Study The Effect Of Lorentzian And Gaussian Illumination On Line Spread Function For Elliptical Aperture

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Abstract:

In this research , Line Spread Function (LSF) for elliptical aperture was studied, Also the effect of Lorentzian illumination with different values of (Γ), full width half maximum (FWHM), and Gaussian illumination with different values of standard deviation were studied, In case of diffraction limited system and with present of coma aberration . The results showed that the type of the filters reduce the effect of coma aberration where the Strehl ratio is increasing and also the shifting of the function when (ψ=90) for coma aberration is decreasing.

Keywords: Line spread function, coma aberration, Lorentzian Filter, Gaussian Filter ,elliptical Aperture

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دراسة تأثير مرشح لورنز والمرشح الكاوسي على دالة الانتشار الخطية لفتحة بيضوية

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الخلاصة

تم في هذا البحث دراسة دالة الانتشار الخطية لفتحة بيضوية, وكذلك تأثير كل من مرشح لورنز ولقيم مختلفة من معامل عرض النصف للدالة (Γ) والمرشح الكاوسي لقيم مختلفة من معامل الانحراف المعياري (a), في حالة النظام المحدد بالحيود وكذلك في حالة وجود الزيغ المذنبي. وبينت النتائج ان المرشحين يقللان من تأثير الزيغ المذنبي وذلك بزيادة نسبة ستريل لمنحني الدالة وكذلك نقصان مقدار الازاحة الحاصلة للدالة عندما (ψ=90) للزيغ المذنبي.

1. Introduction

The light rays is used in a Geometrical optics to express image formation by spherical surfaces, lenses, mirrors, and optical instruments. The quality of their images may be no perfect, but the departures of perfection, known as aberrations, may be tolerable for their purpose.

Since that aberrations cannot be eliminated entirly with an optical system, it becomes a matter for choice which aberrations are the most nuisance and which can most easily be tolerated. For example, aphotograph with distortion may be more displeasing to the eye than one with some blurring due to spherical aberration or coma[Smith et.al.,(2007)].

To reduce the effect of coma aberration in Line Spread Function, i.e. high resolution power and low secondary peaks, which improve of image quality, we used Lorentzian and Gaussian filters.

Many studies on the filters have been previously adopted, where in 2007, Saralees Nadarajah and Samuel Kotz, used a truncated model of the bivariate Lorentzian distribution [Nadarajah et.al.,(2007)]. In 2008, Yi-rong Yang et.al, delimitted synthesis functions (Lorentzian, Gaussian and exponential) that express the double-pass LSF [Yang et.al.,(2008)] and in the same year, Maksay Ştefan, et.al. suggested a method of truncation in which the limits are determined using the least squares method, they obtained by simulating the two-dimensional Lorentzian rule and that of the corresponding truncated one, are presented comparatively. [Ştefan et.al.,(2008)]. While in ٢014, H. Zolfagharnasab and A. R. Naghsh-Nilchi, improved the accuracy of the automatic retinal vessel detection through a new Gaussian function with Lorentzian distribution [Zolfagharnasab et.al.,(2014)]. In 2016, B.H. Alrueshd. studied the Point Spread Function for obscured circular synthetic apertures of Gaussian Filter [Alrueshd,(2016)]. In 201٧, Cosmas Mafusire And Tjaart P.J.Kruger, calculated a Strehl Ratio by using to numerically derive OZ polynomials, and they showed, that the aberrations used to balance the selected aberrations can be associated with specific effects, namely, defocus, and tilt. [Mafusire et.al.,(2017)].

2. Deriving the Equation of Line Spread Function for elliptical Aperture

Line object is defined as a number of points sources positioned side by side on a line that make the line object [Born et.al., (1984)] .

To find the intensity of non coherent linear source must know the distribution of complex amplitude for points located at the coordinates (u,v) in the image plane by using Fourier transformation in 2-Dimension [Malacara et.al., (2004)].

$$F(u,v) = \frac{1}{A}\int_{y}\int_{x} f(x,y)\, e^{2\pi i(ux+vy)} dxdy \qquad (1)$$

The intensity, can be explained by multiplication of function F(u,v) by is complex amplitude [Goodman, (1996)]

$$G(u,v) = |F(u,v)|^2 \qquad (2)$$

To calculate the complex amplitude of the line object ,so that [Hasegaw,(1987),and Manzanares et.al., (1997)]

$$\mathrm{L(u,v)} = \int_{\mathrm{v}} \mathrm{G(u,v)dv} \qquad (3)$$

From eq.(1) and eq. (3) ,one obtains:

$$L(u,v) = \int_{v}\left|\int_{y}\int_{x} f(x,y)\, e^{2\pi i(ux+vy)} dxdy\right|^2 dv \qquad (4)$$

we obtain: $A^* A = |A|^2$ the by using

$$L(u,v) = \int_{v}\int_{y}\int_{x}\int_{y_1}\int_{x_1} f(x,y) f^*(x_1,y_1) \,.\, e^{2\pi i(ux+vy)} \,.\, e^{-2\pi i(ux_1+vy_1)}\, dx_1 dy_1 dxdydv \qquad (5)$$

$$L(u,v) = \int_{y}\int_{x}\int_{y_1}\int_{x_1} f(x,y) f^*(x_1,y_1) \,.\, \mathrm{e}^{2\pi iux} \,.\, \mathrm{e}^{-2\pi iux_1}\, dx_1 dy_1 dxdy \int_{v} e^{2\pi iv(y-y_1)} dv \qquad (6)$$

[Gaskill,(1970)]) Dirac-delta function(from

$$\int_{v} e^{2\pi iv(y-y_1)} dv = \delta(y-y_1) \qquad (7)$$

eq.(6) becomes

$$L(u) = \int_{y}\int_{x}\int_{x_1} f(x,y) \,.\, \mathrm{e}^{2\pi iux} . \mathrm{e}^{2\pi iux_1} dx_1 dxdy \int_{y_1} f^*(x_1,y_1)\, \delta(y-y_1) dy_1 \qquad (8)$$

$$L(u) = \int_{y}\int_{x}\int_{x_1} f(x,y) f^*(x_1,y) \,.\, \mathrm{e}^{2\pi iux} . \mathrm{e}^{2\pi iux_1} dx_1 dxdy \qquad (9)$$

Assume that z = 2πu, equation (9) becomes

$$L(z) = \int_{y}\left[\int_{x} f(x,y). \mathrm{e}^{izx} dx \int_{x_1} f^*(x_1,y). \mathrm{e}^{izx_1}\, \mathrm{d}x_1\right] dy \qquad (10)$$

normalized Line spread function will takes the form: Then

$$LSF = L(z) = N\int_y \left|\int_x f(x,y).\mathrm{e}^{izx}\,dx\right|^2 dy \qquad (11)$$

Where (N): a normalization constant. now, if the aperture is elliptical form then

$$LSF = N\int_{-b}^{b}\left|\int_{-\sqrt{1-\frac{y^2}{b^2}}}^{\sqrt{1-\frac{y^2}{b^2}}} f(x,y).\mathrm{e}^{izx}dx\right|^2 dy \qquad (12)$$

Where that principal axis equals one unit and secondary axis (b) equals (0.7).

3. The Equation Lorentzian and Gaussian Filters

From Eq. (1); f(x,y) represents pupil function which consists of two parts[Goodman, (1996)]:

: Æ! ; L ¸: Æ! ; : Æ ; (13)

(x,y): represents the real amplitude distribution in exit pupil coordinates (x,y), and it is called "pupil transparency" and often chooses equal one unit. The research is by commutating equation lorentzian and Gaussian filters to study the effect of Line spread Function for elliptical aperture.

If there found of Gaussian Filter [JAMES,(2011)]

:TÆU; L A ?1 °·> · 6 · p :sv;

Where a: standard deviation modulus for Gaussian Filter function.

If there found of Lorentzian Filter [Walck, (2007)]

:TÆU; L k 6 o 6 / T6 E@ A · k 6 o 6 / U6 E@ A · :sw;

Where Γ: full width half maximum (FWHM) for Lorentzian Filter function

4. The Equations coma aberration

From Eq. (1):

A ŒvÆ is the wave front of aberration function, W(x.y) is the aberration factor and k is wave number (k =2π/λ). In this study, it has been used coma aberration to studying the effect on Line Spread Function, Equation coma aberration is defined as [Bass et.al., (1995)]

9 L ″ @ 9 5 5 σ N?K Ð L 9 57 G 5 N 7 ?K Ð E 9 59 G 5 N 9 ?K Ð E fi :sx;

In Cartesian coordinate (x , y):

x= r sin Φ, y= r cos Φ

Then, The aberration function can be written as:

$$W(x,y) = W_{131}(x_1^2 + y_1^2)y_1 + W_{151}(x_1^2 + y_1^2)^2 y_1 + \cdots \qquad (17)$$

When the Cartesian coordinates are rotated by an angle □□, the new coordinates become:

$x_1 = x \cos \psi - y \sin \psi$ (18)

$y_1 = x \sin \psi + y \cos \psi$

Where □□is rotation angle of Cartesian coordinates, The aberration function becomes:

9 :T5 ÆU5 ; L 957 5:T6 E U6 ; :T OEª ÆE U ?Kª ;E 959 5:T6 E U6 ;6 :T OEª ÆE U ?Kª ;

E fi :s{;

٥. Results and Discussion

In this work, we are refer that; the equations have been simulatied and sloved using matlab enviromat . two filters were used for illumination, Lorentzian filter of full-width half maximum (Γ) (Γ = 0.2,0.4,0.6,0.8 and 1) and Gaussian filter of standard deviation modulus (a) (a =0.2,0.4,0.6,0.8,1) as follows:

5.1. LSF for Elliptical Aperture with Lorentzian Filter .

5.1.1. LSF for Elliptical Aperture with Lorentzian Filter for a Perfect Optical System

By using the Eq. (12), with the presence of Lorentzian Filter (Eq.15), to find the results which shown in Fig.(1), and we observe that the central intensity (Strehl ratio) of the line Spread function of the elliptical aperture starts with (1) in the case of the Perfect optical system, with or without the Lorentzian filter, due to the normalization function. moreover, it is noticed that there is an increase in the width of the Function curve with decreasing the values of (Γ) of the Lorentzian filter. and also the filter work on reducing the secondary peaks. However, (Γ = 0.2) the Lorentzian filter is make for the rise of the function curve about axis, this affects on the clarity of the image .

5.1.2. LSF for An Elliptical Aperture with Lorentzian Filter at Coma Aberration

a- when (ψ=0)

The Fig. (2- 4) describe the effect of coma aberration for the rotation angle (ψ=0) on a Line Spread Function of an elliptical aperture, it is noticed that the effect of coma aberration (W_{131}=0. 5 λ) on the value central intensity (Strehl ratio) will be (0.968), while with the Lorentzian filter, the value become (0.981) at (Γ =1). It reach to (0.999) at (Γ=0.2), when the coma aberration (W_{131}=1 λ), the value central intensity will be (0.879). where as, with the Lorentzian filter, the value become (0.928) at (Γ =1). It reach to (0.995) at (Γ=0.2). The Lorentzian filter increased central intensity (Strehl ratio) of the line

Spread function, the secondary peaks and the width of curve is decreased, especially at (Γ=0.8,1). When applying the final value of coma aberration (W_{31}=1. 5λ), it is noticed the secondary peaks increase, and the central intensity is (0.754). But, With the Lorentzian filter, starting with (0.853) at (Γ =1) reaching to (0.990) when (Γ =0.2).

b- when (ψ=90)

The effect of coma aberration (W_{131}=0.5 λ, 1λ and1.5λ) for the rotation angle (ψ=90) is explaining in figs. (5- 7) , the central intensity is shifted to the other side with increasing in secondary peaks. But, With Lorentzian filter, it is noticed that all (Γ) values make increase in the central intensity and decrease in the secondary peaks.

When use coma aberration (W_{131}=1 λ, 1.5 λ) , will note in the secondary peaks get shifting and increasing, while with Lorentzian filter get decreasing in the shifting and the values of the secondary peaks, and this is observed clearly (Γ=0.4,0.6).

5.2. LSF for Elliptical Aperture with Gaussian Filter .

5.2.1. LSF for Elliptical Aperture with Gaussian Filter for a Perfect Optical System

In this case, When used Eq. (12), with the presence of Gaussian Filter (Eq.14), the results were shown in Fig.(8), and we observe that the central intensity (Strehl ratio) of the line Spread function of the elliptical aperture starts with (1) in the case of the ideal optical system, with or without the Gaussian filter, because the of normalization function.

Moreover, it is noticed that there is an increase in the width of the Function curve with decreasing in standard deviation modulus(a) of the Gaussian filter. and also the filter work on reducing the secondary peaks.

5.2.2. LSF for An Elliptical Aperture with Gaussian Filter at Coma Aberration

a- when (ψ=0)

The Fig. (9 - 11) describe the effect of the Gaussian filter in the central intensity (Strehl ratio) of the line Spread function, when (W_{131}=0.5), the value central intensity will be (0.968). But, with the Gaussian filter, the central intensity value will be (0.971)at (a=1) and it reaches to (0.999) at (a=0.2), while the coma aberration (W_{131}=1 λ), the value central intensity will be (0.879) and the secondary peaks is increased. But, with the Gaussian filter, is noticed the intensity is (0.890) at (a=1) and reaches up to (0.998) when (a=0.2), the Gaussian filter decreasing the secondary peaks, especially at (a=0.4). When applying the coma aberration (W_{31}=1. 5λ), the central intensity decreases, where be (0.754). but Gaussian filter, It is (0.775) when (a=1) and reaches to (0.994) when (a=0.2), yet it reducing the secondary peaks especially at (a=0.4).

b- when (ψ=90)

when the Gaussian filter applied, with coma aberration (W_{131}=0.5 λ), this shifting decreases as the standard deviation modulus (a) value of the filter decrease, and the secondary peaks decreased, but a less than the case of Lorentzian filter, as in Fig. 12.

When using coma aberration (W_{131}=1 λ) with the Gaussian filter, the central intensity increased. both shifting and the secondary peaks decreased, and this is clearly observed (a=0.4), as in Fig. 13.

The Gaussian filter, it increases the central intensity of the LSF , especially at (a=0.2),with coma aberration (W_{131}=1.5 λ). The curve width of the LSF is better than in the case of Lorentzian filter. The secondary peaks decreased, but a less than the case of Lorentzian filter, as in Fig. 14.

6- Conclusions

1- Lorentzian filter enhanced the function curve about axis, more than Gaussian filter especially at (Γ =0.2).
2- The best case for central intensity, when curve width of the (LSF), and decreaseing secondary peaks for Lorentzian filter is (Γ=0.8 and 1) at (ψ=0).
3- The best case for central intensity (Strehl ratio), when curve width, and decreaseing secondary peaks for Gaussian filter is at (a=0.4) at (ψ=0).
4- The Strehl ratio increases when the coefficients of Lorentzian and Gaussian filters (a and Γ) are decreasing.
5- The LSF shifting of the axis increases which means increasing the curve width and decreasing the central intensity as the aberration coefficients increase (W_{131}=0. 5λ,1 λ and1. 5λ), when(ψ=90) which effects negatively on the resolution power which linturn effects on the image.
6- The function shifting decreases and the central intensity increases as the coefficients of filters (a and Γ) decreased. but, Lorentzian filter is better for decreasing the secondary peaks, while the Gaussian filter is better increasing Strehl ratio, especially at (a=0.2).

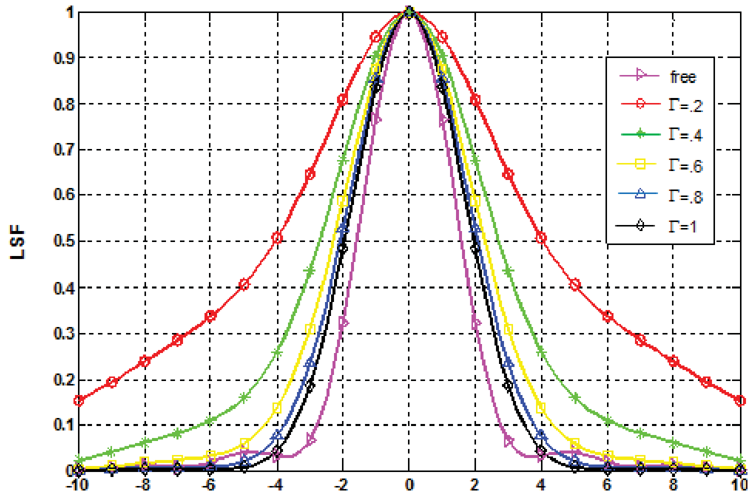

Fig.1. LSF for elliptical aperture of perfect optical system, different values of Lorentzian filter (Γ)

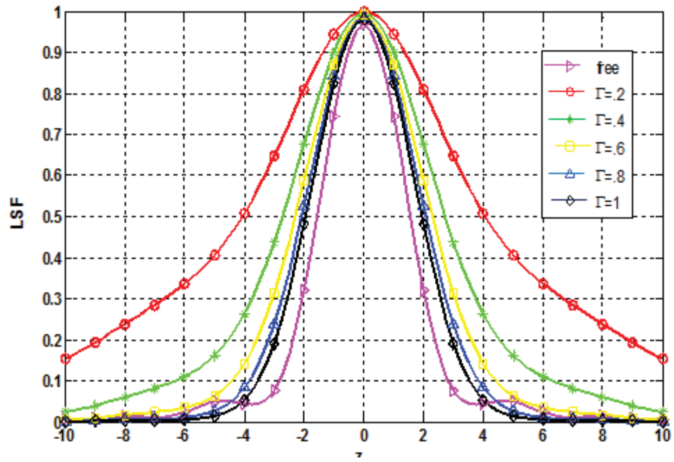

Fig.2. LSF for elliptical aperture of optical system, different values of Lorentzian filter (Γ) when (W_{131}=0.5,ψ =0)

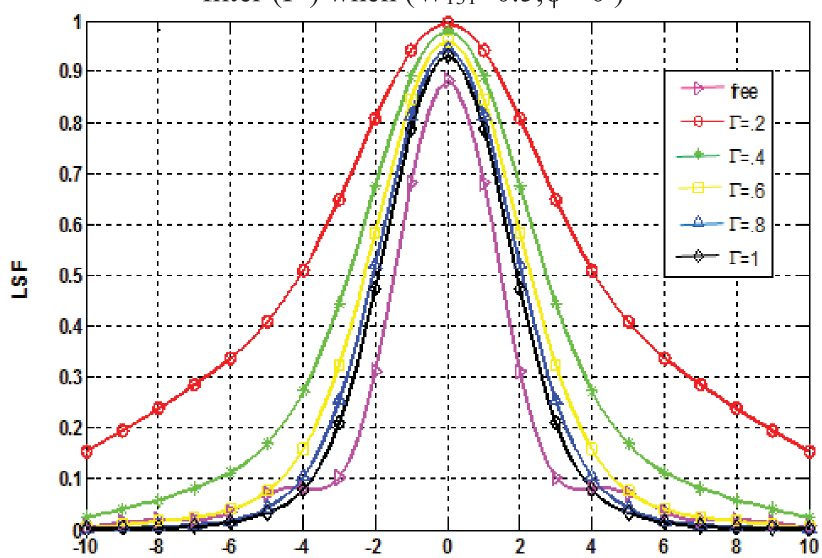

Fig.3. LSF for elliptical aperture of optical system, different values of Lorentzian filter (Γ) when (W_{131}=1,ψ =0)

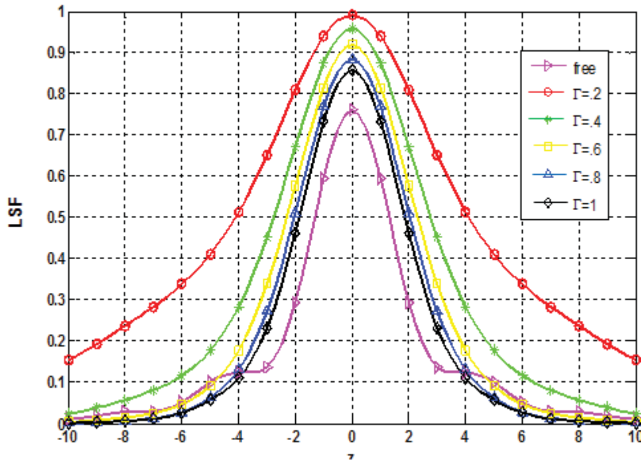

Fig.4. LSF for elliptical aperture of optical system, different values of Lorentzian filter (Γ) when (W_{131}=1.5,ψ =0)

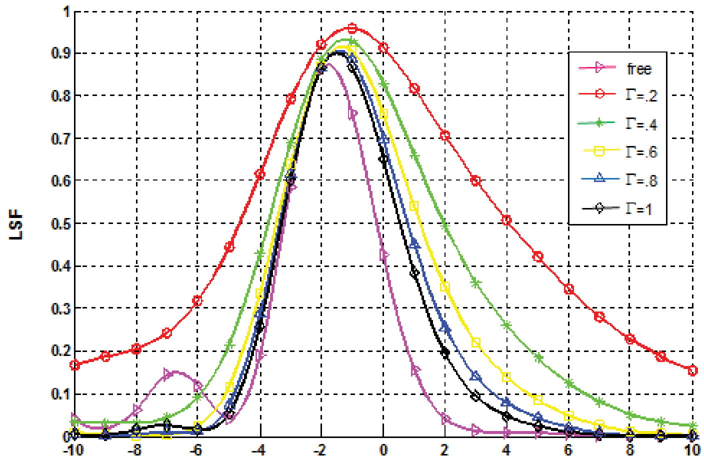

Fig.5. LSF for elliptical aperture of optical system, different values of Lorentzian filter (Γ) when (W_{131}=0.5,ψ =90)

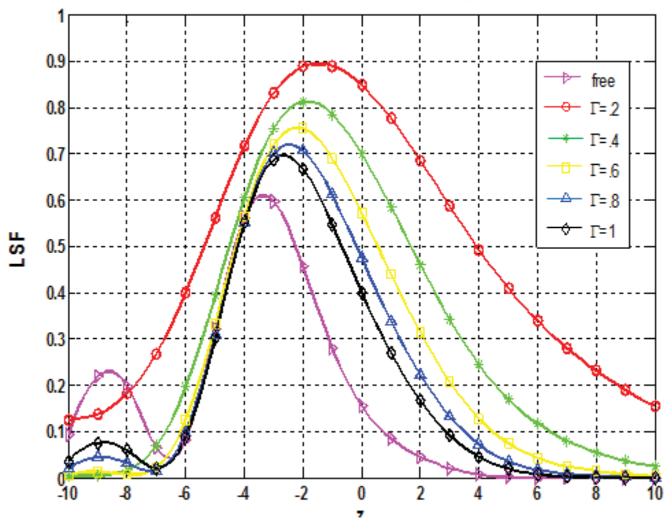

Fig.6. LSF for elliptical aperture of optical system, different values of Lorentzian filter (Γ) when (W_{131}=1,ψ =90)

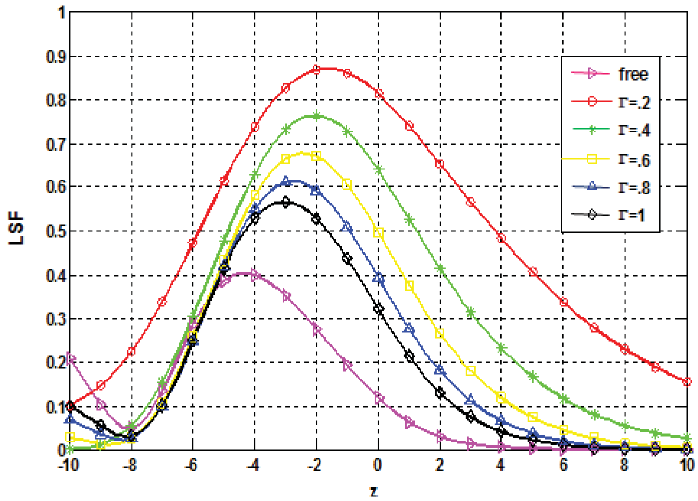

Fig.7. LSF for elliptical aperture of optical system, different values of Lorentzian filter (Γ) when (W_{131}=1.5,ψ =90)

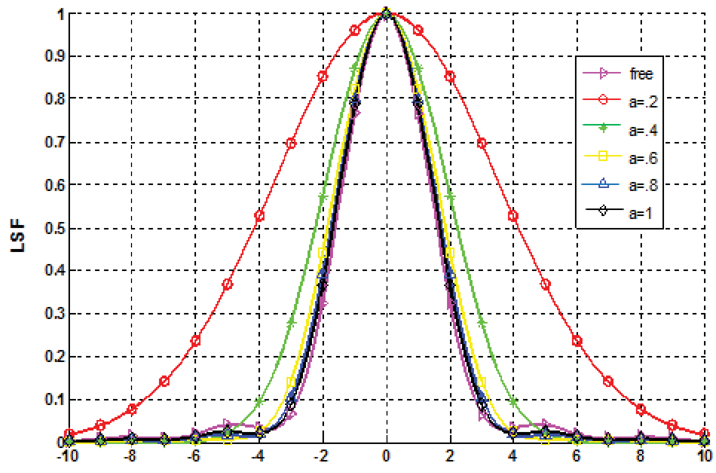

Fig.8. LSF for elliptical aperture of perfect optical system, different values of Gaussian filter (a)

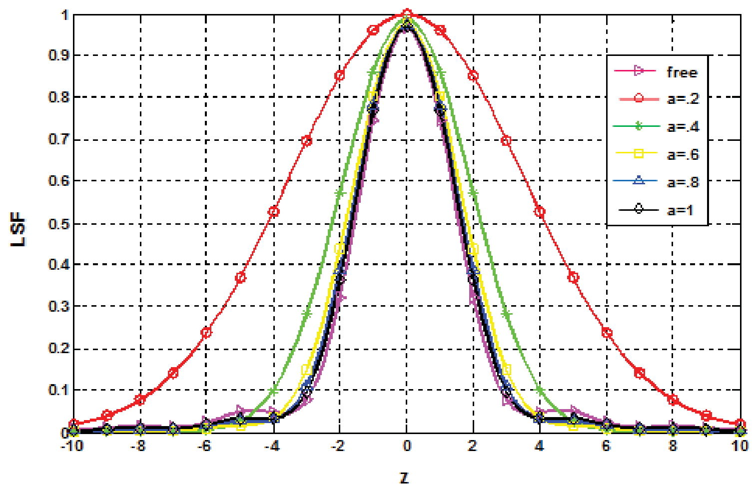

Fig.9. LSF for elliptical aperture of optical system, different values of Gaussian filter (a) when (W_{131}=0.5,ψ =0)

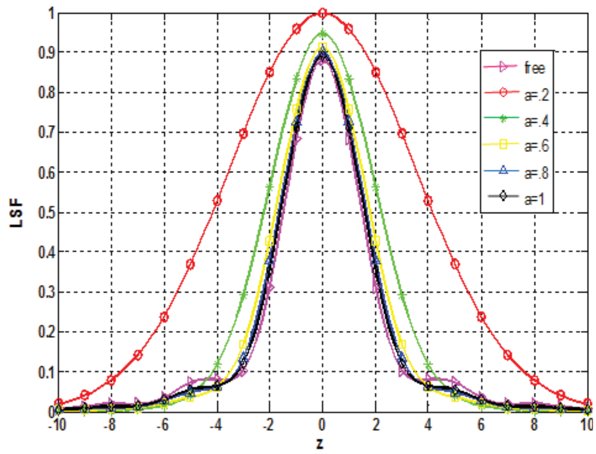

Fig.10. LSF for elliptical aperture of optical system, different values of Gaussian filter (a) when (W_{131}=١,ψ =0)

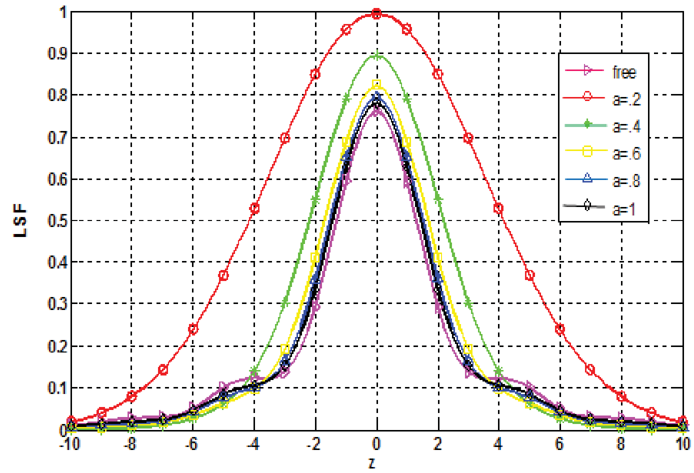

Fig.11. LSF for elliptical aperture of optical system, different values of Gaussian filter (a) when (W_{131}=١.5,ψ =0)

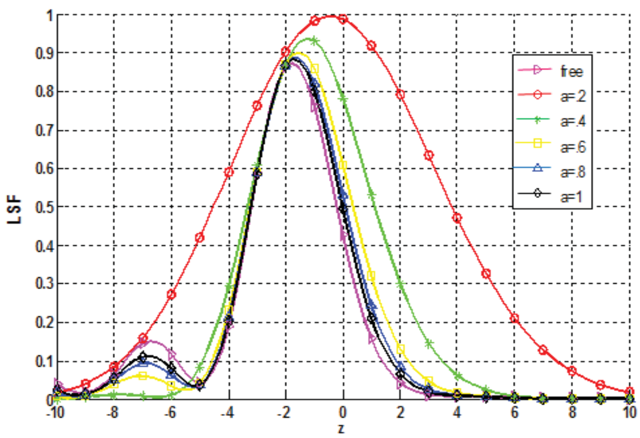

Fig.12. LSF for elliptical aperture of optical system, different values of Gaussian filter (a) when (W_{131}=0.5,ψ =90)

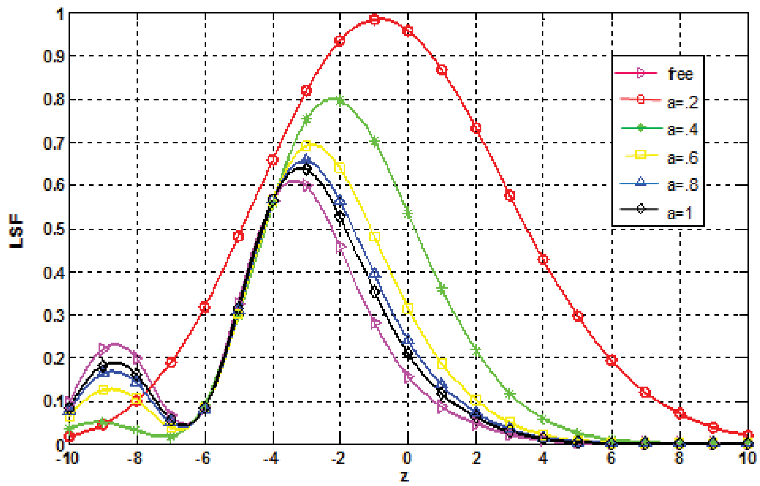

Fig.13. LSF for elliptical aperture of optical system, different values of Gaussian filter (a) when (W_{131}=١,ψ =90)

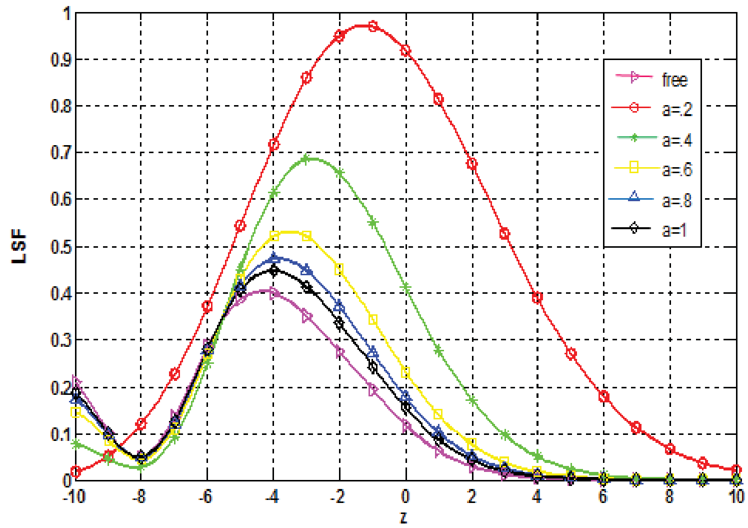

Fig.14. LSF for elliptical aperture of optical system, different values of Gaussian filter (a) when (W_{131}=١.5,ψ =90)

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