ANALYSIS OF THREE DIMENSIONAL NATURAL CONVECTION IN A SQUARE ENCLOSURE+

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Abstract:

The solution of three dimensional Navier-Stokes and energy equations for the problem of natural convection in a square enclosure as a result of differential side heating has been obtained numerically. Two cases were studied for different values of Grashof numbers (10^3, 10^4, 10^6). The effect of Prandtl number and aspect ratio on the flow and thermal field is also verified. Fully elliptic laminar Navier-Stokes equations are solved using primitive variables method with finite volume technique. The results show that the secondary flow region is increased with increasing in Grashof number. Also the results show that the central core region is increased with the increase in Prandtl number. The predicted results are compared with published results and found to be in good agreement.

المستخلص

تم في هذه الدراسة حل معادلات (نفير-ستوكس ومعادلة الطاقة الثلاثية الأبعاد ولمشكلة الحمل الحر الناتج عن التسخين الجزئي لبعض أجزاء الحيز) عدديا.تم دراسة حالتين ولأرقام مختلفة من رقم كراشـــوف (10^3, 10^4, 10^6) .تبين أيضاً تأثير رقم براندتل والنسبةالجانبية على مجال الجريان ومعدل انتقال الحرارة . معادلات نفير-ســـتوكس الطباقية ذات القطع الناقص تم حلها بأستخدام طريقة المتغيرات الأولية مع تقنية الحجم المحدد.أظهرت النتـــائج أن مناطق الجريان الثانوي تزداد مع زيادة رقم كراشوف . بينت النتائج أن منطقة القلب المركزي شبه الساكن تزداد مع زيادة رقم براندتل.تم مقابله النتائج الحالية مع النتائج المنشورة وكان التوافق جيدا.

Nomenclature:

A_p	aspect ratio
Gr	Grashof number
H	enclosure height
Pe	Peclet number
Pr	Prandtl number
T_c	cold wall temperature
T_h	hot wall temperature

Greek

θ	dimensionless temperature
ν	kinematic viscosity
Γ	diffusion coefficient

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Introduction:

The problem of natural convection in an enclosure is still receiving considerable attention due to its importance in many practical fields(solar applications, electronic equipment cooling and air conditioning).Various numerical and experimental studies are available on internal flows driven by buoyancy forces which are shown by references [1-5].In these studies the influence of the important parameters such as Grashof numbers, aspect ratio and the differential side heating is studied extensively. however these studies focus on the two dimensional flow field . Only few studies by references [6-8] cover the three dimensionality of the problem . The present study gives an insight in to the complex three-dimensional flow inside an enclosure.

Problem description

The problem to be considered is shown in Fig.1 (a and b). As shown in the first case (a). The geometry is specified by the height (H),the length (L) and the width (w) . The two vertical walls of the enclosure are maintained at uniform temperatures Tc and Th while the top and bottom walls are maintained adiabatic. In case (b) the top and bottom are maintained at uniform temperatures Th and Tc while the two vertical sides are maintained adiabatic.

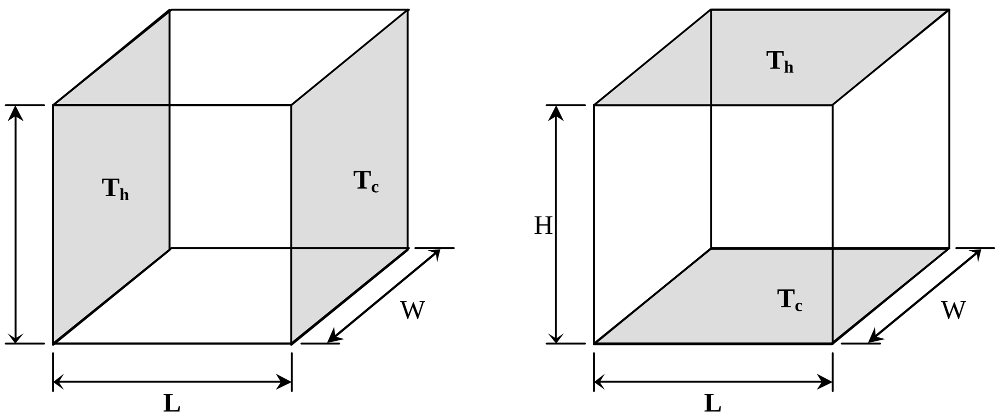


Fig. 1 **Problem description**

Mathematical formulation:

The governing equations of the problem under consideration are the continuity, momentum and energy equation. Constant thermophysical properties are assumed except for the density that appears in buoyancy force. The governing equations were cast in dimensionless form using H, (ν/H), $\rho(\nu/H)^2$ and T_h-T_c to scale the lengths, velocity, pressure and temperature. Thus the dimensionless governing equations can be expressed as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}+W\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial X}+\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}+\frac{\partial^2 U}{\partial Z^2}\right)\ldots\ldots\ldots\ldots(2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}+W\frac{\partial V}{\partial Z}=-\frac{\partial P}{\partial Y}+\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+\frac{\partial^2 V}{\partial Z^2}\right)+Grq\ldots\ldots\ldots\ldots(3)$$

$$U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}+W\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\left(\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Y^2}+\frac{\partial^2 W}{\partial Z^2}\right)\ldots\ldots\ldots(4)$$

$$U\frac{\partial q}{\partial X}+V\frac{\partial q}{\partial Y}+W\frac{\partial q}{\partial Z}=\frac{1}{Pe}\left(\frac{\partial^2 q}{\partial X^2}+\frac{\partial^2 q}{\partial Y^2}+\frac{\partial^2 q}{\partial Z^2}\right)\ldots\ldots\ldots\ldots(5)$$

where $q=(T-T_c)\,/(T_h-T_c)$

Boundary conditions

The boundary conditions for the above set of equations are as follows. On the walls the normal and tangential velocities are: U = V = 0. , W = 0.
$\frac{\partial P}{\partial N}$ = 0; the specified temperatures on the walls are: T_h = 0.5 and T_c = 0.5
and the adiabatic walls are : $\frac{\partial q}{\partial N}=0$,where N vector is normal to the wall

Numerical solution

The governing equations (1) to (5) form non-linear partial differential equations that must be solved subjected to the considered boundary conditions. These equations are discretised and then solved by using successive over relaxation method (S.O.R) method with finite volume technique[9]. The pressure and velocities are linked by the simple algorithm. The staggered grids are used in this study. The grid points are distributed in a uniform manner. full details of the used method are found in ref.[9,10]. The non dimensional local heat flux through a boundary may be represented by Nusselt number, Nu = $\frac{\partial q}{\partial N}$

<u>Results and discussion</u>:

The first case :

Fig.(2) shows the three dimensional flow field for different values of Grashof number. The values of velocities increase with increasing of Grashof number. As shown in this figure the flow movs up the hot wall across the top of the enclosure to the cold wall . Secondary flow regions were found to be along the upper part of the hot wall and the lower part of the cold wall. As shown in all the figures there is a semi quiescent core region separating the boundary layers on the hot and cold sides. The thickness of the boundary layer on the hot and cold side decreases with increasing of Grashof number because of the increase in velocity.

Fig.(3-a) exhibits the three dimensional flow field for Gr=10^6. In this figure the gradient in color indicates the higher values of temperatures in regions near the hot wall, the fluid in the core region is stratified with uniform temperature. Fig.(3-c-d) shows the effect of aspect ratio on the flow field. It can be seen that the values of temperatures in central core region are increased with increasing aspect ratio.

The effect of Prandtl number on the flow field is shown in Fig.(4). It is evident that the Prandtl number increases affected flow field significantly where the secondary flow regions is increased with the increase in the Prandtl number. The central core region area is also increased with the increase of the Prandtl number due to the increase in buoyancy forces.

Also the effect of Prandtl number on the thermal field is studied in Fig.(5). As shown in this figure the values of temperature are increased with increasing of Prandtl number and the convection seems to be dominant in figure (5-b).

The second case:

The three-dimensional pattern of the flow field is studied in Fig. (6) for different values of Grashof numbers. It is evident that the secondary flow regions are decreased compared with the fist case. Also the values of the velocities are increased with increasing in Grashof number.

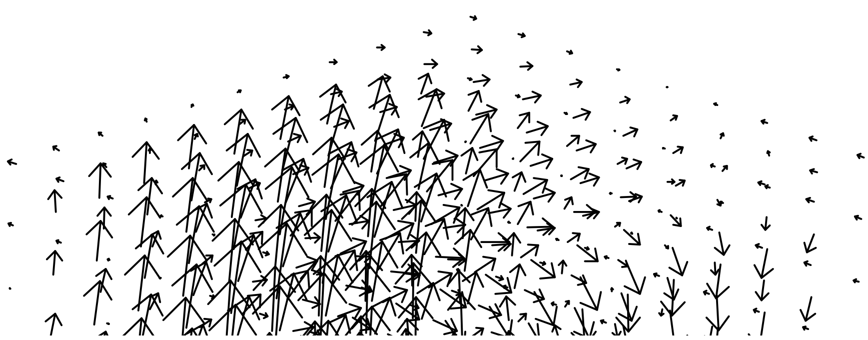
Fig.(7) exhibits the obtained pattern of the three-dimensional thermal field for different values of Grashof numbers .It is evident that the core region in (b) is nearly filled with the thermal layer more rapidly than in (a).

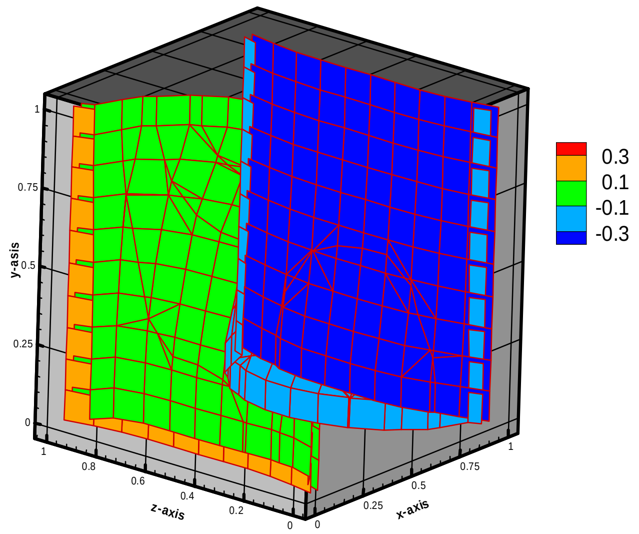
Fig.(8) shows the comparison between the calculatied and the published results. From this figure the values of Nusselt number increase along the hot side especially in the core region due to the large gradient in temperatures and then decrease at the boundary due to the decrease in temperature gradient

Conclusions:

The following conclusions can be drawn from this study

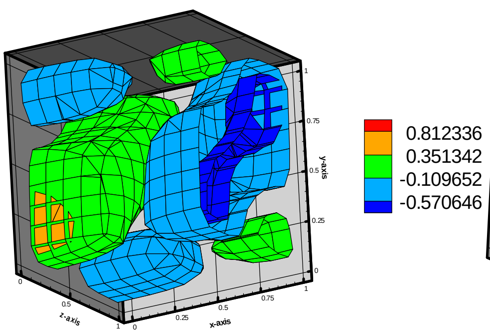
- The secondary flow region is increased with the increase in Grashof numbers.
- The thermal layer fills the core region more rapidly with the increase in Prandtl numbers
- The aspect ratio affects the flow and thermal field significantly because the values of temperature in the central core region is increased with the increase of aspect ratio.

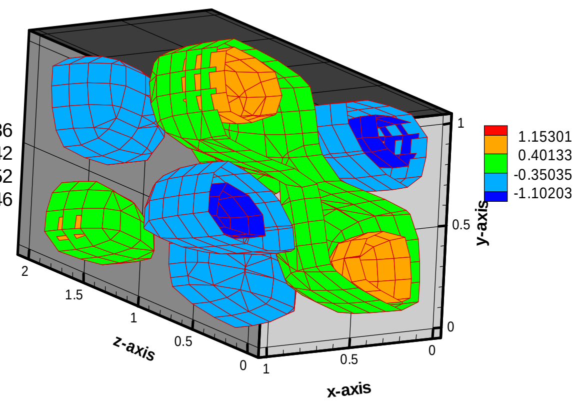




(a) A_p=

Fig.(3) a. Three dimensional isothermal contours





(c) A_p=

(d) A_p=2

Fig.(3) c.& d. Three dimensional flow field contours

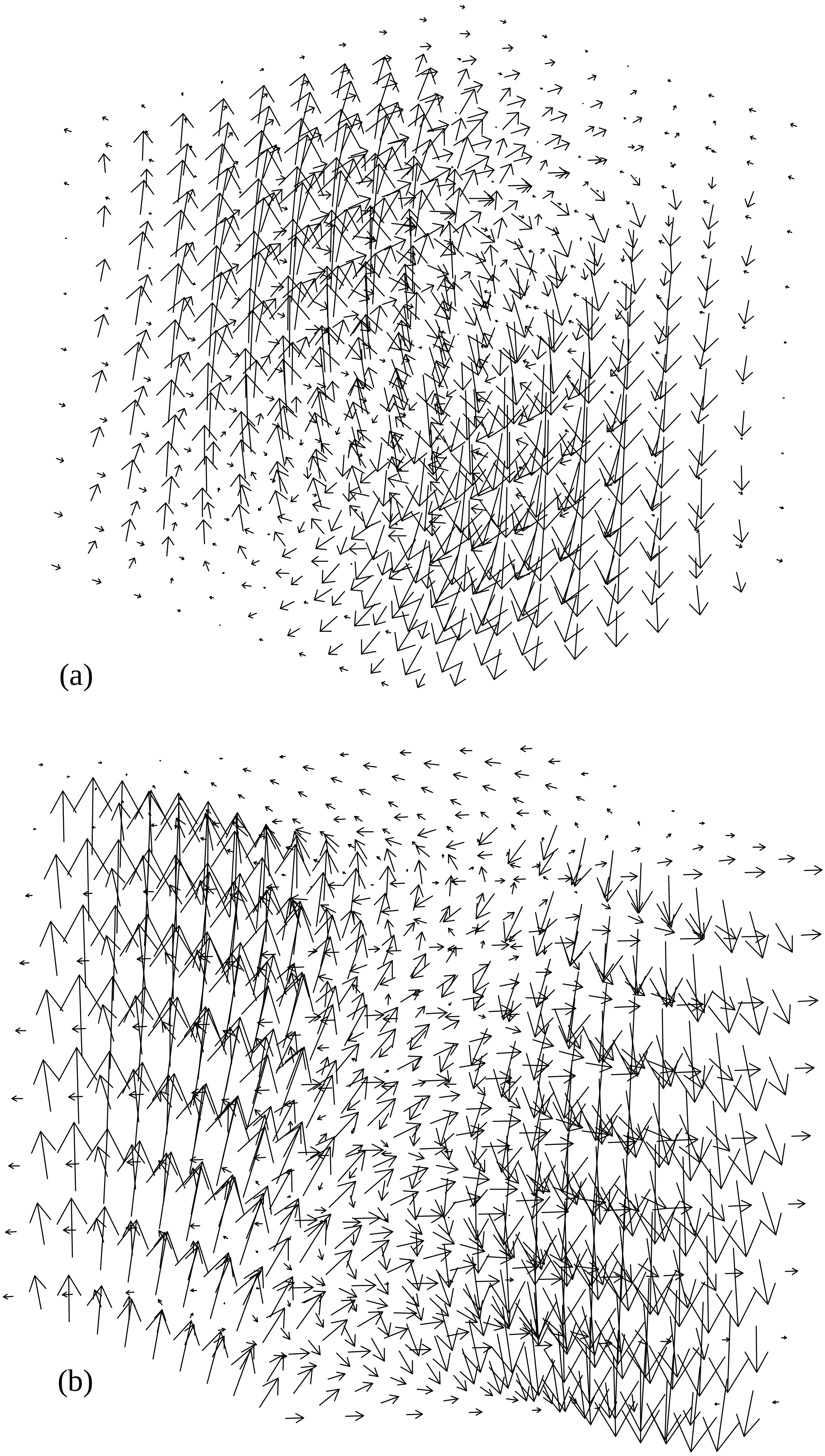
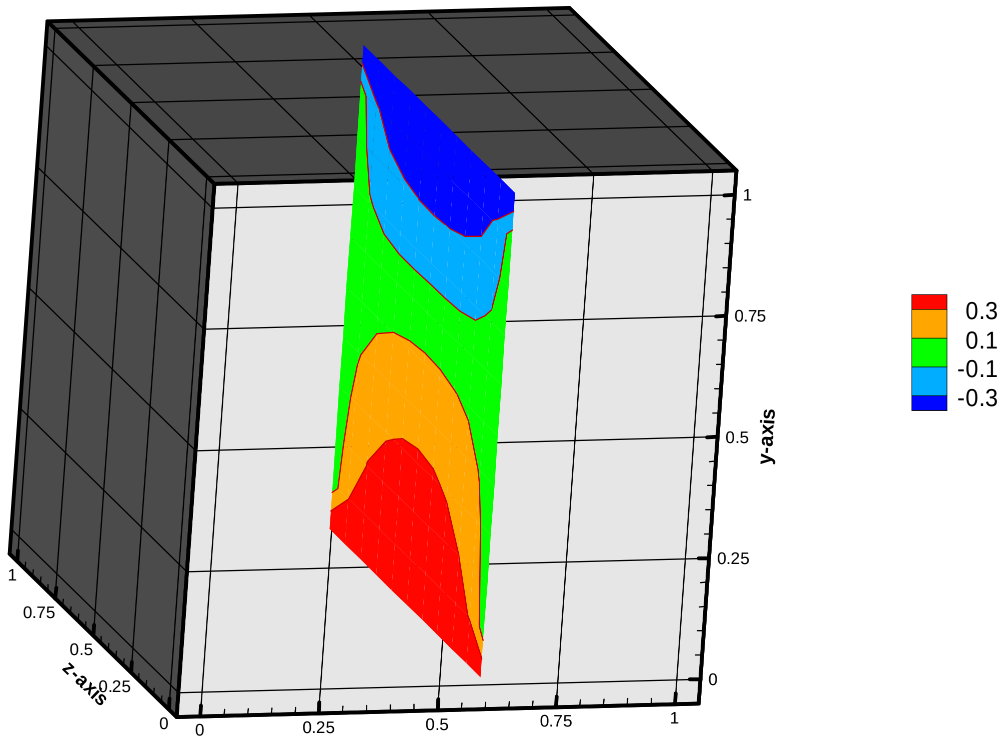
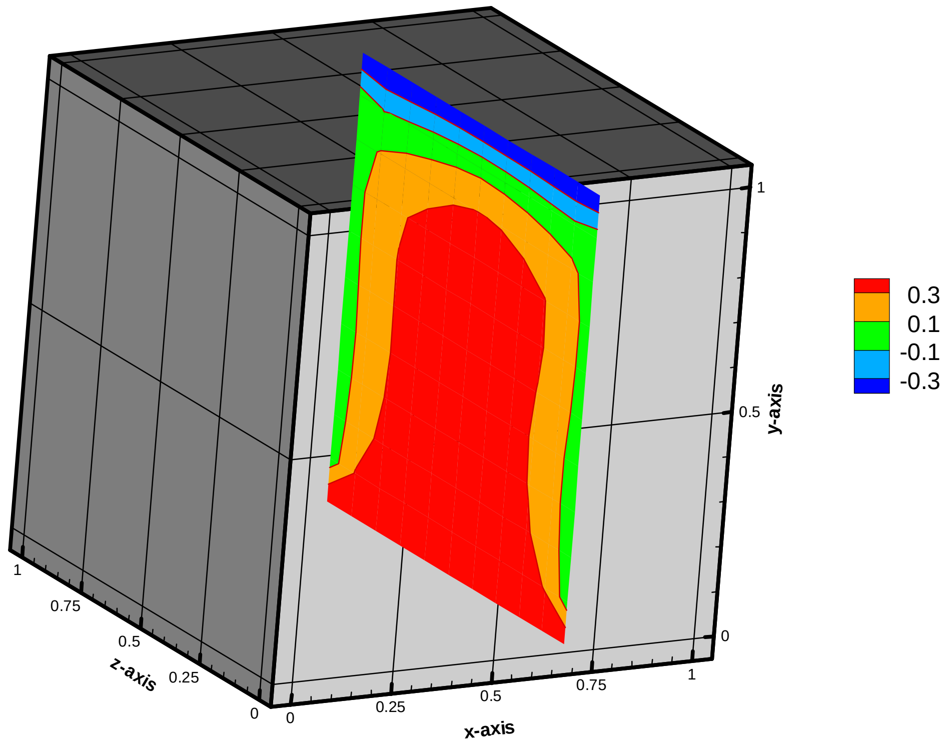


Fig.(4) Velocity vector field (a. Pr =1., b. Pr = 5)



(a) Pr = 1



(b) Pr = 5

Fig. (5) Effect of prandtl number on the thermal field

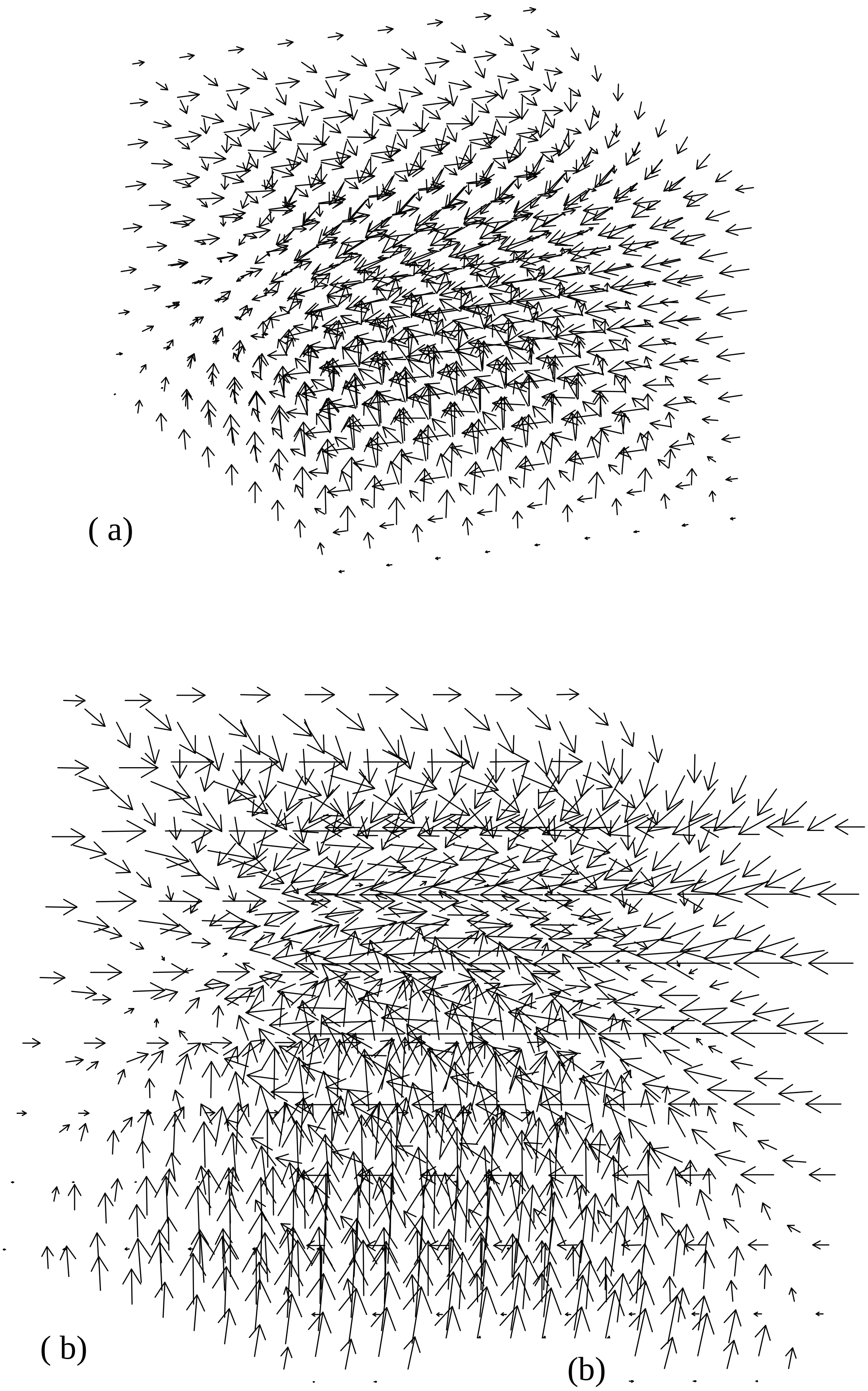
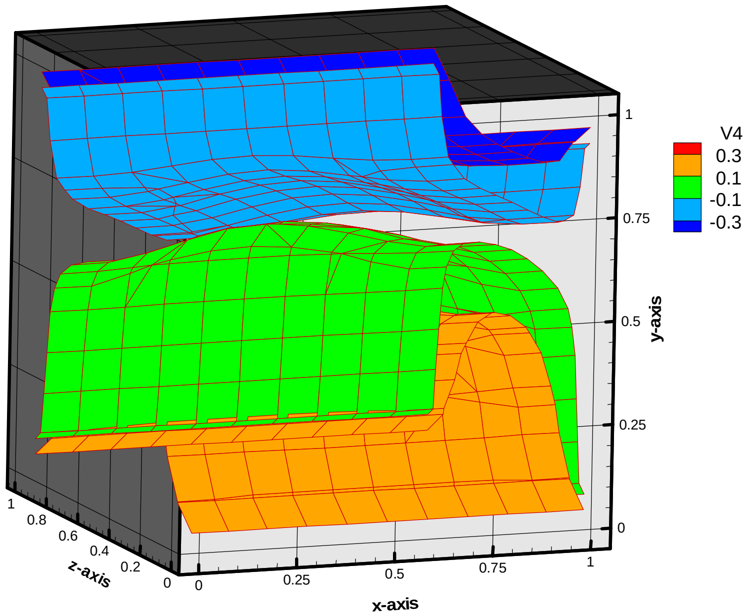


Fig.(6) Three dimensional velocity vectors (a.Gr = 10^3, b.Gr =10^6



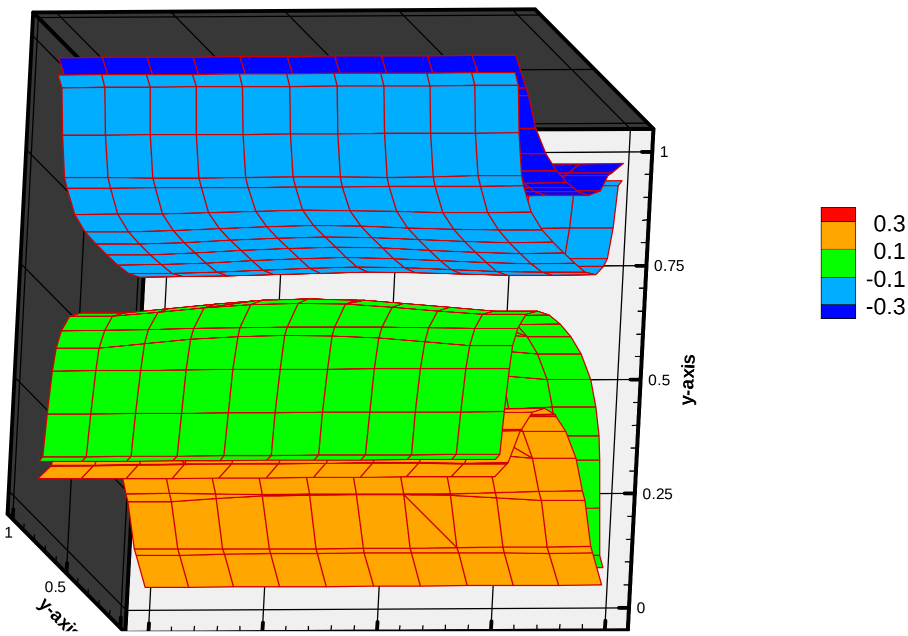


Fig.(7) Three dimensional isotherm contours (a.Gr = 10^3, b. Gr = 10^6)

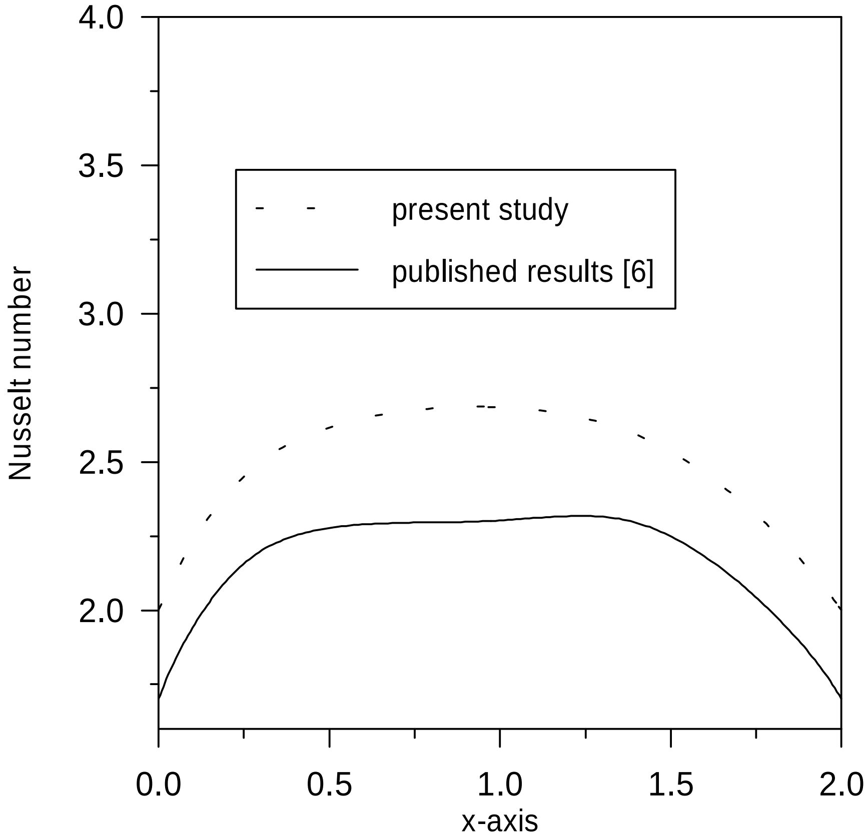


Fig.(8) Comparison between the present calculations and published results

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