The Rational Valued Characters Table of the group $Q_{2p}\times D_4$ when p is a Prime Number

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1.Abstract :

The main purpose of this paper is to find the rational valued characters table of the group $Q_{2p}\times D_4$ when p is a prime number, which is denoted by $\equiv^*(Q_{2p}\times D_4)$, where Q_{2p} is denoted to quaternion group of order 4p, such that for each positive integer p, there are two generators x and y for Q_{2p} satisfies $Q_{2p}=\{x^h\ y^k,\ 0\leq h\leq 2p-1, k=0,1\}$which has the properties $x^{2p}=y^4=I, yx^ry^{-1}=x^{-r}$ and D_4 is the dihedral group of order 8 is generate by a rotation r of order 4 and reflection s of order 2. The eight elements of D_4 can be written as: $\{I^*,r,r^2,r^3,s,sr,sr^2,sr^3\}$with properties $sr^ks = r^{-k}$, k = 0,1,2,3.

المستخلص:

الهدف الرئيسي لهذا البحث هو ايجاد جدول الشواخص النسبي للزمرة $Q_{2p}\times D_4$ عندما p عدد اولي ونرمز له بالرمز $\equiv^*(Q_{2p}\times D_4)$،عندما Q_{2p} هي الزمرة الرباعية من الرتبة 4p لكل عدد صحيح p ، لها مولدين x ، y وتحقق $Q_{2p}=\{x^h\ y^k, 0\leq h\leq 2p-1, k=0,1\}$ وتمتلك الخواص التالية $\{x^{2p}=y^4=I, yx^ry^{-1}=x^{-r}\}$ والزمرة D_4 من الرتبة 8 تتولد بواسطة دوران r من الرتبة 4 و انعكاس s من الرتبة 2 لها 8 عناصر تكتب $\{I^*,r,r^2, r^3,s,sr,sr^2,sr^3\}$.

Keywords:

Character, rational, group, Q_{2p}, D_4

2.Introduction :

Let F be a field and G be a group .A matrix representation of G is a homomorphism $T:G\rightarrow GL(n,F)$, n is called the degree of representation T. T is called a unit representation (principal) T(g)=1 for all g∈G [3].

Where $0\leq k\leq 2p-1$.The rest characters of irreducible representations T_h of degree 2 are denoted by μ_h such that: $\mu_h(x^k) = \upsilon^{hk} + \upsilon^{-hk} = e^{2\pi ihk/2p} + e^{-2\pi ihk/2p} = 2\cos(\pi hk/p)$

Then , the general form of the characters table of

Let T be a matrix representation of G over the field F.The character χ of a matrix representation T is the mapping χ: $G\rightarrow F$ defined by $\chi(g)$ = tr(T(g)), for all g ∈G. The degree of T is called th degree of χ.

Recall that the trace of an nxn matrix A is the sum of main diagonal elements.i.e $tr(A)=\sum_{i=1}^{n} a_{ii}$ [9].

Let G be a finite group ,two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes [2].

In 1980, M.S. Kirdar [6] studied "The rational valued characters table of the cyclic group C_n".

In 1995, N.R. Mahamood [7] studied"The rational valued characters of Q_{2n}.

In 1994, H. H. Abass [1] studied "The rational valued characters table of dihedral group D_n.

In this work we find the rational valued characters table of the qroup $Q_{2p}\times D_4$when p is a prime number.

3.Preliminaries

In this section we find the rational characters table of the group Q_{2p} when p is a prime number and the rational characters table of the group D_4 .

<u>3.1 The characters Table of the Quaternion Group Q_{2p} when p is a prime number</u> [7]:

There are two types of irreducible characters of Q_{2p} one of them is the characters of linear representation R_1, R_2, R_3 and R_4 which are denoted by λ_1 , λ_2 , λ_3 and λ_4 respectively as in the following table :

Q_{2p} when p is a prime number is given in the following table:

	x^k	$x^k y$
λ_1	1	1
λ_2	1	-1
λ_3	$(-1)^k$	$i\,(-1)^k$
λ_4	$(-1)^k$	$i\,(-1)^{k+1}$

Table(1)

$\mu_h(x^k y) = 0$, When $0 \le k \le 2p-1$, $1 \le h \le p-1$ and $\upsilon = e^{2\pi i/2p}$

$\equiv(Q_{2p})=$

CL_α	[I]	$[x^2]$	$[x^4]$	$\cdots$	$[x^{p-1}]$	$[x^p]$	[x]	$[x^3]$	$\cdots$	$[x^{p-2}]$	[y]	[xy]
$\lvert CL_\alpha\rvert$	1	2	2	$\cdots$	2	1	2	2	$\cdots$	2	p	p
$\lvert C_{Q_{2p}}(CL\alpha)\rvert$	4p	2p	2p	$\cdots$	2p	4p	2p	2p	$\cdots$	2p	4	4
λ_1	1	1	1	$\cdots$	1	1	1	1	$\cdots$	1	1	1
μ_2	2	$\upsilon^4+\upsilon^{2p-4}$	$\upsilon^8+\upsilon^{2p-8}$	$\cdots$	$\upsilon^{2(p-1)}+\upsilon^2$	2	$\upsilon^2+\upsilon^{2(p-1)}$	$\upsilon^6+\upsilon^{2p-6}$	$\cdots$	$\upsilon^{2(p-2)}+\upsilon^4$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
μ_{p-1}	2	$\upsilon^{2(p-1)}+\upsilon^2$	$\upsilon^{4(p-1)}+\upsilon^4$	$\cdots$	$\upsilon^{p+1}+\upsilon^{p-1}$	2	$\upsilon^{p-1}+\upsilon^{p+1}$	$\upsilon^{p-3}+\upsilon^{p+3}$	$\cdots$	$\upsilon^2+\upsilon^{2(\text{prtty } 4-1)}$	0	0
λ_2	1	1	1	$\cdots$	1	1	1	1	$\cdots$	1	-1	-1
μ_1	2	$\upsilon^2+\upsilon^{2(p-1)}$	$\upsilon^4+\upsilon^{4(p-1)}$	$\cdots$	$\upsilon^{p-1}+\upsilon^{p+1}$	-2	$\upsilon+\upsilon^{2p-1}$	$\upsilon^3+\upsilon^{2p-3}$	$\cdots$	$\upsilon^{p-2}+\upsilon^{p+2}$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
μ_{p-2}	2	$\upsilon^{2p-4}+\upsilon^4$	$\upsilon^{2p-8}+\upsilon^8$	$\cdots$	$\upsilon^2+\upsilon^{2(p-1)}$	-2	$\upsilon^{p-2}+\upsilon^{p+2}$	$\upsilon^{p-6}+\upsilon^{p+6}$	$\cdots$	$\upsilon^{(p-2)^2}+\upsilon^{p^2-4}$	0	0
λ_3	1	1	1	$\cdots$	1	-1	-1	-1	$\cdots$	-1	i	-i
λ_4	1	1	1	$\cdots$	1	-1	-1	-1	$\cdots$	-1	-i	i

Table(2)

The characters table of matrix from degree(p+3)×(p+3)where $\upsilon=e^{2\pi i/2p}$, $\upsilon^p = -1$

3.2 The Characters Table of The Dihedral Group D_4 [1] :

The characters table of dihedral group D_4 is given by

$\equiv(D_4)=$

CL_α	$[I^*]$	$[r^2]$	[r]	[s]	[sr]
$\lvert CL_\alpha\rvert$	1	1	2	2	2
$\lvert C_{D_4}(CL_\alpha)\rvert$	8	8	4	4	4
λ_1'	1	1	1	1	1
λ_2'	1	1	-1	1	-1
λ_3'	1	1	1	-1	-1
λ_4'	1	1	-1	-1	1
λ_5'	2	-2	0	0	0

Table(3)

3.3The Rational Valued Characters Table:

Definition(3.3.1) [4]:

The group generated by all characters on the field of complex numbers C is called the group of the generalized characters of G, and it is denoted by R(G).

Definition(3.3.2) [4]:

The intersection of cf(G,Z) with R(G) forms an abelian group is called the group of Z-valued generalized characters of G, denoted by $\bar{R}$(G).

Definition(3.3.3) [3]:

A rational valued character ρ of G is a character whose values are in Z,which is ρ(g)∈Z for all g∈ G.

Corollary (3.3.4) [6]:

The rational valued characters $\rho_i = \sum_{\sigma \epsilon Gal(Q(\lambda_i)/Q)} \sigma(\lambda_i)$ form a basis for $\bar{R}(G)$,whereλ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of a cyclic subgroup of G.

Proposition(3.3.5) [6]:

The number of all rational valued characters of finite G is equal to the number of all distinct Γ-classes.

Proposition(3.3.6) [7]:The rational character table of Q_{2p}, when p is a prime number is

$\equiv^*(Q_{2p})$ =

Γ-classes	[1]	$[x^2]$	$[x^p]$	[x]	[y]
ρ_1	1	1	1	1	1
ρ_2	p-1	-1	p-1	-1	0
ρ_3	1	1	1	1	-1
ρ_4	p-1	-1	1-p	1	0
ρ_5	2	2	-2	-2	0

Table(4)

Proposition (3.3.7) [1]:

The rational valued character table of the dihedral group D_4

$\equiv^*(D_4)$ =

CL_α	$[I^*]$	$[r^2]$	[r]	[s]	[sr]
$\lvert CL_\alpha \rvert$	1	1	2	2	2
$\lvert C_{D_4}(CL_\alpha) \rvert$	8	8	4	4	4
ρ_1'	1	1	1	1	1
ρ_2'	1	1	-1	1	-1
ρ_3'	1	1	1	-1	-1
ρ_4'	1	1	-1	-1	1
ρ_5'	2	-2	0	0	0

Table (5)

4 The Main Results

In this section we find the general form of the characters table of the group $Q_{2p}\times D_4$ and the rational characters table of this group when p is a prime number.

4.1 Characters Table of the Group $Q_{2p}\times D_4$ when p is a Prime Number:

According to (3.1),(3.2) each irreducible character λ_i of Q_{2p} , defines four characters $\lambda_{(i,1)}$, $\lambda_{(i,2)}$, $\lambda_{(i,3)}$, $\lambda_{(i,4)}$ and $\lambda_{(i,5)}$. such that $\lambda_{(i,1)} = \lambda_i \lambda_1'$, $\lambda_{(i,2)} = \lambda_i \lambda_2'$, $\lambda_{(i,3)} = \lambda_i \lambda_3'$, $\lambda_{(i,4)} = \lambda_i \lambda_4'$ $\lambda_{(i,5)} = \lambda_i \lambda_5'$ of $Q_{2m}\times D_4$.

Then $\equiv(Q_{2p}\times D_4) = \equiv(Q_{2p}) \otimes \equiv(D_4)$.

Then , the general form of the characters table of $Q_{2p}\times D_4$ when p is a prime number is given in the following table:

$\equiv(Q_{2p}\times D_4)$ =

CL_α	$[I,I']$	$[I,r^2]$	$[I,r]$	$[I,s]$	$[I,sr]$	$[x^2, I']$	$[x^2, r^2]$	$[x^2,r]$	$[x^2,s]$	$[x^2,sr]$	$\cdots$	$[x^{p-1}, I']$	$[x^{p-1},r^2]$	$[x^{p-1},r]$	$[x^{p-1},s]$	$[x^{p-1},sr]$	$[x^p, I']$	$[x^p, r^2]$	$[x^p, r]$	$[x^p, s]$	$[x^p, sr]$
$\lvert CL_\alpha\rvert$	1	1	2	2	2	2	2	4	4	4	$\cdots$	2	2	4	4	4	1	1	2	2	2
$\lvert C_{Q_{2p}\times D_4}(CL\alpha)\rvert$	32p	32p	16p	16p	16p	16p	16p	8p	8p	8p	$\cdots$	16p	16p	8p	8p	8p	32p	32p	16p	16p	16p
$\lambda_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	$\cdots$	1	1	1	1	1	1	1	1	1	1
$\lambda_{(1,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	$\cdots$	1	1	-1	1	-1	1	1	-1	1	-1
$\lambda_{(1,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	$\cdots$	1	1	1	-1	-1	1	1	1	-1	-1
$\lambda_{(1,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	$\cdots$	1	1	-1	-1	1	1	1	-1	-1	1
$\lambda_{(1,5)}$	2	-2	0	0	0	2	-2	0	0	0	$\cdots$	2	-2	0	0	0	2	-2	0	0	0
$\mu_{(2,1)}$	2	2	2	2	2	υ^4	υ^4	υ^4	υ^4	υ^4	$\cdots$	υ^2	υ^2	υ^2	υ^2	υ^2	2	2	2	2	2
$\mu_{(2,2)}$	2	2	-2	2	-2	υ^4	υ^4	$-\upsilon^4$	υ^4	$-\upsilon^4$	$\cdots$	υ^2	υ^2	$-\upsilon^2$	υ^2	υ^2	2	2	-2	2	-2
$\mu_{(2,3)}$	2	2	2	-2	-2	υ^4	υ^4	υ^4	$-\upsilon^4$	$-\upsilon^4$	$\cdots$	υ^2	υ^2	υ^2	$-\upsilon^2$	$-\upsilon^2$	2	2	2	-2	-2
$\mu_{(2,4)}$	2	2	-2	-2	2	υ^4	υ^4	$-\upsilon^4$	$-\upsilon^4$	υ^4	$\cdots$	υ^2	υ^2	$-\upsilon^2$		$-\upsilon^2$	2	2	-2	-2	2
$\mu_{(2,5)}$	4	-4	0	0	0	$2\upsilon^4$	$-2\upsilon^4$	0	0	0	$\cdots$	$2\upsilon^2$	$-2\upsilon^2$	0	0	0	4	-4	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mu_{((p-1),1)}$	2	2	2	2	2	υ^2	υ^2	υ^2	υ^2	υ^2	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	2	2	2	2	2
$\mu_{((p-1),2)}$	2	2	-2	2	-2	υ^2	υ^2	$-\upsilon^2$	υ^2	$-\upsilon^2$	$\cdots$	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	υ^{p-1}	$-\upsilon^{p-1}$	2	2	-2	2	-2
$\mu_{((p-1),3)}$	2	2	2	-2	-2	υ^2	υ^2	υ^2	$-\upsilon^2$	$-\upsilon^2$	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$-\upsilon^{p-1}$	2	2	2	-2	-2
$\mu_{((p-1),4)}$	2	2	-2	-2	2	υ^2	υ^2	$-\upsilon^2$	$-\upsilon^2$	υ^2	$\cdots$	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$-\upsilon^{p-1}$	υ^{p-1}	2	2	-2	-2	2
$\mu_{((p-1),5)}$	4	-4	0	0	0	$2\upsilon^2$	$-2\upsilon^2$	0	0	0	$\cdots$	$2\upsilon^{p-1}$	$-2\upsilon^{p-1}$	0	0	0	4	-4	0	0	0
$\lambda_{(2,1)}$	1	1	1	1	1	1	1	1	1	1	$\cdots$	1	1	1	1	1	1	1	1	1	1
$\lambda_{(2,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	$\cdots$	1	1	-1	1	-1	1	1	-1	1	-1
$\lambda_{(2,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	$\cdots$	1	1	1	-1	-1	1	1	1	-1	-1
$\lambda_{(2,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	$\cdots$	1	1	-1	-1	1	1	1	-1	-1	1
$\lambda_{(2,5)}$	2	-2	0	0	0	2	-2	0	0	0	$\cdots$	2	-2	0	0	0	2	-2	0	0	0
$\mu_{(1,1)}$	2	2	2	2	2	υ^2	υ^2	υ^2	υ^2	υ^2	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$
$\mu_{(1,2)}$	2	2	-2	2	-2	υ^2	υ^2	$-\upsilon^2$	υ^2	$-\upsilon^2$	$\cdots$	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	υ^{p-1}	$-\upsilon^{p-1}$	$2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$
$\mu_{(1,3)}$	2	2	2	-2	-2	υ^2	υ^2	υ^2	$-\upsilon^2$	$-\upsilon^2$	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$-\upsilon^{p-1}$	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$	$-2\upsilon^p$
$\mu_{(1,4)}$	2	2	-2	-2	2	υ^2	υ^2	$-\upsilon^2$	$-\upsilon^2$	υ^2	$\cdots$	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$-\upsilon^{p-1}$	υ^{p-1}	$2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$	$-2\upsilon^p$	$2\upsilon^p$
$\mu_{(1,5)}$	4	-4	0	0	0	$2\upsilon^2$	$-2\upsilon^2$	0	0	0	$\cdots$	$2\upsilon^{p-1}$	$-2\upsilon^{p-1}$	0	0	0	$4\upsilon^p$	$4\upsilon^p$	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mu_{((p-2),1)}$	2	2	2	2	2	υ^4	υ^4	υ^4	υ^4	υ^4	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$
$\mu_{((p-2),2)}$	2	2	-2	2	-2	υ^4	υ^4	$-\upsilon^4$	υ^4	$-\upsilon^4$	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$
$\mu_{((p-2),3)}$	2	2	2	-2	-2	υ^4	υ^4	υ^4	$-\upsilon^4$	$-\upsilon^4$	$\cdots$	υ^{p-1}	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$-\upsilon^{p-1}$	$2\upsilon^p$	$2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$	$-2\upsilon^p$
$\mu_{((p-2),4)}$	2	2	-2	-2	2	υ^4	υ^4	$-\upsilon^4$	$-\upsilon^4$	υ^4	$\cdots$	υ^{p-1}	υ^{p-1}	$-\upsilon^{p-1}$	$-\upsilon^{p-1}$	υ^{p-1}	$2\upsilon^p$	$2\upsilon^p$	$-2\upsilon^p$	$-2\upsilon^p$	$2\upsilon^p$
$\mu_{((p-2),5)}$	4	-4	0	0	0	$2\upsilon^4$	$-2\upsilon^4$	0	0	0	$\cdots$	$2\upsilon^{p-1}$	$-2\upsilon^{p-1}$	0	0	0	$4\upsilon^p$	$4\upsilon^p$	0	0	0
$\lambda_{(4,1)}$	1	1	1	1	1	1	1	1	1	1	$\cdots$	1	1	1	1	1	-1	-1	-1	-1	-1
$\lambda_{(4,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	$\cdots$	1	1	-1	1	-1	-1	-1	1	-1	1
$\lambda_{(4,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	$\cdots$	1	1	1	-1	-1	-1	-1	-1	1	1

$\lambda_{(4,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	…	1	1	-1	-1	1	-1	-1	1	1	-1
$\lambda_{(4,5)}$	2	-2	0	0	0	2	-2	0	0	0	…	2	-2	0	0	0	-2	2	0	0	0

Table(6)

The characters table of matrix from degree 5(p+3)×5(p+3) where $\upsilon = e^{2\pi i/2p}$, υ^p = -1, $\upsilon^n = e^{2\pi in/2p}$ $\equiv(Q_{2p}\times D_4)$ =

[x, I]	[x,r²]	[x,r]	[x , s]	[x , sr]	⋯	[xᵖ⁻², I]	[xᵖ⁻²,r²]	[xᵖ⁻²,r]	[xᵖ⁻²,s]	[xᵖ⁻²,sr]	[y , I]	[y,r²]	[y,r]	[y , s]	[y , sr]	[xy, ,I]	[xy,r²]	[xy,r]	[xy ,s]	[xy , sr]
2	2	4	4	4	⋯	2	2	4	4	4	p	p	2p	2p	2p	p	p	2p	2p	2p
16p	16p	8p	8p	8p	⋯	16p	16p	8p	8m	8m	32	32	16	16	16	32	32	16	16	16
1	1	1	1	1	…	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	-1	1	-1	…	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1
1	1	1	-1	-1	…	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
1	1	-1	-1	1	…	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
2	-2	0	0	0	…	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0
υ^2	υ^2	υ^2	υ^2	υ^2	…	υ^4	υ^4	υ^4	υ^4	υ^4	0	0	0	0	0	0	0	0	0	0
υ^2	υ^2	-υ^2	υ^2	-υ^2	…	υ^4	υ^4	υ^4	υ^4	-υ^4	0	0	0	0	0	0	0	0	0	0
υ^2	υ^2	υ^2	-υ^2	-υ^2	…	υ^4	υ^4	υ^4	-υ^4	-υ^4	0	0	0	0	0	0	0	0	0	0
υ^2	υ^2	-υ^2	-υ^2	υ^2	…	υ^4	υ^4	-υ^4	-υ^4	-υ^4	0	0	0	0	0	0	0	0	0	0
$2\upsilon^2$	-$2\upsilon^2$	0	0	0	…	$2\upsilon^4$	-$2\upsilon^4$	0	0	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	υ^{p-1}	…	υ^2	υ^2	υ^2	υ^2	υ^2	0	0	0	0	0	0	0	0	0	0
υ^{p-1}	υ^{p-1}	-υ^{p-1}	υ^{p-1}	-υ^{p-1}	…	υ^2	υ^2	υ^2	υ^2	-υ^2	0	0	0	0	0	0	0	0	0	0
υ^{p-1}	υ^{p-1}	υ^{p-1}	-υ^{p-1}	-υ^{p-1}	…	υ^2	υ^2	υ^2	-υ^2	-υ^2	0	0	0	0	0	0	0	0	0	0
υ^{p-1}	υ^{p-1}	-υ^{p-1}	- υ^{p-1}	υ^{p-1}	…	υ^2	υ^2	-υ^2	-υ^2	υ^2	0	0	0	0	0	0	0	0	0	0
$2\upsilon^{p-1}$	-2 υ^{p-1}	0	0	0	…	$2\upsilon^2$	-$2\upsilon^2$	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	…	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	-1	1	-1	…	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	-1	1
1	1	1	-1	-1	…	1	1	1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1
1	1	-1	-1	1	…	1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
2	-2	0	0	0	…	2	-2	0	0	0	-2	2	0	0	0	-2	2	0	0	0
υ	υ	υ	υ	υ	…	υ^{p-2}	υ^{p-2}	υ^{p-2}	υ^{p-1}	υ^{p-1}	0	0	0	0	0	0	0	0	0	0
υ	υ	- υ	υ	-υ	…	υ^{p-1}	υ^{p-1}	-υ^{p-1}	υ^{p-1}	-υ^{p-1}	0	0	0	0	0	0	0	0	0	0
υ	υ	υ	-υ	- υ	…	υ^{p-1}	υ^{p-1}	υ^{p-1}	-υ^{p-1}	-υ^{p-1}	0	0	0	0	0	0	0	0	0	0
υ	υ	-υ	- υ	υ	…	υ^{p-1}	υ^{p-1}	-υ^{p-1}	-υ^{p-1}	υ^{p-1}	0	0	0	0	0	0	0	0	0	0
2 υ	-2 υ	0	0	0	…	2 υ^{p-1}	-$2\upsilon^{p-1}$	0	0	0	0	0	0	0	0	0	0	0	0	0

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
υ^{p-2}	υ^{p-2}	υ^{p-2}	υ^{p-2}	υ^{p-2}		$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	0	0	0	0	0	0	0	0	0	0
υ^{p-2}	υ^{p-2}	$-\upsilon^{p-2}$	υ^{p-2}	$-\upsilon^{p-2}$		$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$-\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$-\upsilon^{(p-2)^2}$	0	0	0	0	0	0	0	0	0	0
υ^{p-2}	υ^{p-2}	υ^{p-2}	υ^{p-2}	$-\upsilon^{p-2}$		$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$-\upsilon^{(p-2)^2}$	$-\upsilon^{(p-2)^2}$	0	0	0	0	0	0	0	0	0	0
υ^{p-2}	υ^{p-2}	$-\upsilon^{p-2}$	υ^{p-2}	υ^{p-2}		$\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	$-\upsilon^{(p-2)^2}$	$-\upsilon^{(p-2)^2}$	$\upsilon^{(p-2)^2}$	0	0	0	0	0	0	0	0	0	0
$2\upsilon^{p-2}$	$-2\upsilon^{p-2}$	0	0	0		$2\upsilon^{(p-2)^2}$	$-2\upsilon^{(p-2)^2}$	0	0	0	0	0	0	0	0	0	0	0	0	0

The characters table of matrix from degree 5(p+3)×5(p+3) where $\upsilon = e^{2\pi i/2p}$, $\upsilon^p = -1$, $\upsilon^n = e^{2\pi in/2p}$ Table(6)

$\lambda_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	…	1	1	1	1	1	-1	-1	-1	-1	-1
$\lambda_{(3,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	…	1	1	-1	1	-1	-1	-1	1	-1	1
$\lambda_{(3,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	…	1	1	1	-1	-1	-1	-1	-1	1	1
$\lambda_{(3,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	…	1	1	-1	-1	1	-1	-1	1	1	-1
$\lambda_{(3,5)}$	2	-2	0	0	0	2	-2	0	0	0	…	2	-2	0	0	0	-2	2	0	0	0
$\lambda_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	…	1	1	1	1	1	-1	-1	-1	-1	-1
$\lambda_{(3,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	…	1	1	-1	1	-1	-1	-1	1	-1	1
$\lambda_{(3,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	…	1	1	1	-1	-1	-1	-1	-1	1	1
$\lambda_{(3,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	…	1	1	-1	-1	1	-1	-1	1	1	-1
$\lambda_{(3,5)}$	2	-2	0	0	0	2	-2	0	0	0	…	2	-2	0	0	0	-2	2	0	0	0

-1	-1	-1	-1	-1	…	-1	-1	-1	-1	-1	i	i	i	i	i	-i	-i	-i	-i	-i
-1	-1	1	-1	1	…	-1	-1	1	-1	1	i	i	-i	i	-i	-i	-i	i	-i	i
-1	-1	-1	1	1	…	-1	-1	-1	1	1	i	i	i	-i	-i	-i	-i	-i	i	i
-1	-1	1	1	-1	…	-1	-1	1	1	-1	i	i	-i	-i	i	-i	-i	i	i	-i
-2	2	0	0	0	…	-2	2	0	0	0	2i	-2i	0	0	0	-2i	2i	0	0	0
-1	-1	-1	-1	-1	…	-1	-1	-1	-1	-1	-i	-i	-i	-i	-i	i	i	i	i	i
-1	-1	1	-1	1	…	-1	-1	1	-1	1	-i	-i	i	-i	i	i	i	-i	i	-i
-1	-1	-1	1	1	…	-1	-1	-1	1	1	-i	-i	-i	i	i	i	i	i	-i	-i
-1	-1	1	1	-1	…	-1	-1	1	1	-1	-i	-i	i	i	-i	i	i	-i	-i	i
-2	2	0	0	0	…	-2	2	0	0	0	-2i	2i	0	0	0	2i	-2i	0	0	0

Table(6)

4.2 The rational valued characters table of the group $Q_{2p}\times D_4$ when p is a prime number:

Theorem(4.2.1):

$\equiv^*(Q_{2p}\times D_4) = \equiv^*(Q_{2p}) \otimes \equiv^*(D_4)$

Proof :-

Since $D_4 = \{I^*, r, r^2, r^3, s, sr, sr^2, sr^3\}$

Each element in $Q_{2p}\times D_4$ are $g_{qd} = (g_q,g_d)$,such that $\forall\ g_q\in Q_{2p}$, $g_d\in D_4$.

And each irreducible character $\lambda_{(i,j)}$ of $Q_{2p}\times D_4$ is can be written as follows

$\lambda_{(i,j)} = \lambda_i . \lambda_j'$

Where λ_i is an irreducible character of Q_{2p} and λ_j' is the irreducible character of D_4 ,then

$$\lambda_{(i,j)}(g_{qd}) = \begin{cases} \lambda i(g_q) \text{if } j = 1 \text{ and } d \in D_4 \\ \lambda i(g_q) \text{if } j = 2 \text{ and } d \in \{I^*, r^2, s, sr^2\} \\ -\lambda i(g_q) \text{if } j = 2 \text{ and } d \in \{r, r^3, sr, sr^3\} \\ \lambda i(g_q) \text{if } j = 3 \text{ and } d \in \{I^*, r, r^2, r^3\} \\ -\lambda i(g_q) \text{if } j = 3 \text{ and } d \in \{s, sr, sr^2, sr^3\} \\ \lambda i(g_q) \text{if } j = 4 \text{ and } d \in \{I^*, r^2, sr, sr^3\} \\ -\lambda i(g_q) \text{if } j = 4 \text{ and } d \in \{r, r^3, s, sr^2\} \\ 2\lambda i(g_q) \text{if } j = 5 \text{ and } d = I^* \\ -2\lambda i(g_q) \text{if } j = 5 \text{ and } d = r^2 \\ 0 \quad \text{if } j = 5 \text{ and } d \in \{r, r^3, s, sr, sr^2\} \end{cases}$$

From proposition (3.3.4)

$\rho_{(i,j)} = \sum_{\sigma\epsilon Gal(\,Q(\lambda_{(i,j)})/Q)} \sigma(\lambda_{(i,j)})$

where $\rho_{(i,j)}$ is the rational valued character of $Q_{2p}\times D_4$

Then , $\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_{(i,j)}(g_{qd}))/Q)} \sigma(\lambda_{(i,j)}(g_{qd}))$

(I) If j=1 and $d \in D_4$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)) = \rho_i(g_q).1 = \rho_i(g_q).\,\rho'_j(g'_d)$

where ρ_i is the rational valued character of Q_{2p} .

(II) (a) if $j = 2$ and $d \in \{I^*, r^2, s, sr^2\}$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)) = \rho_i(g_q).1 = \rho_i(g_q).\,\rho'_j(g'_d)$.

(b) if $j = 2$ and $d \in \{r, r^3, sr, sr^3\}$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(-\lambda_i(g_q)) = -\sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q))$

$= \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)).-1= \rho_i(g_q).-1= \rho_i(g_q).\,\rho'_j(g'_d)$.

(III) (a) if $j = 3$ and $d \in \{I^*, r, r^2, r^3\}$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)) = \rho_i(g_q).1 = \rho_i(g_q).\,\rho'_j(g'_d)$.

(b) if $j = 3$ and $d \in \{s, sr, sr^2, sr^3\}$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(-\lambda_i(g_q)) = -\sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q))$

$= \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)).-1= \rho_i(g_q).-1= \rho_i(g_q).\,\rho'_j(g'_d)$.

(IV) (a) if $j = 4$ and $d \in \{I^*, r^2, sr, sr^3\}$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)) = \rho_i(g_q).1 = \rho_i(g_q).\,\rho'_j(g'_d)$.

(b) if $j = 4$ and $d \in \{r, r^3, s, sr^2\}$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(-\lambda_i(g_q)) = -\sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q))$

$= \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)).-1= \rho_i(g_q).-1= \rho_i(g_q).\,\rho'_j(g'_d)$.

(V) (a) if $j = 5$ and $d = I^*$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(2\lambda_i(g_q)) = 2\sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q))$

$= \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)).2= \rho_i(g_q).2= \rho_i(g_q).\,\rho'_j(g'_d)$.

(b) if $j = 5$ and $d = r^2$

$\rho_{(i,j)}(g_{qd}) = \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(-2\lambda_i(g_q)) = -2\sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q))$

$= \sum_{\sigma\epsilon Gal(\,Q(\lambda_i(g_q))/Q)} \sigma(\lambda_i(g_q)).-2= \rho_i(g_q).-2= \rho_i(g_q).\,\rho'_j(g'_d)$.

(c) $j = 5$ and $d \in \{r, r^3, s, sr, sr^2\}$

$\rho_{(i,j)}(g_{qd}) =$
$\sum_{\sigma\epsilon Gal(\,Q(\lambda_i\,(g_q))/Q)} \sigma(0.\lambda_i\,(g_q)) =$
$0.\sum_{\sigma\epsilon Gal(\,Q(\lambda_i\,(g_q))/Q)} \sigma(\lambda_i\,(g_q))$
$= \sum_{\sigma\epsilon Gal(\,Q(\lambda_i\,(g_q))/Q)} \sigma(\lambda_i\,(g_q)).0= 0=$
$\rho_i(g_q).\,\rho'_j(g'_d).$

From [I],[II],[III],[IV] and [V] we have

$$\rho_{(i,j)} = \rho_i\,.\,\rho'_j$$

Then $\equiv^*(Q_{2p}\times D_4) = \equiv^*(Q_{2p}) \otimes \equiv^*(D_4)$.

Example(4.2.2):

To find the rational valued characters table of $Q_{14}\times D_4$.
By (3.3.6) and proposition (3.3.7), we have

$\equiv^*(Q_{14}) =$

Γ-classes	[1]	[x^2]	[x^7]	[x]	[y]
ρ_1	1	1	1	1	1
ρ_2	6	-1	6	-1	0
ρ_3	1	1	1	1	-1
ρ_4	6	-1	-6	1	0
ρ_5	2	2	-2	-2	0

and

$\equiv^* D_4 =$

CL_α	[I^*]	[r^2]	[r]	[s]	[sr]
$\lvert CL_\alpha\rvert$	1	1	2	2	2
$\lvert C_{D_4}(CL_\alpha)\rvert$	8	8	4	4	4
ρ'_1	1	1	1	1	1
ρ'_2	1	1	-1	1	-1
ρ'_3	1	1	1	-1	-1
ρ'_4	1	1	-1	-1	1
ρ'_5	2	-2	0	0	0

and by theorem (4.2.1)

$\equiv^*(Q_{14}\times D_4) = \equiv^*(Q_{14}) \otimes \equiv^*(D_4)$.

We get

	[I, I*]	[I, r^2]	[I, r]	[I, s]	[I, sr]	[x^2, I*]	[x^2 , r^2]	[x^2,r]	[x^2, s]	[x^2, sr]	^{7}, I*]	[x^7,r^2]	[x^7, r]	[x^7, s]	[x^7, sr]	[x, I*]	[x,r^2]	[x, r]	[x, s]	[x,sr]	[y, I*]	[y,r^2]	[y, r]	[y , s]	[y, sr]
$\|CL_\alpha\|$	1	1	2	2	2	2	2	4	4	4	1	1	2	2	2	2	2	4	4	4	14	14	28	28	28
$\|C_{Q_{14}\times D4}(CL_\alpha)\|$	224	224	112	112	112	112	112	56	56	56	224	224	112	112	112	112	112	56	56	56	16	16	8	8	8
$\rho_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\rho_{(1,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1
$\rho_{(1,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
$\rho_{(1,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
$\rho_{(1,5)}$	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0
$\rho_{(2,1)}$	6	6	6	6	6	-1	-1	-1	-1	-1	6	6	6	6	6	-1	-1	-1	-1	-1	0	0	0	0	0
$\rho_{(2,2)}$	6	6	-6	6	-6	-1	-1	1	-1	1	6	6	-6	6	-6	6	6	-6	6	-6	0	0	0	0	0
$\rho_{(2,3)}$	6	6	6	-6	-6	-1	-1	-1	1	1	6	6	6	-6	-6	6	6	6	-6	-6	0	0	0	0	0
$\rho_{(2,4)}$	6	6	-6	-6	6	-1	-1	1	1	-1	6	6	-6	-6	6	6	6	-6	-6	6	0	0	0	0	0
$\rho_{(2,5)}$	12	-12	0	0	0	-2	2	0	0	0	12	-12	0	0	0	12	-12	0	0	0	0	0	0	0	0
$\rho_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
$\rho_{(3,2)}$	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1	-1	1	-1	1
$\rho_{(3,3)}$	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	-1	1	1
$\rho_{(3,4)}$	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	-1	-1	1	1	-1
$\rho_{(3,5)}$	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	-2	2	0	0	0
$\rho_{(4,1)}$	6	6	6	6	6	-1	-1	-1	-1	-1	-6	-6	-6	-6	-6	1	1	1	1	1	0	0	0	0	0
$\rho_{(4,2)}$	6	6	-6	6	-6	-1	-1	1	-1	1	-6	-6	6	-6	6	1	1	-1	1	-1	0	0	0	0	0
$\rho_{(4,3)}$	6	6	6	-6	-6	-1	-1	-1	1	1	-6	-6	-6	6	6	1	1	1	-1	-1	0	0	0	0	0
$\rho_{(4,4)}$	6	6	-6	-6	6	-1	-1	1	1	-1	-6	-6	6	6	-6	1	1	-1	-1	1	0	0	0	0	0
$\rho_{(4,5)}$	12	-12	0	0	0	-2	2	0	0	0	-12	12	0	0	0	2	-2	0	0	0	0	0	0	0	0
$\rho_{(5,1)}$	2	2	2	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0
$\rho_{(5,2)}$	2	2	-2	2	-2	2	2	-2	2	-2	-2	-2	2	-2	2	-2	-2	2	-2	2	0	0	0	0	0
$\rho_{(5,3)}$	2	2	2	-2	-2	2	2	2	-2	-2	-2	-2	-2	2	2	-2	-2	-2	2	2	0	0	0	0	0
$\rho_{(5,4)}$	2	2	-2	-2	2	2	2	-2	-2	2	-2	-2	2	2	-2	-2	-2	2	2	-2	0	0	0	0	0
$\rho_{(5,5)}$	4	-4	0	0	0	4	-4	0	0	0	-4	4	0	0	0	-4	4	0	0	0	0	0	0	0	0

Table(7)

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