Comparing Different Fuzzy Reliability Estimation of Exponential Rayleigh Probability Distribution

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Abstract

This paper deals with constructing mixed probability distribution, from mixing Exponential (2α) and Rayleigh with (β), after the mixed P.D.F is found and also the mixed C D F, and mixed Reliability are Exponential distribution, and Rayleigh distribution found, we work on estimating its two parameters (α, β) by three different methods, which are maximum likelihood and Moments and percentiles method.

Then the fuzzy Reliability estimators are compared and the results of comparison is explained in special tables. The comparison is done using simulation procedure, and all the results are explained in tables.

Keywords: Maximum Likelihood Estimator (MLE), Moments Method (MOM), Ordinary Lest Square method (OLS), Exponential distribution, Rayleigh distribution, ki = 0.3 ,0.6 (fuzzy factor), n= 25 ,50 , and 75 (sample size).

Introduction:

The Rayleigh distribution was introduced by Lord Rayleigh (1880) in connection with problems in the field of acoustics. Reliability analysis is a tool kit of statistical procedures for analyzing time-to-failure data. Usually, reliability analysis is carried out using the classical or Bayesian statistical analysis of parametric reliability models. Reliability analysis datasets are represented with a single univariate or multivariate statistical distributions such as Exponential, Rayleigh, inverse Weibull, Pareto and Burr distributions (2003)(2011).

Due to the expansion in the field of probability distribution in all type, were it is unique or mixed or compound, the research er's introduce many expellants research in Reliability and hazard Rate function, In (2017)(A.C. Mkolesia and M.Y. Shatalov) attempting to estimate parameters of a Rayleigh distribution numerical iterative methods or routines are frequently employed. In this work, an exact method on the constant minimization of the goal function is proposed.

An application for the Rayleigh distribution is the analysis of wind velocity (2016). In (2017) Rama Shanker and etl. discuss three parameters Lindley, and studied their different estimators of parameter and reliability function, all mathematical and statistical properties were discussed. The aim of this research is to build mixed failure to time model from exponential and Gamma distribution where this model is necessary when the observation of time to failure cannot be represented by single probability distribution, which is recognized by scatter diagram, so two types of distributions need to be mixed which are exponential and Gamma using certain proportion as weights function, and the sum of this weight equals one.

The mixture distributions have vital role in practical applications for research that deal with economics, medicine, agriculture, life testing, and reliability for classical reliability theory. There are several methods and models in which the parameters assume precise, but in real world application, due to vague and randomness affect the life times distribution. In (2017) two researchers M.A. Hussian and Esssam A. discussed fuzzy exponential distribution and discussed how to compute reliability in case of stress- strength model and ranked set sampling. The Rayleigh distribution has a number of applications in settings where magnitudes of normal variables are important (2009).

In this researcher we continue the work in the field of fuzzy Reliability and work on comparing three deferent estimators of fuzzy Reliability function, and explain the results of comparison of simulation by tables.

Theoretical Parts:

Let:

$$f(x) = (2\alpha + \beta x)\, e^{-\left(2\alpha x + \frac{\beta}{2}x^2\right)} \qquad x > 0, \beta > 0, \alpha > 0 \qquad \dots\dots(1)$$

be Exponential distribution with scale parameter (β).

And the corresponding cumulative distribution function (C D F) is

$$G(x) = 1 - e^{-\left(2\alpha x + \frac{\beta}{2}x^2\right)} \qquad x > 0, \beta > 0\ , \alpha > 0 \qquad \dots\dots(2)$$

While Reliability function is:

$$R_x(x) = e^{-\left(2\alpha x + \frac{\beta}{2}x^2\right)} \qquad x > 0, \beta > 0, \alpha > 0 \qquad \dots\dots(3)$$

$$h\left(x^{(x)}\right) = 2\alpha + \beta x \qquad \alpha > 0, \beta > 0\ , x > 0 \qquad \dots\dots(4)$$

The two parameters (α, β) are estimated by Maximum Likelihood Estimator (MLE), Moments Method (MOM), and Ordinary Lest Square method (OLS).

$$\hat{R}_x(\ ki, \alpha, \beta i) = e^{-\left(2\hat{\alpha} ki\ xi + \frac{\hat{\beta}}{2}(ki\ xi)^2\right)} \qquad xi > 0, \beta > 0, \alpha > 0 \qquad \dots\dots(5)$$

After the two parameter (α, β) are estimated then $\hat{R}_x(ki\ xi)$ are compared using given values of (ki) and (α, β). There are different values for $(\alpha = 1.5\ ,3)$, $(\beta = 2.5\ ,4)$ and $(\hat{k}i = 0.3, 0.6)$.

Estimation Methods:

The two parameters (α, β) are estimated by Maximum Likelihood and Moments Method, and Ordinary Lest Square Method.

1. Estimation by Maximum Likelihood (MLE):

Let $x_1\ , x_2\ , x_3, \dots \dots\dots, x_n$ be ar.s from p.d.f in equation (6), then:

$$L = \prod_{i=1}^{n} f(xi, \alpha, \beta)$$

$$= \prod_{i=1}^{n} (2\alpha + \beta xi)\ e^{-\sum_{i=1}^{n}\left(2\alpha xi + \frac{\beta}{2}xi^2\right)} \qquad \dots\dots(6)$$

$$log\ log\ L = \sum_{i=1}^{n} log\ log\ (2\alpha + \beta xi) - \sum_{i=1}^{n}\left(2\alpha xi + \frac{\beta}{2}xi^2\right)$$ Then:

$$\frac{\partial\ log\ log\ L}{\partial \alpha} = \sum_{i=1}^{n} \frac{2}{(2\alpha + \beta xi)} - 2\sum_{i=1}^{n} xi$$

$$put\ \frac{\partial\ log\ log\ L}{\partial\alpha}=0\ \ \rightarrow\ \ n\ \underline{x}=\sum_{i=1}^{n}\ (2\alpha+\beta xi)^{-1}\qquad \ldots\ldots(7)$$

Is the MLE for α and from:

$$\frac{\partial\ log\ log\ L}{\partial\beta}=\sum_{i=1}^{n}\ \frac{xi}{(2\alpha+\beta xi)}-\sum_{i=1}^{n}\ \frac{xi^2}{2}=0$$

$$\sum_{i=1}^{n}\ xi\left(2\hat{\alpha}+\hat{\beta}xi\right)^{-1}=\frac{1}{2}\sum_{i=1}^{n}\ xi^2\qquad \ldots\ldots(8)$$

Were equations (7) and (8) are solved numerically to find $\hat{\alpha}_{MLE}, \hat{\beta}_{MLE}$

2. Estimation by Moment Method (MOM):

The Moments estimators of two parameters (β), of mixed (Exp-Rayleigh) obtained from solving equations:

$\acute{\mu}_r=E(x^r)$ with

$$E(x^r)=\frac{\sum_{i=1}^{n}\ xi^r}{n}, for\ r=1,2$$

Then from

$$E(x)=\frac{\sum_{i=1}^{n}\ xi}{n}$$

$$E(x^2)=\frac{\sum_{i=1}^{n}\ xi^2}{n}$$

Since we have two parameters (β) and solving resulted equation:

$$\frac{\sum_{i=1}^{n}\ xi}{n}=\frac{\Gamma\left(\frac{1}{\beta}+1\right)\ \Gamma(\alpha+1)}{\Gamma\left(\frac{1}{\beta}+1+\alpha\right)}\qquad \ldots\ldots(9)$$

$$\frac{\sum_{i=1}^{n}\ xi^2}{n}=\frac{\Gamma\left(\frac{2}{\beta}+1\right)\Gamma(\alpha+1)}{\Gamma\left(\frac{1}{\beta}+1+\alpha\right)}\qquad \ldots.(10)$$

These two equations solved numerically to find $(\hat{\alpha}_{MOM,} \hat{\beta}_{MOM})$ and $\hat{\alpha}\geq 1$ and $\hat{\beta}\geq 1$ integer and solving equation (9) and (10) numerically to obtain $\left(\hat{\alpha}_{MOM\,,}\hat{\beta}_{MOM}\right)$.

3. Estimation by Ordinary Least Square Method (OLS):

The estimators($\hat{\alpha}_{ols,} \hat{\beta}_{ols}$)by this method obtained from minimizing total sum square of the difference between sample value (i) and the expected value E(pi).

$$\sum_{i=}^{n}\ ei^2=\sum_{i=1}^{n}\ [pi-E(pi)]^2$$

$$= \sum_{i=1}^{n} \left[1 - e^{-\left(2\alpha x+\frac{\beta}{2}x^2\right)} - \frac{i}{n+1}\right]^2 \qquad \dots\dots(11)$$

in equation (11), then deriving $\frac{\partial T}{\partial \alpha}$ $, and$ $\frac{\partial T}{\partial \beta}$ and solving ($\frac{\partial T}{\partial \alpha} = , \frac{\partial T}{\partial \beta} = 0$) by iterative procedure we can obtain ($\hat{\alpha}_{ols\,,}\ \hat{\beta}_{ols}$).

$$\frac{\partial \sum_{i=}^{n} ei^2}{\partial \alpha} = 2\sum_{i=1}^{n} \left[1 - e^{-\left(2\alpha x+\frac{\beta}{2}x^2\right)} - \frac{i}{n+1}\right]\left[-\left(2\alpha x + \frac{\beta}{2}x^2\right) e^{-\left(2\alpha x+\frac{\beta}{2}x^2\right)} - xi\right]$$

$$= \sum_{i=1}^{n} xi\, e^{-2\hat{\alpha} xi} - \sum_{i=1}^{n} xi\, e^{-4\hat{\alpha} xi} - \sum_{i=1}^{n} \frac{i}{n+1}\, xi\, e^{-2\hat{\alpha} xi} \quad \dots.(12)$$

And:

$$\frac{\partial \sum_{i=}^{n} ei^2}{\partial \beta} = 2\sum_{i=1}^{n} \left[1 - e^{-\left(2\alpha xi+\frac{\beta}{2}xi^2\right)} - \frac{i}{n+1}\right]\left[-\left(2\alpha x + \frac{\beta}{2}x^2\right) e^{-\frac{\beta}{2}xi^2} - \frac{xi^2}{2}\right] \quad \dots(13)$$

Then from $\frac{\partial T}{\partial \alpha} = 0$, $\frac{\partial T}{\partial \beta} = 0$, We find $\hat{\alpha}_{ols}$,$\hat{\beta}_{ols}$

Simulation Procedures:

The fuzzy Reliability function of mixed Exponential distribution, and Rayleigh distribution are compared by three different method which are Maximum Likelihood and second by Moments third one is Ordinary Lest Square.

$Since \quad G(xi) = 1 - e^{-\left(2\alpha xi+\frac{\beta}{2}x^2\right)}$

$Zi = 1 - e^{-\left(2\alpha xi+\frac{\beta}{2}x^2\right)}$

Let $e^{-\left(2\alpha xi+\frac{\beta}{2}xi^2\right)} = 1 - Zi$

$-\left(2\alpha xi + \frac{\beta}{2}xi^2\right) = log\ log\ (1 - zi)$

$2\alpha xi + \frac{\beta}{2}xi^2 = log(1 - zi)^{-1}$

$2\alpha xi + \frac{\beta}{2}xi^2 = Ui$

Which are:

$$2\alpha \sum_{i=1}^{n} xi + \frac{\beta}{2}\sum_{i=1}^{n} xi^2 = \sum_{i=1}^{n} ui$$

$$2\alpha\sum_{i=1}^{n} xi+\frac{\beta}{2}\sum_{i=1}^{n} xi^3=\sum_{i=1}^{n} uixi$$

And the method of estimation are Maximum Likelihood and second by Moments Method and third by Ordinary Lest Square.

Here we have two parameters (α, β) as well as (Ki) fuzzy factor.

The sample size taken n= (25, 50, 75) and the results are compared by mean square error.

There are different values for $(\alpha = 0.5\ ,1\ )$, $(\beta = 0.5\ ,1.2)$ and $(\hat{k}i = 0.3\ ,0.6)$.

And each experiment is repeated $R = 500$

All the results of comparisons of fuzzy reliability functions, are explained in different Tables (1, 2, 3, 4, 5, 6, 7, 8) and according to different sets of initial values. And different combination parameters $(\ ki, \alpha, \beta)$, and the best fuzzy reliability estimation is highest one, and all the results in tables.

***Table* 1. *Estimator of Fuzzy Reliability when* $\beta = 1.5\ , \alpha = 1.5, \tilde{k} = 0.3$**

n	t_i	Real Ri	$\hat{R}_{Mle}$	$\hat{R}_{mom}$	$\hat{R}_{Ols}$	Best
25	1.5	0.4889	0.4886	0.3396	0.3384	Mle
	2.5	0.5032	0.4577	0.3974	0.3976	Mom
	3.5	0.5082	0.5029	0.4345	0.4361	Mom
	4.5	0.5324	0.5319	0.4806	0.4802	Mom
	5.5	0.5614	0.5536	0.4961	0.4952	Mle
	6.5	0.5662	0.5702	0.5008	0.5068	Mle
	7.5	0.5782	0.5778	0.4242	0.5136	Mle
	8.5	0.5899	0.5853	0.5308	0.5242	Mle
	9.5	0.6032	0.5864	0.5306	0.5963	Ols
	10.5	0.6208	0.6019	0.5301	0.5308	Ols
50	1.5	0.4889	0.4006	0.3262	0.3116	Mle
	2.5	0.5032	0.4143	0.3844	0.3749	Mle
	3.5	0.5082	0.4743	0.4266	0.4166	Mle
	4.5	0.5324	0.5202	0.4538	0.5452	Ols
	5.5	0.5614	0.5485	0.4726	0.4657	Ols
	6.5	0.5662	0.5664	0.4891	0.4812	Ols
	7.5	0.5782	0.5709	0.5012	0.4936	Ols
	8.5	0.5899	0.5873	0.5109	0.5123	Ols
	9.5	0.6032	0.5993	0.5123	0.5224	Ols
	10.5	0.6208	0.6103	0.5266	0.5148	Ols
75	1.5	0.4889	0.4889	0.3263	0.3102	Ols
	2.5	0.5032	0.5032	0.3815	0.3802	Ols
	3.5	0.5082	0.5082	0.4214	0.4212	Ols
	4.5	0.5324	0.5324	0.4493	0.4483	Ols
	5.5	0.5614	0.5614	0.4687	0.4699	Ols

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
	6.5	0.5662	0.5662	0.5856	0.4852	Mom
	7.5	0.5782	0.5782	0.5980	0.4873	Mom
	8.5	0.5899	0.5899	0.5981	0.4982	Mom
	9.5	0.6032	0.5032	0.5166	0.5080	Mom
	10.5	0.6208	0.6208	0.5221	0.5162	Mom

$Table\ 2.\ Estimator\ of\ Fuzzy\ Reliability\ when\ \boldsymbol{\beta = 1.5}\ , \boldsymbol{\alpha = 1.5}, \boldsymbol{\widetilde{k} = 0.6}$

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
25	1.5	0.2668	0.2667	0.3126	0.2955	Mom
	2.5	0.3173	0.3504	0.3507	0.3342	Mle
	3.5	0.3453	0.3768	0.3769	0.3642	Mom
	4.5	0.3667	0.3861	0.4108	0.3802	Mom
	5.5	0.3826	0.4033	0.4226	0.4002	Mom
	6.5	0.3947	0.4266	0.4319	0.4115	Mom
	7.5	0.4045	0.4338	0.4296	0.4303	Mle
	8.5	0.4132	0.4409	0.4463	0.4375	Mom
	9.5	0.4262	0.4467	0.4402	0.4426	Mle
	10.5	0.4468	0.4458	0.4522	0.4006	Mle
50	1.5	0.2668	0.2867	0.2982	0.4107	Ols
	2.5	0.3173	0.3280	0.3368	0.4188	Ols
	3.5	0.3453	0.3543	0.3634	0.4257	Ols
	4.5	0.3667	0.3768	0.3829	0.4288	Ols
	5.5	0.3826	0.3826	0.3978	0.4269	Ols
	6.5	0.3947	0.4052	0.4062	0.5021	Ols
	7.5	0.4045	0.4152	0.4192	0.5223	Ols
	8.5	0.4132	0.4235	0.4272	0.5312	Ols
	9.5	0.4262	0.4304	0.4338	0.5422	Ols
	10.5	0.4468	0.4354	0.4396	0.5472	Ols
75	1.5	0.2668	0.2967	0.3225	0.2863	Mom
	2.5	0.3173	0.3367	0.3506	0.3367	Mom
	3.5	0.3453	0.3563	0.2983	0.2956	Mle
	4.5	0.3667	0.3724	0.3347	0.3363	Mle
	5.5	0.3826	0.3776	0.3523	0.3542	Mle
	6.5	0.3947	0.4052	0.3649	0.3846	Mle
	7.5	0.4045	0.4125	0.3977	0.4006	Mle
	8.5	0.4132	0.4235	0.4097	0.4126	Mle
	9.5	0.4262	0.4506	0.4183	0.4305	Mle
	10.5	0.4468	0.4364	0.4272	0.4434	Ols

$Table\ 3.\ Estimator\ of\ \ Fuzzy\ Reliability\ when\ \boldsymbol{\beta = 1.5}\ , \boldsymbol{\alpha = 3}, \boldsymbol{\widetilde{k} = 0.3}$

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
25	1.5	0.4856	0.4886	0.4102	0.2998	Mle
	2.5	0.5033	0.45577	0.5333	0.3546	Mom
	3.5	0.3566	0.5029	0.5641	0.3552	Mom
	4.5	0.5324	0.5319	0.5521	0.3762	Mom
	5.5	0.5614	0.5536	0.5712	0.5532	Ols
	6.5	0.5662	0.5701	0.5872	0.4505	Mom

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
	7.5	0.5782	0.6776	0.6011	0.4612	Mle
	8.5	0.5899	0.5021	0.5502	0.4613	Mle
	9.5	0.6032	0.4843	0.4073	0.5625	Ols
	10.5	0.6208	0.6019	0.4762	0.4673	Mle
50	1.5	0.4856	0.4066	0.4892	0.4778	Mle
	2.5	0.5033	0.4743	0.5201	0.4892	Mom
	3.5	0.3566	0.5201	0.5496	0.2912	Mom
	4.5	0.5324	0.5485	0.5311	0.3190	Mle
	5.5	0.5614	0.5864	0.5862	0.3464	Mle
	6.5	0.5662	0.5709	0.5004	0.3765	Mle
	7.5	0.5782	0.5873	0.6102	0.3802	Mom
	8.5	0.5899	0.6103	0.6246	0.4025	Mom
	9.5	0.6032	0.6224	0.6372	0.4066	Mom
	10.5	0.6208	0.6324	0.6524	0.4267	Mle
75	1.5	0.4856	0.5868	0.6552	0.4305	Mom
	2.5	0.5033	0.5664	0.6561	0.6302	Mom
	3.5	0.3566	0.5824	0.7021	0.6227	Mom
	4.5	0.5324	0.4048	0.5133	0.4015	Mom
	5.5	0.5614	0.4130	0.5244	0.4026	Mom
	6.5	0.5662	0.4206	0.5336	0.3995	Mom
	7.5	0.5782	0.4256	0.5456	0.3998	Mom
	8.5	0.5899	0.4346	0.3546	0.3999	Mom
	9.5	0.6032	0.4451	0.4548	0.4260	Mom
	10.5	0.6208	0.4566	0.4600	0.4444	Mom

$\textit{Table 4. Estimator of Fuzzy Reliability when } \beta = 1.5\ , \alpha = 3, \tilde{k} = 0.6$

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
25	1.5	0.2668	0.2998	0.2809	0.2806	Mle
	2.5	0.3162	0.3546	0.3201	0.3202	Mle
	3.5	0.3664	0.3552	0.4466	0.4461	Mom
	4.5	0.3825	0.3761	0.4462	0.4465	Ols
	5.5	0.3904	0.4233	0.5013	0.5016	Ols
	6.5	0.4048	0.4505	0.5026	0.5066	Ols
	7.5	0.4130	0.4606	0.5031	0.5036	Ols
	8.5	0.4200	0.4612	0.5036	0.5033	Mom
	9.5	0.4256	0.4613	0.5036	0.5036	Mom
	10.5	0.4346	0.4625	0.5047	0.5067	Mom
50	1.5	0.2668	0.4362	0.2902	0.2868	Mle
	2.5	0.3162	0.3282	0.3192	0.3286	Ols
	3.5	0.3664	0.3445	0.3464	0.3566	Ols
	4.5	0.3825	0.3606	0.3765	0.3776	Ols
	5.5	0.3904	0.3621	0.3802	0.3927	Ols
	6.5	0.4048	0.4225	0.4025	0.4251	Ols
	7.5	0.4130	0.3162	0.4252	0.4152	Mom
	8.5	0.4200	0.3664	0.4326	0.4235	Mom
	9.5	0.4256	0.3904	0.4331	0.4325	Mom
	10.5	0.4346	0.4048	0.4362	0.4364	Ols
75	1.5	0.2668	0.4132	0.2868	0.4364	Ols
	2.5	0.3162	0.4201	0.3284	0.4308	Ols

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
	3.5	0.3664	0.4256	0.3563	0.3606	Mle
	4.5	0.3825	0.4346	0.3766	0.3728	Mle
	5.5	0.3904	0.4421	0.3927	0.3607	Mle
	6.5	0.4048	0.5001	0.4051	0.4606	Mle
	7.5	0.4130	0.5161	0.4162	0.4502	Mle
	8.5	0.4200	0.5172	0.4235	0.4241	Mle
	9.5	0.4256	0.6026	0.4364	0.4366	Mle
	10.5	0.4346	0.6043	0.4372	0.4452	Mle

$Table\ 5.\ Estimator\ of\ Fuzzy\ Reliability\ when\ \beta = 4\ , \alpha = 1.5, \widetilde{k} = 0.3$

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
25	1.5	0.2666	0.2948	0.2098	0.2809	Mle
	2.5	0.3162	0.3736	0.3564	0.3223	Mle
	3.5	0.3825	0.3871	0.3762	0.4462	Ols
	4.5	0.3904	0.4032	0.4242	0.5023	Ols
	5.5	0.4048	0.4155	0.4505	0.5026	Ols
	6.5	0.4130	0.4255	0.4606	0.5032	Ols
	7.5	0.4200	0.4336	0.5036	0.5466	Ols
	8.5	0.4222	0.4339	0.4612	0.5022	Ols
	9.5	0.4256	0.4409	0.4613	0.5036	Ols
	10.5	0.4468	0.4625	0.4661	0.5047	Ols
50	1.5	0.2666	0.2868	0.2974	0.2921	Mle
	2.5	0.3162	0.3482	0.3199	0.3364	Mle
	3.5	0.3825	0.3563	0.3277	0.3464	Mle
	4.5	0.3904	0.3766	0.3464	0.3566	Mle
	5.5	0.4048	0.3977	0.3765	0.3765	Mle
	6.5	0.4130	0.4052	0.4062	0.4147	Ols
	7.5	0.4200	0.4235	0.4267	0.4326	Ols
	8.5	0.4222	0.4327	0.4315	0.4366	Ols
	9.5	0.4256	0.4365	0.4472	0.4406	Ols
	10.5	0.4468	0.5485	0.4762	0.4566	Mle
75	1.5	0.2666	0.4336	0.5202	0.4073	Mom
	2.5	0.3162	0.4802	0.5486	0.4762	Mom
	3.5	0.3825	0.4816	0.5711	0.4892	Mom
	4.5	0.3904	0.4926	0.5862	0.5201	Mom
	5.5	0.4048	0.5212	0.6002	0.5496	Mom
	6.5	0.4130	0.5412	0.6102	0.5711	Mom
	7.5	0.4200	0.5617	0.6223	0.5862	Mom
	8.5	0.4222	0.6166	0.6446	0.6002	Mom
	9.5	0.4256	0.6277	0.6279	0.6102	Mom
	10.5	0.4468	0.6306	0.6326	0.6246	Mom

$$Table\ 6.\ Estimator\ of\ Fuzzy\ \ Reliability\ when\ \beta = 4\ , \alpha = 1.5\ \ , \tilde{k} = 0.6$$

n	t_i	Real Ri	$\hat{R}_{Mle}$	$\hat{R}_{mom}$	$\hat{R}_{Ols}$	Best
25	1.5	0.4886	0.4886	4336	0.2998	Mle
	2.5	0.5032	0.4577	0.4832	0.3546	Mom
	3.5	0.5082	0.5029	0.5326	0.3552	Mom
	4.5	0.5324	0.5319	0.5624	0.4063	Mom
	5.5	0.5514	0.5936	0.5822	0.4371	Mle
	6.5	0.5662	0.5702	0.5672	0.4828	Mle
	7.5	0.5782	0.5776	0.5882	0.5202	Mom
	8.5	0.5899	0.5821	0.6094	0.5398	Mom
	9.5	0.6032	0.5843	0.6196	0.5422	Mom
	10.5	0.6208	0.6019	0.6203	0.5398	Mom
50	1.5	0.4886	0.4066	0.4792	0.5422	Ols
	2.5	0.5032	0.4743	0.4896	0.5712	Ols
	3.5	0.5082	0.5282	0.5132	0.5872	Ols
	4.5	0.5324	0.5306	0.5103	0.6011	Ols
	5.5	0.5514	0.5526	0.5322	0.6102	Ols
	6.5	0.5662	0.5607	0.5641	0.6203	Ols
	7.5	0.5782	0.5682	0.5521	0.6208	Ols
	8.5	0.5899	0.5694	0.5654	0.6122	Ols
	9.5	0.6032	0.5701	0.5902	0.6145	Ols
	10.5	0.6208	0.5723	0.6203	0.6306	Ols
75	1.5	0.4886	0.4889	0.4063	0.6442	Ols
	2.5	0.5032	0.5032	0.4372	0.5872	Mom
	3.5	0.5082	0.5086	0.4829	0.6011	Mom
	4.5	0.5324	0.5324	0.5202	0.6102	Ols
	5.5	0.5514	0.5614	0.5398	0.6342	Ols
	6.5	0.5662	0.5662	0.5712	0.5806	Ols
	7.5	0.5782	0.5804	0.5872	0.5960	Ols
	8.5	0.5899	0.6208	0.6011	0.5682	Mle
	9.5	0.6032	0.6311	0.6102	0.6022	Mle
	10.5	0.6208	0.6824	0.6204	0.6103	Mle

$$Table\ 7.\ Estimator\ of\ Fuzzy\ Reliability\ when\ \beta = 4\ , \alpha = 3\ , \tilde{k} = 0.3$$

n	t_i	Real Ri	$\hat{R}_{Mle}$	$\hat{R}_{mom}$	$\hat{R}_{Ols}$	Best
25	1.5	0.3206	0.3116	0.3226	0.3117	Mom
	2.5	0.3743	0.3262	0.3749	0.3748	Mom
	3.5	0.4165	0.3762	0.4161	0.4166	Mom
	4.5	0.4462	0.3852	0.4452	0.4452	Mom
	5.5	0.4667	0.3864	0.4657	0.4647	Mom
	6.5	0.4942	0.4268	0.4812	0.4722	Mom
	7.5	0.5032	0.4538	0.4836	0.4732	Mom
	8.5	0.5043	0.4726	0.5023	0.4836	Mom
	9.5	0.5126	0.4862	0.5224	0.5123	Mom
	10.5	0.5194	0.5012	0.5324	0.5224	Mom
50	1.5	0.3206	0.5108	0.5364	0.5322	Mom
	2.5	0.3743	0.5242	0.5582	0.5148	Mom
	3.5	0.4165	0.3263	0.5592	0.5253	Mom
	4.5	0.4462	0.3815	0.3102	0.5442	Mom
	5.5	0.4667	0.4214	0.3622	0.5462	Ols

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
	6.5	0.4942	0.4493	0.3804	0.5472	Ols
	7.5	0.5032	0.4665	0.4212	0.5462	Ols
	8.5	0.5043	0.5166	0.4483	0.5177	Ols
	9.5	0.5126	0.5232	0.4699	0.5362	Ols
	10.5	0.5194	0.3116	0.4852	0.5405	Ols
75	1.5	0.3206	0.3758	0.4852	0.5416	Ols
	2.5	0.3743	0.4172	0.4982	0.5432	Ols
	3.5	0.4165	0.4454	0.5082	0.5442	Ols
	4.5	0.4462	0.4662	0.5226	0.5462	Ols
	5.5	0.4667	0.4703	0.5237	0.5471	Ols
	6.5	0.4942	0.5845	0.5306	0.5472	Mle
	7.5	0.5032	0.5666	0.5416	0.5473	Mle
	8.5	0.5043	0.5632	0.5422	0.5477	Mle
	9.5	0.5126	0.6222	0.6016	0.5532	Mle
	10.5	0.5194	0.6326	0.6166	0.6216	Mle

***Table* 8. *Estimator of Fuzzy Reliability when* $\boldsymbol{\beta = 4 \quad, \alpha = 3 \quad, \widetilde{k} = 0.6}$**

n	t_i	Real Ri	$\widehat{R}_{Mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{Ols}$	Best
25	1.5	0.3668	0.0.3068	0.3261	0.4316	Ols
	2.5	0.4062	0.3676	0.3686	0.3116	Mom
	3.5	0.4153	0.4165	0.4224	0.3758	Ols
	4.5	0.4226	0.4562	0.4463	0.4172	Mle
	5.5	0.5311	0.4665	0.4678	0.4454	Mle
	6.5	0.6062	0.4842	0.4857	0.4662	Mom
	7.5	0.6172	0.5032	0.4685	0.4803	Ols
	8.5	0.6326	0.5036	0.4980	0.4826	Mle
	9.5	0.6736	0.5126	0.5082	0.4837	Mle
	10.5	0.6566	0.5194	0.5166	0.5056	Mle
50	1.5	0.3668	0.3206	0.4231	0.5132	Ols
	2.5	0.4062	0.3747	0.4454	0.5162	Ols
	3.5	0.4153	0.4165	0.4662	0.5224	Ols
	4.5	0.4226	0.4462	0.4803	0.5304	Ols
	5.5	0.5311	0.4667	0.4945	0.5406	Ols
	6.5	0.6062	0.5043	0.5056	0.6106	Ols
	7.5	0.6172	0.5129	0.5130	0.6226	Ols
	8.5	0.6326	0.5194	0.5221	0.6236	Ols
	9.5	0.6736	0.5244	0.5306	0.6306	Ols
	10.5	0.6566	0.5366	0.5321	0.6224	Ols
75	1.5	0.3668	0.7263	0.5104	0.6086	Mle
	2.5	0.4062	0.7815	0.5702	0.6246	Mle
	3.5	0.4153	0.6214	0.5212	0.6308	Mle
	4.5	0.4226	0.6493	0.4463	0.6321	Mle
	5.5	0.5311	0.6687	0.4677	0.6322	Mle
	6.5	0.6062	0.6856	0.4853	0.6344	Mle
	7.5	0.6172	0.6883	0.4667	0.6404	Mle
	8.5	0.6326	0.5082	0.5062	0.6412	Ols
	9.5	0.6736	0.5166	0.5162	0.6422	Ols
	10.5	0.6566	0.5232	0.5227	0.6455	Ols

Conclusion:

From the results of simulation for comparing the fuzzy reliability function of mixed Exp-Rayleigh we find that the first best one is the Ordinary Lest Square estimator and the second one is Moments, and the third one is Maximum Likelihood.

The results of comparison indicates that the first best fuzzy Reliability is the Ordinary Lest Square estimator and the second one is Moments, and the third one is Maximum Likelihood Here are the results of preference of fuzzy Reliability estimators for all tables percentage of preference of three different methods estimators:

Tables	$\hat{R}_{Mle}$	$\hat{R}_{Mom}$	$\hat{R}_{Ols}$
Table (1)	$\frac{8}{30} = 0.266$	$\frac{8}{30} = 0.266$	$\frac{14}{30} = 0.466$
Table (2)	$\frac{11}{30} = 0.366$	$\frac{8}{30} = 0.266$	$\frac{11}{30} = 0.333$
Table (3)	$\frac{9}{30} = 0.3$	$\frac{19}{30} = 0.633$	$\frac{2}{30} = 0.066$
Table (4)	$\frac{11}{30} = 0.366$	$\frac{7}{30} = 0.233$	$\frac{12}{30} = 0.4$
Table (5)	$\frac{8}{30} = 0.266$	$\frac{10}{30} = 0.333$	$\frac{12}{30} = 0.4$
Table (6)	$\frac{6}{30} = 0.233$	$\frac{9}{30} = 0.3$	$\frac{15}{30} = 0.5$
Table (7)	$\frac{5}{30} = 0.166$	$\frac{14}{30} = 0.466$	$\frac{11}{30} = 0.366$
Table (8)	$\frac{12}{30} = 0.4$	$\frac{2}{30} = 0.066$	$\frac{16}{30} = 0.533$

From the summary of results in this Table, we concluded that Ordinary Lest Square is best with percentage $\frac{93}{240} = 38\%$, and then Moments is best with $\frac{77}{240} = 0.32\%$, and Maximum Likelihood also is best with $\frac{70}{240} = 0.29\%$.

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