Then$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ) is a fuzzy V-B-ideal of ψ.

Proof :

Let {$Ά_\epsilon$(ǎ), ε ∈ γ} be a family of fuzzy V-B-ideals of ψ.

i. Let ǎ∈ψ.

$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ)= inf{ A_ϵ(ọ), ε ∈ γ }

≥ in f{A_ϵ (ǎ), ε ∈ γ }

[Since A_ϵ is a fuzzy V-B-ideal,for every ε ∈ γ. By definition (2.1)(i)]

$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ)⇒$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$ (ọ) ≥$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ)

ii. Let ǎ∈ V and č∈ψ

$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ) = inf{ A_ϵ(ọ), ε ∈ γ }

≥ in f{min{A_ϵ(ǎ⊛č),A_ϵ(č)}, ε ∈ γ

= min{ inf{ A_ϵ (ǎ⊛č), ε ∈ γ }, inf{ $Ά_\epsilon$(č), ε ∈ γ } }

= min{ $\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ⊛č), $\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(č)}

⇒$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ)

≥ min{$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(ǎ⊛č),$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$(č)}

⇒$\bigcap_{\epsilon\in\gamma}$$Ά_\epsilon$ is a fuzzy V-B-ideal of ψ.

Theorem (2.9):

Let{$Ά_\epsilon$(ǎ), ε ∈ γ} be chain of fuzzy V-B-ideals of a V-B-BH-algebraψ. Then $\bigcup_{\varepsilon\in\gamma}$ $Ά_\epsilon$ is a fuzzy V-B-ideal of ψ.

Proof :

Let {$Ά_\epsilon$: ε ∈ γ } be a chain of fuzzy V-B-ideals of ψ.

i. Let ǎ∈ψ.

$\bigcup_{\varepsilon\in\gamma}$ $Ά_\epsilon$(ọ)=sup{ $Ά_\epsilon$ (ọ), ε ∈ γ }

≥ sup{$Ά_\epsilon$ (č), ε ∈ γ }[By definition (2.1)(i)]

= $\bigcup_{\varepsilon\in\gamma}$ $Ά_\epsilon$(ǎ)⇒$\bigcup_{\varepsilon\in\gamma}$ $Ά_\epsilon$ (ọ) ≥$\bigcup_{\varepsilon\in\gamma}$ $Ά_\epsilon$(ǎ)$for\ every$ ǎ∈ψ.

ii. Let ǎ∈ V, č∈ψ.

$\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$ (ă) = sup{ Ά$_\epsilon$(ă), ε ∈ γ } ≥ sup{ min{Ά$_\epsilon$ (ă⍟č),Ά$_\epsilon$ (č)}, ε ∈ γ }

[By definition (2.1)(ii)]

But { Ά$_\epsilon$, ε ∈ γ } is a chain then there exists $\sigma\in\gamma$such that

sup{ min{Ά$_\epsilon$(ă ⍟č), Ά$_\epsilon$(č)}, ε ∈ γ } = min{Ά$_\sigma$(ă⍟č), A$_j$(č)}

=min{sup{Ά$_\epsilon$ (ă ⍟č), ε ∈ γ }, sup{Ά$_\epsilon$ (č), ε ∈ γ }}

⇒$\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$(ă)) ≥ min{Ά$_\sigma$(ă ⍟č) , Ά$_\sigma$(č)}≥min{sup{Ά$_\epsilon$(ă ⍟č), ε ∈ γ }, sup{Ά$_\epsilon$ (č), ε ∈ γ }}

= min{ $\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$(ă ⍟č) ,$\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$ (č) }

⇒$\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$(ă) ≥ min{ $\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$(ă ⍟č),$\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$(č)}

⇒$\bigcup_{\varepsilon\in\gamma}$ Ά$_\epsilon$ is a fuzzy V-B-ideal of ψ.

Theorem (2.10):

Let Ά *be a nonempty subset of a*V-B-BH-algebra$\psi$and let ฿be a fuzzy set in ψdefined by

฿(ă) =$\begin{cases}\rho_1 & ă\in Ά\\ \rho_2 & \text{otherwise}\end{cases}$

where $\rho_1 > \rho_2$ in [0 , 1] and Ά=ψ$_฿$ = {ă∈ ψ|฿(ă) = ฿(ọ)}.Then ฿is a fuzzy V-B-ideal of ψif and only if Ά is a V-B-ideal of ψ.

Proof :

Let ฿ *be a fuzzy*V − B − ideal *of* ψ.

i. ฿ (ọ) $= \rho_1$⇒ ọ ∈ Ά.[Since ฿(ọ)≥฿(ă)for every ă∈ψ]

ii. *Let* ă ∈ *V and* ă ⍟ č, č ∈ Ά

⇒฿(ă) ≥ *min*{฿ ((ă⍟ č), ฿ (č)} = ρ_1 [By definition (2.1)(ii)]

⇒฿ (ă) $= \rho_1$

⇒ ă ∈ Ά ⇒Ά *is a* V − B − ideals *of* ψ.

Conversely,

Let Ά *be a* V − B − ideal *of* ψ.

i.*Since* ọ ∈ Ά ⇒฿(ọ) = ρ_1 ≥ ฿(ă), *for every* ă ∈ ψ.

ii. *Let* ă ∈ *V and* č ∈ ψ.

Then we have four cases.

*Case*1: *If* č ∈ Ά *and* ă ⍟č ∈ Ά

⇒ ă ∈ Ά. [*By definition* (1.6)(*ii*)]

⇒฿(ă) = ρ_1⇒฿(ă) ≥ *min*{฿ (ă ⍟č), ฿ (č)}.

*Case*2 : *If* č ∈ Ά *and* ă⍟č ∉ Ά

⇒฿(č) = ρ_1 *and* ฿ (ă ⍟ č) = ρ_2

⇒min{฿(ă ⍟č), ฿(č)} = ρ_2⇒฿(ă) ≥ *min*{฿ (ă ⍟č), ฿ (č)}.

*Case*3 : *If* č ∉ Ά *and* ă⍟č ∈ Ά

⇒฿(č) = ρ_2*and* ฿ (ă ⍟ č) = ρ_1

⇒min{฿(ă ⍟č), ฿(č)} = ρ_2⇒฿(ă) ≥ *min*{฿ (ă ⍟č), ฿ (č)}.

*Case*4 : *If* č ∉ Ά *and* ă⍟ č ∉ Ά

⇒฿ (č) = ρ_2 *and* ฿ (ă ⍟ č) = ρ_2

⇒*min*{฿ (ă ⍟č), ฿ (č)} = ρ_2⇒฿ (ă) ≥ *min*{฿ (ă⍟č), ฿ (č)}.

⇒฿ *is a fuzzy* V − B − ideals *of* ψ.

Proposition (2.11):

Let Ω: (ψ, ⍟, ọ) → (ξ, ⍟′, ọ′) *be a V − B − BH − epimorphism. If* Ά *is a fuzzy* V − B − ideal *of* ψ, *then* Ω(Ά) *is a fuzzy*Ω (V − B) − ideal *of* ξ.

Proof :

Let 'A $be\ a\ fuzzy$ V − B − ideals $of\ \psi$.

i. Let č $\in\ \xi\ such\ that$ č $= \Omega$(ă).

for some ă $\in\ \psi$.

$(\Omega$ ('A))(ọ′) $= sup\{$ 'A (ă) /ă $\in \Omega^{-1}$(ọ′)}

= 'A (ọ)≥'A (ă)

$[By\ definition\ (2.1)(i)]$

= (Ω ('A))(Ω(ă)) = (Ω ('A))(č)

⇒(Ω ('A))(ọ′) ≥(Ω (ψ))(č)

i. Let č1 $\in\ \Omega(V)$, č2 $\in\ \xi$,

$there\ exists$ ă1 $\in V\ and$ ă2 $\in \psi$

$such\ that$

č1 = Ω (ă$_1$) and č$_2$ = Ω (ă$_2$)

⇒(Ω('A))(č$_1$) =

$sup\{$ 'A (ă) / ă $\in \Omega^{-1}$(č$_1$)}

(Ω('A))(č$_1$) ≥ 'A (ă$_1$)

≥ min{ 'A (ă$_1$⊛ă$_2$), 'A (ă$_2$)}

$[By\ definition\ (2.1)(ii)]$

= min{(Ω ('A))(Ω (ă$_1$⊛ă$_2$)) ,

(Ω ('A))(ă$_2$) }

= min{(Ω ('A))(Ω (ă$_1$)⊛′ Ω (ă$_2$)),

(Ω ('A))(Ω (ă$_2$) }

= min{(Ω ('A))(č$_1$⊛′ č$_2$),

(Ω ('A))(č$_2$) }

⇒ (Ω ('A))(č$_1$)

≥ min\{(Ω ('A))(č$_1$⊛′ č$_2$), (Ω ('A))(č$_2$)}

⇒Ω('A)$is\ a\ fuzzy$

Ω (V − B) − ideal $of\ \xi$.

Theorem (2.12):

$Let\ \Omega: (\psi, ⊛, ọ) \rightarrow (\xi, ⊛', ẹ)\ be\ a\ V - B$ $-BH - epimorphism, and\ let$ ฿ $be\ a\ fuzzy$ V − B − ideal $of\ \xi\ such\ that\ \Omega^{-1}$(V − B)$is\ a$ $B - algebra. Then\ \Omega^{-1}$(฿)$is\ a\ fuzzy\ \Omega^{-1}$(V −B) − ideal $of\ \psi$.

Proof :

i. Let ă $\in\ \psi. Since\ \Omega$(ă) $\in\ \xi$

and ฿ $is\ a\ fuzzy$ V − B − ideals $of\ \xi$,

$(\Omega^{-1}$(฿))(ọ) = ฿(Ω(ọ))

= ฿(0′) ≥ ฿(Ω(ă))

= (Ω^{-1}(฿))(ă)

ii. Let ă $\in\ \Omega^{-1}(V)$, č $\in\ \psi$.

Ω^{-1}(฿)(ă) = ฿(Ω(ă))

$[\ By\ definition\ (1.15)]$

≥ min{ ฿ (Ω(ă) ⊛′ Ω(č)), ฿(Ω(č))}

$[By\ definition(3.1)(ii)]$

= min\{ ฿ (Ω (ă ⊛č)), ฿ (Ω (č))}

= min\{Ω^{-1}(฿)(ă ⊛ č), Ω^{-1}(฿)(č)}

⇒Ω^{-1}(฿)(ă) ≥ min{Ω^{-1}(฿)

(ă ⊛ č), Ω^{-1}(฿)(č)}.

$then\ \Omega^{-1}$(฿) $is\ a$

$fuzzy\ \Omega^{-1}$(V − B) − ideal $of\ \psi$.

REFERENCES

[1] C.H. Park, "Interval-valued Fuzzy Ideal in BH-algebras", Advancein fuzzy set and systems1(3), pp.231–24ọ, 2ọọ6.

[2] D.Dubois and H. Prade, "Fuzzy Sets and Systems", Academic Press. INC. (London) LTD.AcademicPress.fifth Avenue,198ọ.

[3] E.M.kim and S.S.Ahn,"On Fuzzy n-fold Strong Ideals of BH-algebras",J.Appl.Math.&InformaticsVol.3ọ,No.3–4,pp.665-676,2ọ12 .

[4] H. H. Abass and L. S. Mahdi ,"A BH-Algebra Related to B-Algebra", European Journal of Scientific Research,Vol. 128 No 2,pp. 134-14ọ, 2ọ15.

[5] J. Neggers and H. S. Kim ,"On B-algebras", Mathematical bechnk,54, pp.21-29, 2ọọ2.

[6] K. ISEKI ,"An algebra related with a propositional calculus", Proc. Japan Acad. 42, pp.26-29, 1966.

[7] L.A. Zadeh, "Fuzzy Sets", Information and control, Vol. 8, PP. 338-353, 1965.

[8] S.S.Ahn and H.S.Kim,"R-Maps and L-Maps in BH-algebras",Journal of the Chungcheong Mathematical Society, Vol. 13, No. 2, 2ọọọ.

[9] Y. B. Jun, E. H. Roh and H. S. Kim ,"On BH-algebras", Scientiae Mathematicae Vol. 1,No 3, pp.347–354, 1998.