Coupled And Common Coupled Fixed Point Theorems In Soft Metric Andfuzzy Soft Metric Spaces

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Abstract:

In this paper, we prove coupled fixed point theorems for somemappings by using the concepts q-contractive soft mapping andR-weakly commutingsoft mappings in soft metric spaces andfuzzysoft metric spaces.

Keywords: soft metric space, fuzzy soft metric space, coupled fixed point theorems, common coupled fixed point theorem.

1. Introduction

The concept of soft set was introduced in 1999 by D.Molodtsov [6]. In 2001,Maji [8] defined the concept of fuzzy soft setdepending on definition of fuzzy set. In 2013,Das and Samanta [3]introduced the concept of soft metric space by using soft point and soft real number.In 2015,Thangaraj Beaulaa and R.Rajab[12]introduced the concept of fuzzy softmetric space. In 2016,Yazara [9]proved some fixed point theorem of soft contractive mapping on complete soft metric space. In2016, Sumit Mohinta and SamantaVariant[11] proved some fixed point theorems in soft metric space by using semi soft compatible mapping.In 2009, V.Lakshmikantham, and Lj.B. ´Ciri´c [5] proved coupled fixed point theorem for nonlinear contractions in partially ordered metric spaces. In this paper, we prove new coupled fixed point theorem and common coupled fixed point theorem by usingq-contractive soft mapping and R-weakly commuting soft mappings in soft metric and fuzzy soft metric spaces.

2. Preliminaries

Definition(2.1)[6]

Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if F is a function from E into the set of all subsets of X , i.e $F: E \to P(X)$, where $P(X)$ is the power set of X . The set of all soft set over X is denoted by $SS(X, E)$.

Definition(2.2)[7]

Let (F, E) ,$(G, E) \in SS(X, E)$.

1) We say that the soft set (F, E) is a soft subset of (G, E) if $F(e) \subseteq G(e)$, for every $e \in E$. symbolically, we write $(F, E) \cong (G, E)$.

2) We say that the soft sets (F, E) and(G, E) are soft equal if $(F, E) \cong (G, E)$and $(G, E) \cong (F, E)$. symbolically, we write$(F, E) = (G, E)$.

3) The Nnull soft set $(F, E)$is soft set such that $\forall e \in E, F(e) = \emptyset$ and denoted by$\tilde{\emptyset}$.

4) The absolute soft set(F, E) is soft set such that $\forall e \in E, F(e) = X$ and denoted by $\tilde{X}$.

5) The complement of a soft set (F, E) is soft set denoted by $(F, E)^c$and define by

$(F, E)^C = (F^c, E)$, $F^c: E \to P(X)$ is a function given by $F^c(e) = X - F(e), \forall e \in E$.

F^c is called the soft complement function of F. clearly, $(F^c)^c$ is the same as F and $((F, E)^c)^c = (F, E)$. clearly, $\tilde{X}^c = \tilde{\emptyset}$ and $\tilde{\emptyset}^c = \tilde{X}$.

6) The difference of two soft sets (F, E) and(G, E) over X ; denoted by $(H, E) = (F, E)/(G, E)$such that$H(e) = F(e)/G(e)$ for all $e \in E$.

Definition (2.3)[7]

Let I be an arbitrary index set and <: gÆ ;ª — =C : Æ ;

1) The soft intersectionof these soft sets is the soft set : Æ ;— : Æ ;where the function ª 7 : ¢defined as follows : : ;L Œ <g : ;ª — ¬for every — We write : Æ ;L Œ <: gÆ ;ª — .=

2) The soft union of these soft sets is the soft set : Æ ;— : Æ ;where the function ª 7 : ; defined as follows : : ;L ° <g : ;ª — ¬for every — We write : Æ ;L ° <: gÆ ;ª — .=

Definition(2.4)[3]

A soft set :(Æ′ ;over :is said to be a soft point if there exactly A— such that (:Ais a singleton, say <T=and (:Aæ;L ¸for all Aæ— ′ F <Asuch a soft point is denoted by T. The set of all soft points of the universal set :is denoted by 52 ::.;

Definition(2.5)[3]

Two soft points T Œare said to be equal if AL =and T L U

Thus , T M U ; T M Uor AM =

Definition(2.6)[2]

Let R be the set of real numbers Æ$:4 ¡be the collection of all nonempty bounded subsets of R and A be a set of parameters. Then a mapping (# " $:4 ¡is called a soft real set. It is denoted by :(Æ#;. Inparticular if □: Æ#¡is a singleton soft set, then identifying (F,A) with the corresponding soft element, it will be called a soft real number and denoted by NŒÆPÀP Let ¶ # ¢denoted the set of all non-negative soft real numbers ÍÆare soft real numbers where Í:A;L rÆ:A;L ¸for all A—#Æ respectively.

Definition(2.7)[3]

For two soft real numbers

1) N`Q if N:A;Q P:Afor all A—#

2) N`R if N:A;R P:Afor all A—#

3) N`O if N:A;O P:Afor all A—#

4) N`P if N:A;P P:Afor all A—#

Definition(2.8)[3]

Let X be an initial universal set and A be anon empty set of parameters . Let Íbe the absolute soft set , S (L Í¢collection of all soft point of Íand ¶ # ¢denoted the set of all non-negative soft real numbers ,we defined soft metric space soft as follow:

A mapping $\tilde{d}: Sp(\tilde{X}) \times Sp(\tilde{X}) \to \mathbb{R}(A^*)$ satisfying for all $\tilde{x}_e, \tilde{x}_a, \tilde{x}_z \in Sp(\tilde{X})$

1) $\tilde{d}(\tilde{x}_e, \tilde{x}_a) \geq$ 0

2) $\tilde{d}(\tilde{x}_e, \tilde{x}_a) = 0 \leftrightarrow \tilde{x}_e = \tilde{x}_a$

3) $\tilde{d}(\tilde{x}_e, \tilde{x}_a) = \tilde{d}(\tilde{x}_a, \tilde{x}_e)$

4) $\tilde{d}(\tilde{\square}_e, \tilde{x}_z) \leq \tilde{d}(\tilde{x}_e, \tilde{x}_a) + \tilde{d}(\tilde{x}_a, \tilde{x}_z)$

Then $(\tilde{X}, \tilde{d})$ is called soft metric space .

Definition(2.9)[3]

Let$(\tilde{X}, \tilde{d})$ be a soft metric space and $\{\tilde{x}_{en}\}$ be a soft sequence in $\tilde{X}$.Then

1) A soft sequence { T in is said to be soft convergent to T if for every « r there exist G 0such that @:T Æ T ;" J R G

2) A soft sequence { T in is said to be soft Cauchy sequence if forevery «rÆ

There exist G 0such that @:T Æ T ;" JÆ R G

3) A soft metric space is said to be complete if every soft Cauchy sequence is convergent in Ø

Proposition(2.10)

Let k be a soft metric space.Let <T be a sequence in such that

@:T Æ>5 ÆT Æ ;Q M@:T Æ ÆT Æ?5 ;

Where M — >rÆs;ÆJ — 0Then <T is Cauchy sequence.

Proof

Let <T be a sequence in By condition

@:T Æ>5 ÆT Æ ;Q M@:T Æ ÆT Æ?5 ;Q M@:T Æ?5 ÆT Æ?6 ;Q fi Q M@:T 5 ÆT 4 ;

For J O IThen

@:T Æ ÆT ;Q @:T Æ ÆT Æ>5 ;E @:T Æ>5 ÆT Æ>6 ;

Q @:T Æ ÆT Æ>5 ;E @:T Æ>5 ÆT Æ>6 ;E @:T Æ>6 ÆT Æ>7 ;

:

:

Q @:T Æ ÆT Æ>5 ;E @:T Æ>5 ÆT Æ>6 ;E @:Ø Æ>6 ÆT Æ>7 ;E fi E @:T ?5 ÆT ;

Q M@:T 5 ÆT 4 ;E M@:T 5 ÆT 4 ;E fi E M ?5 @:T 5 ÆT 4 ;

L M>M@?5 E ME fi E M ?@:T 5 ÆT 4 ;

$$L \frac{M@}{s\ F\ M} @:T_5 ÆT_4 ;$$

As $n \to \infty$, $\frac{q^n}{1-q}\tilde{d}(\tilde{x}_{e1}, \tilde{x}_{e0}) \to 0$ since $q \in [0,1)$.Then

$$\lim_{n\to\infty} \tilde{d}(\tilde{x}_{en}, \tilde{x}_{em}) = 0$$

Therefore $\{\tilde{x}_{en}\}$ is Cauchy sequence .

Proposition(2.11)

Let $(\tilde{X}, \tilde{d})$ be a soft metric space.if there exists $q \in [0,1)$such that

$\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q\tilde{d}(\tilde{x}_{\square}, \tilde{x}_a)$ for all $\tilde{x}_e, \tilde{x}_a \in \tilde{X}$.Then$\tilde{x}_e = \tilde{x}_a$.

Proof

Since$\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q\tilde{d}(\tilde{x}_e, \tilde{x}_a)$ for all $\tilde{x}_e, \tilde{x}_a \in \tilde{X}$

Then $q\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q\left(q\tilde{d}(\tilde{x}_e, \tilde{x}_a)\right)$

$$\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q^2\tilde{d}(\tilde{x}_e, \tilde{x}_a)$$

Again for n time

$$\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq q^2\tilde{d}(\tilde{x}_e, \tilde{x}_a) \leq \cdots \leq q^n\tilde{d}(\tilde{x}_e, \tilde{x}_a)$$

Since $q \in [0,1)$, then for $n \in N$, such that

$\tilde{d}(\tilde{x}_e, \tilde{x}_a) = 0$ as $n \to \infty$

Hence$\tilde{x}_e = \tilde{x}_a$.

Definition (2.12)[14]

Let X be a non-empty set and I be the closed interval $I = [0,1]$ of real numbers. A fuzzy set H in X (or a fuzzy subset from X) is a function from X into $I = [0,1]$.If H is a fuzzy set in X then H is described as characteristic function which connects every $x \in X$ to real number $H(x)$ in the interval I. $H(x)$ is the grade of membership function to x inH. H can be described completely as": $H = \{(x, H(x)): x \in X,\ 0 \leq H(x) \leq 1\}$ or $H = \left\{\frac{H(x)}{x} : x \in X\right\}$,

where $H(x)$ is called the membership function for the fuzzy set H. The family of all fuzzy sets in X is denoted by I^X.

Definition(2.13)[10]

Let $*$ be a binary operation on the set $I = [0,1]$, i.e $*: [0,1] \times [0,1] \to [0,1]$ is a function, then $*$ is said to be t-norm (triangular-norm) on the set I if $*$ satisfies the following axioms :

(1) $*$ is commutative and associative

(2) $a * 1 = a$ for all $a \in [0,1]$

(3)if $b, c \in I$ such that $b \leq c$, then $a * b \leq a * c$ for all $a \in I$

In addition, if $*$ is continuous then $*$is called a continuous t-norm.

Definition(2.14)[4]

A triple (X, M,) is a fuzzy metric space.If X is an arbitrary set, is continuous t-norm and M is a fuzzy set on : H : H :rÆ» satisfying the following conditions, for all TÆUÆV — ; suchthat t, s Ì

s / :TÆUÆP; P r

t / :TÆUÆP; L s T L U

u / :TÆUÆP; L / :UÆTÆP;

v / :TÆUÆP; / :UÆVÆO; Q / :TÆVÆP E O;

w / :TÆUÆ ; >rÆ» ; \ >rÆsis continuous.

Definition(2.15)[12]

Let X be an initial universal set and A be anon empty set of parameters . Let be the absolute fuzzy soft set , L:Ì collection of all soft points of and (# denoted the set of all non-negative soft real numbers,we defined fuzzy soft metric space using soft point as following :

A mapping ªL :Ì; H LkÌo ¶ 9 : ; \ >rÆ satisfy the following for all

T ÆT ÆT —Ì

s;/ :T ÆT ÆP; L r BK ¼H HP Q r

t;/ :T ÆT ÆP; L s ^ T L T P P r

u;/ :T ÆT ÆP; L @:T ÆT ÆP; P P r

v;/ :T ÆT ÆP E O; R @:T ÆT ÆP; @:T ÆT ÆO; PÆO P r

5) / :T ÆT Æ ;ª:rÆ» ; \ >rÆcontinous.

Then :ÌÆ/ Æ called fuzzy soft metric space .

Definition(2.16)[13]

Let(ÌÆ/ Æbe a fuzzy soft metric space and { T Æ be a fuzzy soft sequence in Ìthen

1) A fuzzy soft sequence { T Æ } in is said to be fuzzy soft convergent to TIf for «there G 0such that / :T Æ ÆT ÆP; P s F Æ J R G

2) A fuzzy soft sequence { T Æ =} in Ìis said to be fuzzy soft Cauchy if for «there G 0such that / :T Æ ÆT Æ ÆP; P s F Æ JÆI R G

3) A fuzzy soft metric space is said to be complete if every fuzzy soft Cauchy sequence is convergent in Ì

3.coupled and common coupled fixed point theorem in soft metric spaces

Definition(3.1)[1]

Let $(X, \leq)$ be a partially ordered set and $F: X \times X \to X$ be a mapping. The mapping F is said to have the mixed monotone property if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for any $x, y \in X$,

$$T_5 \text{ÆT}_6 - : \text{ÆU}_5 \text{ Q } T_6 \quad (:T_5 \text{ÆU}; \text{Q } (:T_6 \text{ÆU};$$

And

$$U_5 \text{ÆU}_6 - : \text{ÆU}_5 \text{ Q } U_6 \quad (:T\text{ÆU}_6 ; \text{Q } (:T\text{ÆU}_5 ;$$

Definition (3.2)[1]

An element :TÆU;— : is called a coupled fixed point of 6 : : \ : if

$$T \text{ L } 6:T\text{ÆU}; \quad U \text{ L } 6:U\text{ÆT};$$

Definition (3.3)[4]

An element :TÆU;— : is called a common coupled fixed point of 6ª: \ : and 5 : : \ : if T L 5:TÆU; L 6:T; U L 5:UÆT; L 6:U;

Definition(3.4)

Let :ÆÆ@be a soft metric space.The soft mapping :BÆ ;ª: \ is said to be M-contractive soft mapping if there exists M — >1) such that @::BÆ ;T Æ:BÆ ;T ; Q M@:T ÆT for all T ÆT — and M is called contractive constant.

Theorem(2.5)

Let :k:ÆÆ@oÆQ be a partial ordered complete soft metric space and Bª:h: \ be a M-contractive function with M-contractive constant and Bhas a mixed monotone property,then Bhas a unique coupled fixed point .

Proof

Let T ÆT$_4$ — and set

$$T_5 \text{ L } B:T \text{ ÆT}_4 ;$$

$$T_5 \text{ L} \square :T \text{ ÆT}_4 ;$$

$$T_6 \text{ L } B:T_5 \text{ÆT}_5 ; \text{L } B^6 :T \text{ ÆT}_4 ; \text{L } BB:T \text{ ÆT}_4 ; \text{L } BT_5$$

$$T_6 \text{ L } B:T_5 \text{ÆT}_5 ; \text{L } B^6 :T \text{ ÆT}_4 ; \text{L } BB:T \text{ ÆT}_4 ; \text{L } BT_5$$

:

$$T_{Æ>5} \text{ L } B:T_Æ \text{ÆT}_Æ ; \text{L } B^{Æ>5} :T \text{ ÆT}_4 ; \text{L } BB^Æ :T \text{ ÆT}_4 ; \text{L } BT_Æ$$

$$T_{Æ>5} \text{ L } B:T_Æ \text{ÆT}_Æ ; \text{L } B^{Æ>5} :T \text{ ÆT}_4 ; \text{L } BB^Æ :T \text{ ÆT}_4 ; \text{L } BT_Æ$$

With

$\tilde{x}_{e0} \leq \tilde{x}_{e1} = f(\tilde{x}_{eo}, \tilde{x}_{a0})$ and $\tilde{x}_{a0} \geq \tilde{x}_{a1} = f(\tilde{x}_{ao}, \tilde{x}_{e0})$

Due to the mixed monotone property of f, we have

$\tilde{x}_{e0} \leq \tilde{x}_{e1} \leq \tilde{x}_{e2}$ and $\tilde{x}_{a0} \geq \tilde{x}_{a1} \geq \tilde{x}_{a2}$

In general

$$\tilde{x}_{e0} \leq \tilde{x}_{e1} \leq \cdots \leq \tilde{x}_{en} \leq \tilde{x}_{en+1} \leq \cdots$$

$$\tilde{x}_{a0} \geq \tilde{x}_{a1} \geq \cdots \geq \tilde{x}_{an} \geq \tilde{x}_{an+1} \geq \cdots$$

$$\tilde{d}(\tilde{x}_{e2}, \tilde{x}_{e1}) = \tilde{d}\big(f(\tilde{x}_{e1}, \tilde{x}_{a1}), f(\tilde{x}_{eo}, \tilde{x}_{a0})\big)$$

$$\leq q\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{eo}, \tilde{x}_{a0})\big)$$

i.e

$\tilde{d}(\tilde{x}_{e2}, \tilde{x}_{e1}) \leq q\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{eo}, \tilde{x}_{a0})\big)$ and

$$\tilde{d}(\tilde{x}_{a2}, \tilde{x}_{a1}) = \tilde{d}\big(f(\tilde{x}_{a1}, \tilde{x}_{e1}), f(\tilde{x}_{ao}, \tilde{x}_{e0})\big)$$

$$\leq q\tilde{d}\big((\tilde{x}_{a1}, \tilde{x}_{e1}), (\tilde{x}_{ao}, \tilde{x}_{e0})\big)$$

i.e

$\tilde{d}(\tilde{x}_{a2}, \tilde{x}_{a1}) \leq q\tilde{d}\big((\tilde{x}_{a1}, \tilde{x}_{e1}), (\tilde{x}_{ao}, \tilde{x}_{e0})\big)$

$\tilde{d}(\tilde{x}_{e2}, \tilde{x}_{e1}) + \tilde{d}(\tilde{x}_{a2}, \tilde{x}_{a1}) \leq q\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{eo}, \tilde{x}_{a0})\big) + q\tilde{d}\big((\tilde{x}_{a1}, \tilde{x}_{e1}), (\tilde{x}_{ao}, \tilde{x}_{a0})\big)$

Again

$\tilde{d}(\tilde{x}_{en+2}, \tilde{x}_{en+1}) = \tilde{d}\big(f(\tilde{x}_{en+1}, \tilde{x}_{an+1}), f(\tilde{x}_{en}, \tilde{x}_{an})\big)$

$$= \tilde{d}\big(f^n(\tilde{x}_{a1}, \tilde{x}_{e1}), f^n(\tilde{x}_{ao}, \tilde{x}_{e0})\big)$$

$$\leq q^n\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{eo}, \tilde{x}_{a0})\big)$$

$\tilde{d}(\tilde{x}_{an+1}, \tilde{x}_{an}) = \tilde{d}\big(f(\tilde{x}_{an}, \tilde{x}_{en}), f(\tilde{x}_{an-1}, \tilde{x}_{en-1})\big)$

$$= \tilde{d}\big(f^n(\tilde{x}_{a1}, \tilde{x}_{e1}), f^n(\tilde{x}_{ao}, \tilde{x}_{e0})\big)$$

$$\leq q^n\tilde{d}\big((\tilde{x}_{a1}, \tilde{x}_{e1}), (\tilde{x}_{ao}, \tilde{x}_{a0})\big)$$

$\tilde{d}(\tilde{x}_{en+1}, \tilde{x}_{en}) + \tilde{d}(\tilde{x}_{an+1}, \tilde{x}_{an}) \leq q^n\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{eo}, \tilde{x}_{a0})\big) + q^n\tilde{d}\big((\tilde{x}_{a1}, \tilde{x}_{e1}), (\tilde{x}_{ao}, \tilde{x}_{a0})\big)$

Let $\tilde{d}(\tilde{x}_{en+1}, \tilde{x}_{en}) + \tilde{d}(\tilde{x}_{an+1}, \tilde{x}_{an}) = d_n$ then $d_n \leq qd_n$

Similarly, we can prove $d_n \leq qd_{n-1} \leq \cdots \leq q^n d_0$

We now verify that $\{\tilde{x}_{en}\}$is cauchy

$$\tilde{d}(\tilde{x}_{en}, \tilde{x}_{em}) \leq \tilde{d}(\tilde{x}_{en}, \tilde{x}_{an+1}) + \tilde{d}(\tilde{x}_{an+1}, \tilde{x}_{an+2}) + \cdots + \tilde{d}(\tilde{x}_{em-1}, \tilde{x}_{em})$$

$$\leq q^n\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{e0}, \tilde{x}_{a0})\big) + q^{n+1}\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{e0}, \tilde{x}_{a0})\big) + \cdots +$$

$$q^{m-1}\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{e0}, \tilde{x}_{a0})\big)$$

$$= [q^n + q^{n+1} + \cdots + q^{m-1}]\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{e0}, \tilde{x}_{a0})\big)$$

$$\leq \frac{q^{m-n}}{1-q}\tilde{d}\big((\tilde{x}_{e1}, \tilde{x}_{a1}), (\tilde{x}_{e0}, \tilde{x}_{a0})\big)$$

Since $q \in [0,1)$ then

@k:T Æ T ;Æ:T Æ T ;oQ M ?Æ @k:T Æ T ;Æ:T Æ T ;oL r
ÆÆ \¶ s F M ÆÆ \¶

@k: Ø Æ T ;Æ:T Æ T ;oL r
ÆÆ \¶

Similarly, @k:T Æ T ;Æ:T Æ T ;oL r
Æ\¶

Hence <T = <T = .Since k: @ is complete.Then there exists
Æ Æ L Æ

T, T — such that
Æ Æ

T \ T, T \ T as J \ »
Æ Æ

Now

T \ T
Æ

B:T Æ T ;L T
Æ?5 Æ?5
Æ\¶

B :T Æ T ;L T
Æ\¶ Æ?5 Æ?5

B:T Æ T ;L T

And

T \ T
Æ

B:T Æ T ;L T
Æ?5 Æ?5
Æ\¶

B :T Æ T ;L T
Æ\¶ Æ?5 Æ?5

B:T Æ T ;L T

Uniqueness

Let T L B:T Æ T for some T — then
Æ Æ

@:T Æ T ;L @:B:T ;ÆB:T ;;Q M@:T Æ T ;L M@:B:T ;ÆB:T ;;Q M @:T Æ T ;
Æ Æ Æ

Q fi Q M @:T Æ T ;\ r =O J \ »
Æ

Since M— :rÆ;Æ @:T Æ T ;L r
Æ

T L T

Theorem(3.6)

Let $((\tilde{X},\tilde{d}),\leq)$ be a partial ordered complete soft metric space and $f:\tilde{X}\to\tilde{X}, g:\tilde{X}x\tilde{X}\to\tilde{X}$ be two functions of $\tilde{X}$such that

(a) $g(\tilde{X}x\tilde{X})\subseteq f(\tilde{X})$

(b) there exists $q\in[0,1)$such that for all $\tilde{x}_e,\tilde{x}_a\in\tilde{X}$

@:C:T ÆT ;ÆC:T ÆT ;;Q M@:BT ÆBT ;

(c) Bcontinuous on L

d) BÆC 4weakly commuting on L

e) $T_Æ \setminus T$, $T_Æ \setminus T$ J \ »implies @:$T_Æ$ Æ$T_Æ$; \ @:T ÆT ;

If there exists T ÆT$_4$ — Lwith BT$_4$ Q C:T ÆT$_4$; and BT$_4$ R C:T ÆT$_4$;

Then BÆCthave a unique common coupled fixed point in L

Proof

Let T ÆT$_4$ — Lsuch that □ T$_4$ Q C:T ÆT$_4$; and BT$_4$ R C:T ÆT$_4$from (a) we can take T$_5$ — Lsuch that for J L sÆÆ

BT$_5$ L C:T ÆT$_4$;

BT$_5$ L C:T ÆT$_4$;

BT$_6$ L C:T$_5$ÆT$_5$;L C^6 :T ÆT$_4$;L CC:T ÆT$_4$;

BT$_6$ L C:T$_5$ÆT$_5$;L C^6 :T ÆT$_4$;L CC:T ÆT$_4$;

:

:

BT$_{Æ>5}$ L C:T$_Æ$ÆT$_Æ$;L C$^{Æ>5}$:T ÆT$_4$;L CC^Æ :T ÆT$_4$;

BT$_{Æ>5}$ L C:T$_Æ$ÆT$_Æ$;L C$^{Æ>5}$:T ÆT$_4$;L CC^Æ :T ÆT$_4$;

Now we will prove that for all J R r

BT$_Æ$ Q BT$_{Æ>5}$ L C:T$_Æ$ÆT$_Æ$;and BT$_Æ$ R BT$_{Æ>5}$ L C:T$_Æ$ÆT$_Æ$; :s;

Since BT$_4$ Q BT$_5$ L C:T ÆT$_4$; and BT$_4$ R BT$_5$ L C:T ÆT$_4$;then (1) is hold For J L rwe assume that (1) is hold for J P r

Due to the mixed monotone property of Cand BT$_Æ$ Q BT$_{Æ>5}$and BT$_Æ$ R BT$_{Æ>5}$we have

BT$_Æ$ Q BT$_{Æ>5}$ L C:T$_Æ$ÆT$_Æ$;and C:T$_{Æ>5}$ÆT$_{Æ>5}$;Q C:T$_Æ$ÆT$_Æ$;= BT$_{Æ>5}$

BT$_{Æ>5}$ Q BT$_{Æ>6}$ L C:T$_{Æ>5}$ÆT$_{Æ>5}$;and C:T$_{Æ>6}$ÆT$_{Æ>6}$;Q C:T$_{Æ>5}$ÆT$_{Æ>5}$;= BT$_{Æ>6}$

BT$_4$ Q BT$_5$ Q fi Q BT$_Æ$ Q BT$_{Æ>5}$ Q fi

BT$_5$ R BT$_5$ R fi R BT$_Æ$ R□ T$_{Æ>5}$ R fi

From (b) we get

$$\tilde{d}(g(\tilde{x}_{en}, \tilde{x}_{an}), g(\tilde{x}_{en+1}, \tilde{x}_{an+1})) \leq q\tilde{d}(f\tilde{x}_{en}, f\tilde{x}_{en+1}) = q\tilde{d}(g(\tilde{x}_{en-1}, \tilde{x}_{an-1}), g(\tilde{x}_{en}, \tilde{x}_{an}))$$

$$\tilde{d}(g(\tilde{x}_{an}, \tilde{x}_{en}), g(\tilde{x}_{an+1}, \tilde{x}_{en+1})) \leq q\tilde{d}(f\tilde{x}_{an}, f\tilde{x}_{en+1}) = q\tilde{d}(g(\tilde{x}_{an-1}, \tilde{x}_{en-1}), g(\tilde{x}_{an}, \tilde{x}_{en}))$$

$$\tilde{d}\big(g(\tilde{x}_{en}, \tilde{x}_{an}), g(\tilde{x}_{en+1}, \tilde{x}_{an+1})\big) + \tilde{d}\big(g(\tilde{x}_{an}, \tilde{x}_{en}), g(\tilde{x}_{an+1}, \tilde{x}_{en+1})\big)$$

$$\leq q\tilde{d}\big(g(\tilde{x}_{en-1}, \tilde{x}_{an-1}), g(\tilde{x}_{en}, \tilde{x}_{an})\big) + q\tilde{d}(g(\tilde{x}_{an-1}, \tilde{x}_{en-1}), g(\tilde{x}_{an}, \tilde{x}_{en}))$$

Let $\tilde{d}\big(g(\tilde{x}_{en}, \tilde{x}_{an}), g(\tilde{x}_{en+1}, \tilde{x}_{an+1})\big) = d_n$

$d_n \leq qd_{n-1}$ which implies $d_n \leq qd_{n-1} \leq \cdots . \leq q^n d_0$

From proposition (2.10)we get

$\{g(\tilde{x}_{en}, \tilde{x}_{an})\}$,$\{g(\tilde{x}_{an}, \tilde{x}_{en})\}$is Cauchy sequence in $(\tilde{X}, \tilde{d})$.

Since $(\tilde{X}, \tilde{d})$ is complete

$$f\tilde{x}_{en} = g(\tilde{x}_{en-1}, \tilde{x}_{an-1}) \to \tilde{x}_e \quad as\ n \to \infty, \text{for some} \tilde{x}_e \in \tilde{X}$$

$$f\tilde{x}_{an} = g(\tilde{x}_{an-1}, \tilde{x}_{en-1}) \to \tilde{x}_a \quad as\ n \to \infty, \text{for some} \tilde{x}_a \in \tilde{X}$$

Let us assume$g$ is continuous then

$$gg(\tilde{x}_{en}, \tilde{x}_{an}) \to g(\tilde{x}_e, \tilde{x}_a) as\ n \to \infty$$

$$gf\tilde{x}_{en} \to g(\tilde{x}_e, \tilde{x}_a) as\ n \to \infty$$

$$gg(\tilde{x}_{an}, \tilde{x}_{en}) \to g(\tilde{x}_a, \tilde{x}_e) as\ n \to \infty$$

$$gf\tilde{x}_{an} \to g(\tilde{x}_a, \tilde{x}_e) as\ n \to \infty$$

$$\tilde{d}(gg(\tilde{x}_{en}, \tilde{x}_{an}), g(\tilde{x}_e, \tilde{x}_a)) \leq q\tilde{d}(fg(\tilde{x}_{en}, \tilde{x}_{an}), f\tilde{x}_e)$$

Since $fg(\tilde{x}_{en}, \tilde{x}_{an}) \to f\tilde{x}_e\ as\ n \to \infty$

$$\tilde{d}(fg(\tilde{x}_{en}, \tilde{x}_{an}), f\tilde{x}_e) \to 0\ \ as\ n \to \infty$$

$$\tilde{d}(gg(\tilde{x}_{en}, \tilde{x}_{an}), g(\tilde{x}_e, \tilde{x}_a)) = 0$$

$$gg(\tilde{x}_{en}, \tilde{x}_{an}) \to g(\tilde{x}_e, \tilde{x}_a) \quad \text{as} \quad n \to \infty$$

Since $f\tilde{x}_{en} = g(\tilde{x}_{en-1}, \tilde{x}_{an-1})$

$$\to gf\tilde{x}_{en} = gg(\tilde{x}_{en}, \tilde{x}_{an}) \to g(\tilde{x}_e, \tilde{x}_a) as\ n \to \infty$$

$$f\tilde{x}_{an} = g(\tilde{x}_{an-1}, \tilde{x}_{en-1})$$

$$\to gf\tilde{x}_{an} = gg(\tilde{x}_{an}, \tilde{x}_{en}) \to g(\tilde{x}_a, \tilde{x}_e) as\ n \to \infty$$

Since g, f R_weakly commuting. Then we have a real $R > 0$such that

$$\tilde{d}\big(gf\tilde{x}_{en}, fg(\tilde{x}_{en}, \tilde{x}_{an})\big) \leq R\tilde{d}(g(\tilde{x}_{en}, \tilde{x}_{an}), f\tilde{x}_{en})$$

Taking limit as $n \to \infty$

$$\lim_{n\to\infty} \tilde{d}\big(gf\tilde{x}_{en}, fg(\tilde{x}_{en}, \tilde{x}_{an})\big) = 0$$

$$\lim_{n\to\infty} gf\tilde{x}_{en} = \lim_{n\to\infty} fg(\tilde{x}_{en}, \tilde{x}_{an})$$

Since$gf\tilde{x}_{en} \to g(\tilde{x}_e, \tilde{x}_a) as\, n \to \infty$

Then $fg(\tilde{x}_{en}, \tilde{x}_{an}) \to g(\tilde{x}_e, \tilde{x}_a) as\, n \to \infty$

Therefore

$\tilde{d}(g(\tilde{x}_{en}, \tilde{x}_{an}), gg(\tilde{x}_{en}, \tilde{x}_{an})) \leq$

$q\tilde{d}(f\tilde{x}_{en}, fg(\tilde{x}_{en}, \tilde{x}_{an}))$

Taking limit as $n \to \infty$

$$\tilde{d}((\tilde{x}_e, \tilde{x}_a), g(\tilde{x}_e, \tilde{x}_a)) \leq q\tilde{d}(\tilde{x}_e, g(\tilde{x}_e, \tilde{x}_a))$$

We get$g(\tilde{x}_e, \tilde{x}_a) = (\tilde{x}_e, \tilde{x}_a)$

From (a) we get

C:T ÆT ;L :T ÆT ;L B T5 BK N KI TA5 — L

@:C:T5 ÆT5 ;ÆC C:TÆ ÆTÆ ;;Q M@:B:T5 ÆBC:T ÆT ;;

@:C:T5 ÆT5 ;ÆC:T ÆT ;;Q M@:B T5 ÆBC:T ÆT ;;

L r

=OJ \ »

C:T5 ÆT5 ;L C:T ÆT ;

C:T ÆT ;L :T ÆT ;L B T5 L C:T5 ÆT5 ;

Since CÆB 4weakly commuting. Then we have a soft real 4 P such that

@:C:T ÆT ;ÆB T4 ;L @kCB T5 ÆBC:T5 ÆT5 ;O

Q 4@:C:T5 ÆT5 ;ÆB T5 ;

L r

□ :T ÆT ;L B T4 L :T ÆT ;

C:T ÆT ;L B T4 L :T ÆT ;

Therefor :T ÆT is a common coupled fixed point of L

Uniqueness

Let T — such that :T ÆTØ ;L C:T ÆTØ ;L BT

@k:T ÆT ;Æ:T ÆTØ ;O L @kC:T ÆT ;ÆC:T ÆTØ ;O

Q M@:BT4 ÆBT ;

L M@k:T ÆT ;Æ:T ÆTØ ;O

Then fromproposition (2.11)

:T ÆT ;L :T ÆTØ ;

4.coupled and common coupled fixed point theorem in fuzzy soft metric spaces

Theorem (4.1)

Let $(\tilde{X},\leq)$be partial ordered set and Let $f:\tilde{X}x\tilde{X}\to\tilde{X}$ be a mapping has mixed monotone property and continuous of fuzzy soft metric space $(\tilde{X},M,*)$. Let $k\in[0,1)$ and for all $\tilde{x}_e,\tilde{x}_a\in\tilde{X}$

$$\frac{1}{M(f(\tilde{x}_{e1},\tilde{x}_{a1}),f(\tilde{x}_{e2},\tilde{x}_{a2}),\tilde{t})}-1\leq kmax\{\frac{1}{M((\tilde{x}_{e1},\tilde{x}_{a1}),f(\tilde{x}_{e1},\tilde{x}_{a1}),\tilde{t})}-1,$$

$$\frac{1}{M((\tilde{x}_{e2},\tilde{x}_{a2}),f(\tilde{x}_{e2},\tilde{x}_{a2}),\tilde{t})}-1\}$$

If there exists $\tilde{x}_{e0},\tilde{x}_{a0}\in\tilde{X}$ such that $\tilde{x}_{e0}\leq f(\tilde{x}_{e0},\tilde{x}_{a0})$ and $\tilde{x}_{a0}\geq f(\tilde{x}_{a0},\tilde{x}_{e0})$ Then f has a coupled fixed point .

Proof

Let $\tilde{x}_{e1}\in\tilde{X}$, such that $\tilde{x}_{e1}=f(\tilde{x}_{e0},\tilde{x}_{a0})$

and $\tilde{x}_{en+1}=f^{n+1}(\tilde{x}_{e0},\tilde{x}_{a0}),\forall n\geq 1$

With

$\tilde{x}_{e0}\leq\tilde{x}_{e1}=f(\tilde{x}_{eo},\tilde{x}_{a0})$ and $\tilde{x}_{a0}\geq\tilde{x}_{a1}=f(\tilde{x}_{ao},\tilde{x}_{e0})$

Due to the mixed monotone property of f, we have

$\tilde{x}_{e0}\leq\tilde{x}_{e1}\leq\tilde{x}_{e2}$ and $\tilde{x}_{a0}\geq\tilde{x}_{a1}\geq\tilde{x}_{a2}$

In general

$$\tilde{x}_{e0}\leq\tilde{x}_{e1}\leq\cdots\leq\tilde{x}_{en}\leq\tilde{x}_{en+1}\leq\cdots$$

$$\tilde{x}_{a0}\geq\tilde{x}_{a1}\geq\cdots\geq\tilde{x}_{an}\geq\tilde{x}_{an+1}\geq\cdots$$

$$\frac{1}{M(\tilde{x}_{en},\tilde{x}_{en+1},\tilde{t})}-1=\frac{1}{M(f(\tilde{x}_{en-1},\tilde{x}_{an-1}),(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}-1$$

$$\leq kmax\{\frac{1}{M(\tilde{x}_{en-1},f(\tilde{x}_{en-1},\tilde{x}_{an-1}),\tilde{t})}-1,\frac{1}{M(\tilde{x}_{en},f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}-1\}$$

Let $t_n=max\{\frac{1}{M(\tilde{x}_{en-1},f(\tilde{x}_{en-1},\tilde{x}_{an-1}),\tilde{t})}-1,\frac{1}{M(\tilde{x}_{en},f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}-1\}$

If $\lim_{n\to\infty}\frac{5}{M(\tilde{x}_{en-1},\tilde{x}_{an-1},\tilde{t})}=1$ then

$$\frac{s}{M(\tilde{x}_{en},f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}>0\quad\text{and}\quad\frac{s}{M(f(\tilde{x}_{en},\tilde{x}_{an}),f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}>0\quad\text{and}\quad\frac{s}{M(\tilde{x}_{en},f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}=0$$

This contradiction .

Thus, $\lim_{n\to\infty}\frac{5}{M(\tilde{x}_{en-1},\tilde{x}_{an-1},\tilde{t})}=1$

$$\frac{s}{M(\tilde{x}_{en},f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}>0\quad\text{and}\quad\frac{s}{M(f(\tilde{x}_{en},\tilde{x}_{an}),f(\tilde{x}_{en},\tilde{x}_{an}),\tilde{t})}>0$$

$$\left\{\frac{1}{M(\tilde{x}_{en-1},f(\tilde{x}_{en-1},\tilde{x}_{an-1}),\tilde{t})}-1\right\}=\left\{\frac{1}{M(f(\tilde{x}_{en-2},\tilde{x}_{an-2}),f(\tilde{x}_{en-1},\tilde{x}_{an-1}),\tilde{t})}-1\right\}$$

$$\leq kmax\{\frac{1}{M(\tilde{x}_{en-2},f(\tilde{x}_{en-2},\tilde{x}_{an-2}),\tilde{t})}-1,$$

$$\frac{1}{M(\tilde{x}_{en-1},f(\tilde{x}_{en-1},\tilde{x}_{\square n-1}),\tilde{t})}-1\}$$

$\left\{\frac{1}{M(\tilde{x}_{en-1},f(\tilde{x}_{en-1},\tilde{x}_{an-1}),\tilde{t})}-1\right\}\leq k\{\frac{1}{M(\tilde{x}_{en-2},f(\tilde{x}_{en-2},\tilde{x}_{an-2}),\tilde{t})}-1\}$for all n

$$\frac{1}{M(\tilde{x}_{en},\tilde{x}_{en+1},\tilde{t})}-1\leq k\left\{\frac{1}{M(\tilde{x}_{en-2},f(\tilde{x}_{en-2},\tilde{x}_{an-2}),\tilde{t})}-1\right\}\leq\cdots$$

$$\leq\ k^n\left\{\frac{1}{M(\tilde{x}_{e0},f(\tilde{x}_{e0},\tilde{x}_{a0}),\tilde{t})}-1\right\}$$

$$\frac{1}{M(\tilde{x}_{an},\tilde{x}_{an+1},\tilde{t})}-1=\frac{1}{M(f(\tilde{x}_{an-1},\tilde{x}_{en-1}),(\tilde{x}_{an},\tilde{x}_{en}),\tilde{t})}-1$$

$$\leq k\square ax\{\frac{1}{M(\tilde{x}_{an-1},f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1,\frac{1}{M(\tilde{x}_{an},f(\tilde{x}_{an},\tilde{x}_{en}),\tilde{t})}-1\}$$

Let $t_n=max\{\frac{1}{M(\tilde{x}_{an-1},f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1,\frac{1}{M(\tilde{x}_{an},f(\tilde{x}_{an},\tilde{x}_{en}),\tilde{t})}-1\}$

If $t_n=\frac{1}{M(\tilde{x}_{an},f(\tilde{x}_{an},\tilde{x}_{en}),\tilde{t})}-1$, then

$$\frac{1}{M(\tilde{x}_{an},\tilde{x}_{en+1},\tilde{t})}-1\leq k\left\{\frac{1}{M(\tilde{x}_{an},f(\tilde{x}_{an},\tilde{x}_{en}),\tilde{t})}-1\right\}=k\{\frac{1}{M(\tilde{x}_{an},\tilde{x}_{en+1},\tilde{t})}-1\}$$

This contradiction .

Thus , $t_n=\frac{1}{M(\tilde{x}_{an-1},f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1$

$$\frac{1}{M(\tilde{x}_{an},\tilde{x}_{en+1},\tilde{t})}-1\leq k\left\{\frac{1}{M(\tilde{x}_{an-1},f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1\right\}$$

$$\left\{\frac{1}{M(\tilde{x}_{an-1},f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1\right\}=\left\{\frac{1}{M(f(\tilde{x}_{an-2},\tilde{x}_{en-2}),f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1\right\}$$

$\leq kmax\{\frac{1}{M(\tilde{x}_{an-2},f(\tilde{x}_{an-2},\tilde{x}_{en-2}),\tilde{t})}-1\}\left\{\frac{1}{M(\tilde{x}_{an-1},f(\tilde{x}_{an-1},\tilde{x}_{en-1}),\tilde{t})}-1\right\}\leq k\{\frac{1}{M(\tilde{x}_{an-2},f(\tilde{x}_{an-2},\tilde{x}_{en-2}),\tilde{t})}-$

1}for all n

$$\frac{1}{M(\tilde{x}_{an},\tilde{x}_{en+1},\tilde{t})}-1\leq k\left\{\frac{1}{M(\tilde{x}_{an-2},f(\tilde{x}_{an-2},\tilde{x}_{en-2}),\tilde{t})}-1\right\}\leq\cdots$$

$$\leq\qquad\qquad k^n\left\{\frac{1}{M(\tilde{x}_{a0},f(\tilde{x}_{a0},\tilde{x}_{e0}),\tilde{t})}-1\right\}$$

$$\left\{\frac{1}{M(\tilde{x}_{en},\tilde{x}_{en+1},\tilde{t})}-1\right\}+\left\{\frac{1}{M(\tilde{x}_{an},\tilde{x}_{en+1},\tilde{t})}-1\right\}$$

$$\leq k\left\{\frac{1}{M(\tilde{x}_{en-2},f(\tilde{x}_{en-2},\tilde{x}_{an-2}),\tilde{t})}-1\right\}+k\{\frac{1}{M(\tilde{x}_{an-2},f(\tilde{x}_{an-2},\tilde{x}_{en-2}),\tilde{t})}-1\}$$

let$d_n=\left\{\frac{1}{M(\tilde{x}_{en},\tilde{x}_{en+1},\tilde{t})}-1\right\}+\left\{\frac{1}{M(\tilde{x}_{an},\tilde{x}_{en+1},\tilde{t})}-1\right\}$ then

$$d_n \leq kd_{n-1} \leq \cdots \leq k^n d_0$$

Hence, for $n<m$, we have

$$\frac{1}{M(\tilde{x}_{en},\tilde{x}_{em},\tilde{t})}-1 \leq \left\{\frac{1}{M(\tilde{x}_{en},\tilde{x}_{en+1},\tilde{t})}-1\right\}+\cdots+\left\{\frac{1}{M(\tilde{x}_{em-1},\ \tilde{x}_{em},\tilde{t})}-1\right\}$$

$$\leq k^n\left\{\frac{1}{M(\tilde{x}_{e0},f(\tilde{x}_{e0},\tilde{x}_{a0}),\tilde{t})}-1\right\}+\cdots+k^{m-1}\left\{\frac{1}{M(\tilde{x}_{e0},f(\tilde{x}_{e0},\tilde{x}_{a0}),\tilde{t})}-1\right\}$$

$$\frac{1}{M(\tilde{x}_{en},\tilde{x}_{em},\tilde{t})}-1 \leq [k^n+\cdots+k^m]\left\{\frac{1}{M(\tilde{x}_{e0},f(\tilde{x}_{e0},\tilde{x}_{a0}),\tilde{t})}-1\right\}$$

Q G ?Æ s
s F G / :T 4 ÆB:T 4 ÆT 4 ;ÆP; F s '

Then <T Æ is Cauchy. Since (ÆÆ/ Æcomplete then there exists T ÆT — Æ
Such that T Æ \ Tand T Æ \ T
since Bontinuous then

T Æ>5 L B:T Æ ÆT Æ ; \ BT
T Æ>5 L B:T Æ ÆT Æ ; \ BT
B:T ÆT ; L T

B:T ÆT ; L T.

Definition (4.2)

Let Band Cbe two mappings of fuzzy soft metric space :ÆÆ/ Æ.;We say that Band Csatisfy the property (E.A) if there exists a sequence T Æsuch that

Æ\¶ BT Æ L Æ\¶ CT Æ L Tfor some T — Æ

Theorem (4.3)

Let B ª Æh Æ\ ÆÆC ª Æ\ Æbe 4weakly commuting maps have mixed monotone property of fuzzy soft metric space kÆÆ/ Æsatisfy the property (E.A) and for all T 5 ÆT 6 — Æ

1) B:Æh Æ; ? C:Æ;

2) $M(f(\tilde{x}_{e1},\tilde{x}_{a1}),f(\tilde{x}_{e2},\tilde{x}_{a2}),k\tilde{t}) \geq M(g\tilde{x}_{e1},g\tilde{x}_{e2},\tilde{t}),\ k \geq 0$

3) $M(f(\tilde{x}_{e1},\tilde{x}_{a1}),f^2(\tilde{x}_{e1},\tilde{x}_{a1}),\tilde{t}) > \max\{M(g\tilde{x}_{e1},gf\tilde{x}_{e1},\tilde{t}),$

$M(f(\tilde{x}_{e1},\tilde{x}_{a1}),g\tilde{x}_{e1},\tilde{t}),M(f^2(\tilde{x}_{e1},\tilde{x}_{a1}),gf(\tilde{x}_{e1},\tilde{x}_{a1}),\tilde{t}),$

$M(f(\tilde{x}_{e1},\tilde{x}_{a1}),gf(\tilde{x}_{e1},\tilde{x}_{a1}),\tilde{t}),M(g\tilde{x}_{e1},f^2(\tilde{x}_{e1},\tilde{x}_{a1}),\tilde{t})$

When ever $f(\tilde{x}_{e1},\tilde{x}_{a1}) \neq f^2(\tilde{x}_{e1},\tilde{x}_{a1})$. Ifthere exists $\tilde{x}_{e0}, \tilde{x}_{a0} \in \tilde{X}$ with $g\tilde{x}_{e0} \leq \square(\tilde{x}_{e0},\tilde{x}_{a0})$ and $g\tilde{x}_{a0} \geq f(\tilde{x}_{a0},\tilde{x}_{e0})$ and if the range of f or g is complete. Then$f$and g have a common coupled fixed point theorem .

Proof

Let $\tilde{x}_{e0}\tilde{x}_{e0} \in \tilde{X}$ such that $g\tilde{x}_{e0} \leq f(\tilde{x}_{e0},\tilde{x}_{a0})$ and $g\tilde{x}_{a0} \geq f(\tilde{x}_{a0},\tilde{x}_{e0})$ and $g\tilde{x}_{en+1} = f^{n+1}(\tilde{x}_{e0},\tilde{x}_{a0})$,$\forall n \geq 1$

$$g\tilde{x}_{an+1} = f^{n+1}(\tilde{x}_{a0},\tilde{x}_{e0})\ ,\forall n \geq 1$$

With

$g\tilde{x}_{e0} \leq g\tilde{x}_{e1} = f(\tilde{x}_{eo},\tilde{x}_{a0})$ and $g\tilde{x}_{a0} \geq g\tilde{x}_{a1} = f(\tilde{x}_{ao},\tilde{x}_{e0})$

Due to the mixed monotone property of f, we have

$g\tilde{x}_{e0} \leq g\tilde{x}_{e1} \leq g\tilde{x}_{e2}$ and $g\tilde{x}_{a0} \geq g\tilde{x}_{a1} \geq g\tilde{x}_{a2}$

In general

$$g\tilde{x}_{e0} \leq g\tilde{x}_{e1} \leq \cdots \leq g\tilde{x}_{en} \leq g\tilde{x}_{en+1} \leq \cdots$$

$$g\tilde{x}_{a0} \geq g\tilde{x}_{a1} \geq \cdots \geq g\tilde{x}_{an} \geq g\tilde{x}_{an+1} \geq \cdots$$

Since the above inquality satisfy the property (E.A) there exists a sequence $\tilde{x}_{en}$ such that

$\lim_{n\to\infty} f(\tilde{x}_{en},\tilde{x}_{an}) = \lim_{n\to\infty} g\tilde{x}_{en} = \tilde{x}_e$ for some $\tilde{x}_e \in \tilde{X}$.

$\lim_{n\to\infty} f(\tilde{x}_{an},\tilde{x}_{en}) = \lim_{n\to\infty} g\tilde{x}_{an} = \tilde{x}_a$ for some $\tilde{x}_a \in \tilde{X}$.

Since $\tilde{x}_a \in f(\tilde{X}x\tilde{X}) \subseteq g(\tilde{X})$.Then there exists some point $\tilde{x}_u,\tilde{x}_v \in \tilde{X}$ such that $g\tilde{x}_u = \tilde{x}_e$ and $g\tilde{x}_v = \tilde{x}_a$

$M(f(\tilde{x}_{en},\tilde{x}_{an}),f(\tilde{x}_u,\tilde{x}_v),k\tilde{t}) \geq M(g\tilde{x}_{en},g\tilde{x}_u,k\tilde{t})$

$M(f(\tilde{x}_{an},\tilde{x}_{en}),f(\tilde{x}_u,\tilde{x}_v),k\tilde{t}) \geq M(g\tilde{x}_{an},g\tilde{x}_v,k\tilde{t})$

Letting $n \to \infty$

$M(g\tilde{x}_u,f(\tilde{x}_u,\tilde{x}_v),k\tilde{t}) \geq M(g\tilde{x}_u,g\tilde{x}_u,\tilde{t})$

$M(g\tilde{x}_v,f(\tilde{x}_v,\tilde{x}_u),k\tilde{t}) \geq M(g\tilde{x}_v,gv,\tilde{t})$

Then we have

$g\tilde{x}_u = f(\tilde{x}_u,\tilde{x}_v)$

$g\tilde{x}_v = f(\tilde{x}_v,\tilde{x}_u)$

Since f and g be R-weakly commuting maps, then there exists $4 P$ such that

/ :BCT$_{Ł}$ ÆCB:T$_{Ł}$ ÆT$_{Ø}$;ÆP; R / FB:T$_{Ł}$ ÆT$_{Ø}$;ÆCT$_{Ł}$ÆP$_{4}$ G L s

/ :BCT$_{Ł}$ ÆCB:T$_{Ł}$ ÆT$_{Ø}$;ÆP; R s

/ :BCT$_{Ł}$ ÆCB:T$_{Ł}$ ÆT$_{Ø}$;ÆP; L s

BCT$_{Ł}$ L CB:T$_{Ł}$ ÆT$_{Ø}$ and BB:T$_{Ł}$ ÆT$_{Ø}$; L BCT$_{Ł}$ L CB:T$_{Ł}$ ÆT$_{Ø}$; L CCT$_{Ł}$

/ :B:T$_{Ł}$ ÆT$_{Ø}$;ÆB^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆP; P </ :CT$_{Ł}$ ÆCB:T$_{Ł}$ ÆT$_{Ø}$;ÆP;Æ

/ :B:T$_{Ł}$ ÆT$_{Ø}$;ÆCT$_{Ł}$ ÆP;Æ/ :B^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆCB:T$_{Ł}$ ÆT$_{Ø}$;ÆP;Æ

/ :B:T$_{Ł}$ ÆT$_{Ø}$;ÆCB:T$_{Ł}$ ÆT$_{Ø}$;ÆP;Æ/ :CT$_{Ł}$ ÆB^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆP;=

/ :B:T$_{Ł}$ ÆT$_{Ø}$;ÆB^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆP; P / :B:T$_{Ł}$ ÆT$_{Ø}$;ÆB^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆP;contradiction

Then / :B:T$_{Ł}$ ÆT$_{Ø}$;ÆB^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆP; P s

/ :B:T$_{Ł}$ ÆT$_{Ø}$;ÆB^{6} :T$_{Ł}$ ÆT$_{Ø}$;ÆP; L s

B:T$_{Ł}$ ÆT$_{Ø}$; L B^{6} :T$_{Ł}$ ÆT$_{Ø}$;

Hence

B:T$_{Ł}$ ÆT$_{Ø}$; L B^{6} :T$_{Ł}$ ÆT$_{Ø}$; L CB:T$_{Ł}$ ÆT$_{Ø}$; L B CT$_{Ł}$ L CCT$_{Ł}$

/ :BCT$_{Ø}$ ÆCB:T$_{Ø}$ ÆT$_{Ł}$;ÆP; R / FB:T$_{Ø}$ ÆT$_{Ł}$;ÆCT$_{Ø}$ÆP$_{4}$ G L s

/ :BCT$_{Ø}$ ÆCB:T$_{Ø}$ ÆT$_{Ł}$;ÆP; R s

/ :BCT$_{Ø}$ ÆCB:T$_{Ø}$ ÆT$_{Ł}$;ÆP; L s

BCT$_{Ø}$ L CB:T$_{Ø}$ ÆT$_{Ł}$ and BB:T$_{Ø}$ ÆT$_{Ł}$; L BCT$_{Ø}$ L CB:Ø$_{Ø}$ ÆT$_{Ł}$; L CCT$_{Ø}$

/ :B:T$_{Ø}$ ÆT$_{Ł}$;ÆB^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆP; P </ :CT$_{Ø}$ ÆCB:T$_{Ø}$ ÆT$_{Ł}$;ÆP;Æ

/ :B:T$_{Ø}$ ÆT$_{Ł}$;ÆCT$_{Ø}$ ÆP;Æ/ :B^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆCB:T$_{Ø}$ ÆT$_{Ł}$;ÆP;Æ

/ :B:T$_{Ø}$ ÆT$_{Ł}$;ÆCB:T$_{Ø}$ ÆT$_{Ł}$;ÆP;Æ/ :CT$_{Ø}$ ÆB^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆP;

/ :B:T$_{Ø}$ ÆT$_{Ł}$;ÆB^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆP; P / :B:T$_{Ø}$ ÆT$_{Ł}$;ÆB^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆPhis contradiction

Then □ :B:T$_{Ø}$ ÆT$_{Ł}$;ÆB^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆP; P s

/ :B:T$_{Ø}$ ÆT$_{Ł}$;ÆB^{6} :T$_{Ø}$ ÆT$_{Ł}$;ÆP; L s

B:T$_{Ø}$ ÆT$_{Ł}$; L B^{6} :T$_{Ø}$ ÆT$_{Ł}$;

Hence

B:T$_{Ø}$ ÆT$_{Ł}$; L B^{6} :T$_{Ø}$ ÆT$_{Ł}$; L CB:T$_{Ø}$ ÆT$_{Ł}$; L B CT$_{Ø}$ L CCT$_{Ø}$

Then B:T$_{Ł}$ ÆT$_{Ø}$ is common coupled fixed point of B and C

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