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New Method for Solving Quadratic Fractional Programing Problems

Mediya B. Mrakhan, Basiya K.Abdulrahim, Rzgar F. Mahmood, Aram Bajalan, Shwan Adnan Ali

Department of Mathematics, College of Education, University of Garmian , Kurdistan Region , Iraq

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Corresponding Author:

Name: Mediya B. Mrakhan

E-mail:

media.bawaxan@Garmian.edu.krd

Tel:

ABSTRACT

In this article, we development a methodology to solve an objective function of a quadratic fractional programming problem (QFPP), and the constraints are linear. We introduce a new method called RBM-Technique, which is directly solve the problem and have same optimal solution. Finally, the nonlinear problem is transformed to a linear programming problem with two constraints and no mor convert to the initial problem. Numerical examples are illustrated to show the efficiency of the method and the computer operation of our algorithms was also discussed by using MATLAB 2013a to solve a constructed numerical example.

1. Introduction

Since they've been used in production planning, financial and corporate organizing, medical services and hospital planning, and other fields, quadratic fractional issues (i.e., ratio of goals with numerator and denominator) have generated a lot more research and interest. Many techniques to solutions such problems are introduced by Beck A., Ben-Tal A., and Teboulle M., [4], Problems with linear fraction maximums (i.e., ratio of goals with numerator and denominator) since they are helpful in product development, financial and business organizing, medical services and hospital planning, they have generated a lot of research and interest. Abdulrahim B. K., used Feasible Directions to Solve Quadratic Fractional Programming Problem [2]. The method, known as the modified objective function technique derived, is used to resolve this linear fractional program by optimizing a series of linear programs and only recalculating the objective function's local gradient. Moreover, many aspects of linear fraction program duality and sensitivity analysis were mentioned. Sulaiman N. A., and Nawkhass M. A. Solving Quadratic Fractional Programming Problem [6]. Fukushima and Hayashi solve Quadratic Fractional Programming Problems with Quadratic Constraints [5]. Abdulrahim B. K. used the Feasible Path Creation and Updated Simplex Method [1] to solve QFPP. In this article, we present RBM-Technique for solving problems with quadratic fractional goals and inequality constraints. In third section, we include a detailed overview of the issue as well as our key findings while section 4 contains the steps of our new algorithm with two examples to numerical examples how our algorithm works. Finally, the key conclusion of this solution procedure is limited by the section.

2. Quadratic Fractional Programming Problem

Definition 2.1: a problem involving quadratic programming [1-3]

If the problem formulation has the following structure:

Maximize (Minimize)$Z = (\alpha + C^T X + \frac{1}{2} X^T G X)$

Conditional on:

$$AX \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} B$$
$$X \ge 0$$

where $A = (a_{ij})_{m\times n}$, matrix of cofficients, $\forall i = 1,2,\dots,m,$ and $j = 1,2,..,n,$ $B = (b_1, b_2, \dots, b_m)^T$, $X = (x_1, x_2, \dots, x_n)^T$, $C^T = (c_1, c_2, \dots, c_n)^T$,
and $G = (g_{ii})_{n\times n}$ is a symmetric square matrix that is positive definite or positive semi-definite, if T is transposed and α is scalar, the requirements are linear, and the optimal solution is quadratic.

Definition 2.2: Fractional Programming Problem with Quadratic Fractions [1]

QFPP's mathematical programming issue can be expressed as follows:

$$\text{Maximize (Minimize) } Z = \frac{(\alpha_1 + C_1^T X + \frac{1}{2} X^T G_1 X)}{(\alpha_2 + C_2^T X + \frac{1}{2} X^T G_2 X)}$$

Subject to :

$$AX \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} X$$

$$X \ge 0$$

where G_1, G_2 are $(n \times n)$ coefficients matrix of G_1, G_2 matrixes that are symmetric. Except transposed(T), where X is an n-dimensional vector of decision variables, all vectors are considered to be column vectors, C_1, C_2 is the n-dimensional vector of constants, α_1, α_2 are scalars and B is m-dimensional vector of the constants. The problem that is being attempted to be resolved in this research has the general structure:

$$\text{Maximize } Z = \frac{(c^T X + \gamma)(e^T X + \delta)}{(d^T x + \beta)(f^T x + \varepsilon)} = \left(\frac{(c^T X + \gamma)}{(d^T x + \beta)}\right)\left(\frac{(e^T X + \delta)}{(f^T x + \varepsilon)}\right) = \frac{Z_1}{Z_2}$$

$$\text{or Maximize } Z = \frac{(c^T X + \gamma)(e^T X + \delta)}{(d^T x + \beta)(f^T x + \varepsilon)} = \left(\frac{(c^T X + \gamma)}{(f^T X + \varepsilon)}\right)\left(\frac{(e^T X + \delta)}{(d^T X + \beta)}\right) = \frac{Z_1}{Z_2}$$

Subject to:

$$AX \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} B$$

$$X \ge 0$$

Where $X \in R^n$, A is an $m \times n$ matrix; c, e, d and f are n −vectors; $B \in R^m$ and $\gamma, \beta, \delta, \varepsilon$ are scalar constants. Moreover $f^T X + \varepsilon,\ d^T X + \beta > 0$ everywhere in X.

3. RBM-Technique and Formulation

The RBM-Technique is a systematic algorithm that involves getting from one simple feasible solution (on vertex) to the other in a specified order in order to increase the value of the optimal solution. If the objective function increases with each jump, no foundation can be replicated, so there is no need to return to a previously covered vertex. The process should lead to the optimized vertex in a finite number of steps because the number of vertices is limited.

For resolving quadratic programming issues, the RBM-Technique method is an optimization (step-by-step) method. It consists of the following components:

Having a trail basic feasible solution to constraint equations.

Testing whether is an optimal solution.

Improving the first trial solution by a set of rules.

And repeating the process till an optimal solution is obtained.

$$\Delta_{ji} = c_{ji} - c_{Bi}^T x_{ij}, i = 1,2,3,4\ j = 1,2, \dots, m+n$$

$$\Delta_{ja} = z_2 \Delta_{j1} - z_1 \Delta_{j2},\ \ \Delta_{jb} = z_4 \Delta_{j3} - z_3 \Delta_{j4}, j = 1,2, \dots, m+n$$

$$z_1 = c_{B1} v_B + \gamma, z_2 = c_{B2} v_B + \delta, z_3 = c_{B3} v_B + \epsilon, z_4 = c_{B4} v_B + \beta$$

$$Z = \frac{z_1 z_2}{z_3 z_4} = \frac{Z_1}{Z_2}$$

$$\Delta_j = \Delta_{ja} \times \Delta_{jb}$$

Here $c_{ji}, i = 1,2,3,4, j = 1,2, \dots, m+n$ are that coefficient of the objective function's essential and non-basic variables and $c_{Bi}, i = 1,2,3,4$ are the simple variables' coefficients in the optimization problem.

Algorithm for RBM-Technique:

The RBM-Technique algorithm for solving QFPP that can be categorized as follows:

Step1: Writing the basic way of the issue, attaching slack and artificial variables to a limitation, and using RBM-Technique to begin.

Step2: Create an estimate the Δ_j by using the formula $\Delta_j = \Delta_{ja} \times \Delta_{jb}$, then enter it in the RBM-Technique table at the start.

Step3: The RBM-Technique method is used to find the solution.

Step4: In step3, verify the solution for feasibility; if this is, go to step5, if not, be using dual RBM-Technique to eliminate the infeasibility.

Step5: Verify that the solution is optimal if all $\Delta_j \le 0$,then the solution is perfect, if not, proceed to step 3.

4. Numerical Examples

Example 4.1:

The following QFPP is taken into consideration:

$$Max. Z = \frac{(4x_1 + 2x_2)(2x_1 + x_2 + 3)}{(x_1 + x_2 + 2)(2x_1 + x_2 + 5)} = \frac{4x_1 + 2x}{2x_1 + x_2 + 5} \times \frac{2x_1 + x_2 + 3}{x_1 + x_2 + 2}$$

Subject to:

$$x_1 + x_2 \le 6$$
$$8x_1 + 4x_2 \le 32$$
$$x_1 \le 3$$
$$x_2 \le 5$$
$$x_1, x_2 \ge 0$$

Solution:

Solving the example 4.1 by RBM-Technique and applied an algorithm in the section 4, we arrived at the initial table shown in table 1.1, and after one iteration, we arrived at the table shown in table 1.2.

Table 1.1: RBM-initial Technique's table for Example 4.1

					c_{j1}	4	2	0	0	0	0	
					c_{j2}	1	1	0	0	0	0	
					c_{j3}	2	1	0	0	0	0	
					c_{j4}	2	1	0	0	0	0	
B.V.	c_{B1}	c_{B2}	c_{B3}	c_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Main-Ratio
x_3	0	0	0	0	3	0	1	1	0	0	0	6
x_4	0	0	0	0	8	0	4	0	1	0	0	4
x_5	0	0	0	0	3	$\boxed{1}$	0	0	0	1	0	3
x_6	0	0	0	0	5	0	1	0	0	0	1	–
$z_1 = 0$					Δ_{j1}	-4	-2	0	0	0	0	
$z_2 = 5$					Δ_{j2}	-2	-1	0	0	0	0	
$Z_1 = z_1/z_2 = 0$					Δ_{ja}	-20	-10	0	0	0	0	
$z_3 = 3$					Δ_{j3}	-2	-1	0	0	0	0	
$z_4 = 2$					Δ_{j4}	-1	-1	0	0	0	0	
$Z_2 = z_3/z_4 = 3/2$					Δ_{jb}	-1	1	0	0	0	0	
$Z = Z_1/Z_2$					Δ_j	20	-10	0	0	0	0	

Table 1.2: RBM-final Technique's table for Example 4. 1

					c_{j1}	4	2	0	0	0	0
					c_{j2}	1	1	0	0	0	0
					c_{j3}	2	1	0	0	0	0
					c_{j4}	2	1	0	0	0	0
B.V.	c_{B1}	c_{B2}	c_{B3}	c_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6
x_3	0	0	0	0	3	1	0	1	0	0	-1
x_4	0	0	0	0	8	8	0	0	1	0	-4
x_2	0	0	0	0	3	1	0	0	0	1	0
x_6	2	1	1	1	5	0	1	0	0	0	1
$z_1 = 10$					Δ_{j1}	-4	0	0	0	0	2
$z_2 = 10$					Δ_{j2}	-1	0	0	0	0	1
$Z_1 = z_1/z_2 = 1$					Δ_{ja}	-30	0	0	0	0	10
$z_3 = 8$					Δ_{j3}	-2	0	0	0	0	1
$z_4 = 7$					Δ_{j4}	-2	0	0	0	0	1
$Z_2 = z_3/z_4 = 8/7$					Δ_{jb}	2	0	0	0	0	-1
					Δ_j	-60	0	0	0	0	-10

Then the solution is $x_1 = 0,\ x_2 = 3$ and $Max.Z = \mathrm{Z}_1 \times Z_2 = \frac{8}{7}$. Since all $\Delta_j \le 0$ then the solution is optimal.

Example 4. 2:

The following QFPP is taken into account:

$$Max.Z = \frac{(6x_1+4x_2+2x_3)(x_1+3x_2+2x_3+1)}{(3x_1+2x_2+x_3+2)(3x_1+2x_2+5)}$$

Subject to:

$$x_1 + 3x_2 + 2x_3 \le 9$$
$$3x_1 + 2x_2 + x_3 \le 8$$
$$2x_1 + x_2 + 3x_3 \le 7$$
$$x_1, x_2, x_3 \ge 0$$

Solution: Similar to the example **4.1**, we obtain the solution are $x_1 = 0,\ x_2 = 3, x_3 = 0$ and $Max.Z = \frac{15}{11}$. since all $\Delta_j \le 0$ then the solution is optimal.

5. Conclusion

This is a quadratic fractional objective function specialization of the RBM-Technique. Algorithm selects a course for development. By optimizing the objective function in that direction, an optimal step is selected. We agree with that, because RBM-Technique deals with variables more than $n = 10$. So far, in [6] to find the optimal solution, changed the QFPP to Quadratic Programming Problem and solved.

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حل مشكلة البرمجة الجزئية التربيعية بطريقة جديدة

ميديا باوقخان مراخان ، بسية كاكةولا عبدالرحيم ، رزكار فريق محمود ، أرام خليل إبراهيم ، شوان عدنان علي

قسم الرياضيات ، كلية التربية ، جامعة كرميان ، إقليم كوردستان ، العراق

الملخص

في هذا البحث ، قمنا بتوسيع الخوارزميات المعطاة وفي مشكلة البرمجة الجزئية التربيعية (QFPP) ، تم تطوير منهجية لحل مشكلة البرمجة الكسرية التربيعية ، والتي تكون وظيفتها الموضوعية مشكلة البرمجة الكسرية غير الخطية والقيود خطية. لدينا تقنية RBM لحل المشكلة ونتيجة جيدة. تمت أيضًا مناقشة تطبيق الكمبيوتر لخوارزمياتنا من خلال حل المثال العددي المركب باستخدام إصدار MATLAB 2013a. أخيرًا ، تنتهي هذه الورقة ببعض الاستنتاجات.