# On Intuitionistic Anti- Fuzzy Ideal With Respect to an Element of a Ring

***Showq Mohammed .E***

***AL Kufa university college of education for girls***

***Department of mathematic***

***Showqm.ibriheem@uokufa.edu.iq***

***Abstract*** --- *In this paper ,we introduce the notion of intuitionistic anti- fuzzy ideal with respect to an element of a ring and denoted them by* $\sigma$ *-IAFI . We will investigate some the properties of this ideal .*

***الخلاصة*** --- *قدمنا في هذا البحث مفهوم المثالية ضد الضبابية بالنسبة لعنصر ما في الحلقة وسوف نرمز لها بالرمز* $\sigma$ *-IAFI وسوف نقدم بعض العلاقات والنتائج الخاصة بهذه المثالية .*

***Key words*** --- ***intuitionistic fuzzy subset , intuitionistic fuzzy ideal, intuitionistic fuzzy subring .***

## INTRODUCTION

The concept of fuzzy set was initiated by Zadenh in 1965 .Since then these ideal have been applied to other algebraic structures such that as semi group ,group ,ring .etc , In 1986 the notion intuitionistic fuzzy sets denoted by K.T. Atanassov[1] ,as generalization of the notion of fuzzy set , in 2012 the concept of intuitionistic Anti- fuzzy ideal (IAFI) presented by P.K sharma [2] in 2011 introduce a new notion called an intuitionistic fuzzy ideal (I FI ) proposed by K.Meena [3] , in 2010 the notion I.F.S denoted by P.K sharma [4] .

### *1 .Preliminaries*

In this section , some elementary aspects that are necessary for this paper are included and widely known results of intuitionistic fuzzy set theory.In this study R is a commutative ring with unity throughout this study .

**Definition 1.1 [4]**

If A an intuitionistic fuzzy subset of R , then the sets $\{(x, \overline{\mu_A}(x)),\ x \in R\}, \{(x, \overline{\lambda_A}(x)),\ x \in R\}$ where the functions $\overline{\mu_A} : R \to [0,1]$ and $\overline{\lambda_A} : R \to [0,1]$ denoted the degree of membership and the degree of non-membership and $\leq 0\,\overline{\mu_A}(x) + \overline{\lambda_A}(x) \leq 1.$ for $x \in R$ are respectively called fuzzy subset and anti-fuzzy subset of R with respect to intuitionistic fuzzy intuitionistic set A. For $\alpha, \beta \in [0,1]$ , we define the following sets

$U_1(A,\alpha) = \{x \in R, \overline{\mu_A}(x) \geq \alpha\}$ *and*

$L_1(A, \beta) = \{x \in R, \overline{\mu_A}(x) \leq \beta\}$

$U_2(A,\alpha) = \{x \in R, \overline{\lambda_A}(x) \geq \alpha\}$ *and*

$L_2(A, \beta) = \{x \in R, \overline{\lambda_A}(x) \leq \beta\}$

**Remark 1.2 [2]**

When $\overline{\mu_A}(x) +, \overline{\lambda_A}(x) = 1$ , $\forall x \in R$ , then A is called fuzzy set we denoted the , (IFS)

$A = \{(x, \overline{\mu_A}(x), \overline{\lambda_A}(x)),\ x \in R\}$ by $A(\overline{\mu_A}(x), \overline{\lambda_A}(x))$

**Definition 1.3 [2]**

Let R be a ring an (IFS)

$A = \{(x, \overline{\mu_A}(x), \overline{\lambda_A}(x)),\ x \in R\}$ of R is said to be intuitionistic fuzzy subring denoted by (IFS) of R if

1. $\overline{\mu}_A(x-y) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$
2. $\overline{\mu}_A(xy) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$
3. $\overline{\lambda}_A(x-y) \leq \max\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}$
4. $\overline{\lambda}_A(xy) \leq \max\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}$

For all $x, y \in R$

**Definition 1.4 [3]**

If A= $\{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)), x \in R\}$ of a ring , then R is said to be the intuitionistic fuzzy ideal denoted by (IFI) of R if

1. $\overline{\mu}_A(x-y) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$
2. $\overline{\mu}_A(xy) \geq \max\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$
3. $\overline{\lambda}_A(x-y) \leq \max\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}$
4. $\overline{\lambda}_A(xy) \leq \min\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}$

For all $x, y \in R$

**Theorem 1.5 [5]**

Let $f : (R,+,.) \rightarrow (R',+',.')$ be subjective ring homomorphism with the following conditions:

1. If B is (IFI) of $R'$, then $f^{-1}(B)$ is the (IFI) of R.
2. If A is (IFI) of R, then $f(A)$ is the (IFI) of $R'$.

**Definition 1.6 [2]**

Let $f : (R,+,.) \rightarrow (R',+',.')$ be a mapping from a ring R into a ring $R'$ if A is (IFS) of R, then the anti- image of A under $f$ is the (IFS) $f(A)$ of $R'$ denoted by

$$f(A)_{(A)} = \begin{cases} (\inf\limits_{x \in f^{-1}(y)} \{\overline{\mu}_A(x), if \ f^{-1}(y) \neq \phi\}, \\ \sup\limits_{x \in f^{-1}(y)} \{\overline{\lambda}_A(\mathrm{x}) \ , if \ f^{-1}(y) \neq \phi\}, \ \forall \ \mathrm{y} \in \mathrm{R}' \\ (1,0) \qquad , \text{ otherwise} \end{cases}$$

**Theorem 1.7 [2]**

Let $f : (R,+,.) \rightarrow (R',+',.')$ be a mapping from a ring R into a ring $R'$ .Then

1. If B is (IFS) of $R'$ ,then $f^{-1}(B^c) = (f^{-1}(\mathbf{B}))^c$ .
2. If A is (IFS) of R ,then $f(A^c) = (f(A))^c$ .

**Definition 1.8 [2]**

An IFS A= $\{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)), x \in R\}$ of a ring R is said to be intuitionistic Anti- fuzzy ideal (IAFI) of R if

1. $\overline{\mu}_A(x-y) \leq \max\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$
2. $\overline{\mu}_A(xy) \leq \min\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$
3. $\overline{\lambda}_A(x-y) \geq \min\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}$
4. $\overline{\lambda}_A(xy) \geq \max\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}$

## 2 *Intuitionistic anti- fuzzy ideal with respect to an element of a ring R*

In this section , we recall some definitions and results which will be used in what follows. Thought the paper ,we shall denoted a notion Intuitionistic anti- fuzzy ideal with respect to an element of a ring R.

**Definition 2.1**

An (IFS) A= $\{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)), x \in R\}$ of a ring R is said to be intuitionistic anti- fuzzy ideal with respect to an element of a ring R denoted by ($\sigma$ - IAFI ) where $\sigma$ is any element in R if

1. $\overline{\mu}_A(\sigma x - \sigma y) \leq \max\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$
2. $\overline{\mu}_A(\sigma xy) \leq \min\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$
3. $\overline{\lambda}_A(\sigma x - \sigma y) \geq \min\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\}$
4. $\overline{\lambda}_A(\sigma xy) \geq \max\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\}$

**Example 2.2**

Consider the ring N={0,1,b,2} with addition and multiplication ,as elucidated in tables .

| + | 0 | 1 | b | 2 |
|---|---|---|---|---|
| 0 | 0 | 1 | b | 2 |
| 1 | 1 | 0 | 2 | b |
| b | b | 2 | 0 | 1 |
| 2 | 2 | b | 1 | 0 |

| . | 0 | 1 | b | 2 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| b | 0 | b | 0 | b |
| 2 | 0 | 2 | 0 | 2 |

The (IFS) A $= \{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)),\ x \in R\}$ is b-IAFI when

$$\overline{\mu}_A(x) = \begin{cases} 0.6 & \text{if } y \in \{0, b\} \\ 0 & \text{if } y \in \{2,1\} \end{cases},$$

$$\overline{\lambda}_A(x) = \begin{cases} 0.4 & \text{if } y \in \{0, b\} \\ 0 & \text{if } y \in \{2,1\} \end{cases}$$

**Proposition (2.3)**

Let $A = \{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)),\ x \in R\}$ be ($\sigma$ -IAFI ) of R and give $A^* = \{(x, \overline{\mu}^*_A(x), \overline{\lambda}^*_A(x)),\ x \in R\}$ be the (IFS) of R defined by

$$\overline{\mu}^*_A(x) = \overline{\mu}_A(x) + 1 - \overline{\mu}_A(0),$$

$$\overline{\lambda}^*_A(x) = \overline{\lambda}_A(x) - \overline{\lambda}_A(0) + 1 \quad , \forall x \in R$$

Then $A^*$ is ($\sigma$ -IAFI ) of R .

Proof

Let $x \in R$

$$\overline{\mu}^*_A(x) = \overline{\mu}_A(x) + 1 - \overline{\mu}_A(0),$$

$$\overline{\lambda}^*_A(x) = \overline{\lambda}_A(x) - \overline{\lambda}_A(0) + 1 ,$$

we have

$$\overline{\mu}^*_A(\sigma x - \sigma y) = \overline{\mu}_A(\sigma x - \sigma y) + 1 - \overline{\mu}_A(0)$$
$$\leq \max\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\} + 1 - \overline{\mu}_A(0)$$
$$= \max\{\overline{\mu}_A(\sigma x) + 1 - \overline{\mu}_A(0), \overline{\mu}_A(\sigma y) + 1 - \overline{\mu}_A(0)\}$$
$$= \max\{\overline{\mu}^*_A(\sigma x), \overline{\mu}^*_A(\sigma y)\} \ldots\ldots \quad (1)$$

$$\overline{\mu}^*_A(\sigma xy) = \overline{\mu}_A(\sigma xy) + 1 - \overline{\mu}_A(0)$$
$$\leq \min\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\} + 1 - \overline{\mu}_A(0)$$
$$= \min\{\overline{\mu}_A(\sigma x) + 1 - \overline{\mu}_A(0), \overline{\mu}_A(\sigma y) + 1 - \overline{\mu}_A(0)\}$$
$$= \min\{\overline{\mu}^*_A(\sigma x), \overline{\mu}^*_A(\sigma y)\} \ldots\ldots \quad (2)$$

$$\overline{\lambda}^*_A(\sigma x - \sigma y) = \overline{\lambda}_A(\sigma x - \sigma y) - \overline{\lambda}_A(0) + 1$$
$$\geq \min\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\} - \overline{\lambda}_A(0) + 1$$
$$= \min\{\overline{\lambda}_A(\sigma x) - \overline{\lambda}_A(0) + 1, \overline{\lambda}_A(\sigma y) - \overline{\lambda}_A(0) + 1\}$$
$$= \min\{\overline{\lambda}^*_A(\sigma x), \overline{\lambda}^*_A(\sigma y)\} \ldots\ldots \quad (3)$$

$$\overline{\lambda}^*_A(\sigma xy) = \overline{\lambda}_A(\sigma xy) - \overline{\lambda}_A(0) + 1$$
$$\geq \max\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\} - \overline{\lambda}_A(0) + 1$$
$$= \max\{\overline{\lambda}_A(\sigma x) - \overline{\lambda}_A(0) + 1, \overline{\lambda}_A(\sigma y) - \overline{\lambda}_A(0) + 1\}$$
$$= \max\{\overline{\lambda}^*_A(\sigma x), \overline{\lambda}^*_A(\sigma y)\} \ldots\ldots \quad (4)$$

From (1),(2),(3) and (4) we determine that $A^*$ is ($\sigma$ -I A F I)

**Theorem 2.4**

Let A be ($\sigma$ -IAFI) of R and $f : [0, \overline{\mu}_A(0)] \to [0,1]$ , $g : [0, \overline{\lambda}_A(0)] \to [0,1]$ be the increasing function then (IFS) $A_f = \{(x, \overline{\mu}_{Af}(x), \overline{\lambda}_{Af}(x)),\ x \in R\}$ denoted by $\overline{\mu}_{Af}(x) = f(\overline{\mu}_A(x))$ , $\overline{\lambda}_{Af}(x) = f(\overline{\lambda}_A(x))$ is the ($\sigma$ -IAFI) of R .

Proof

For all $x, y \in R$

$$\overline{\mu_{Af}}(\sigma x - \sigma y) = f(\overline{\mu_A}(\sigma x - \sigma y))$$

$$\leq \max\{ f(\overline{\mu_A}(\sigma x)), f(\overline{\mu_A}(\sigma y)) $$

$$= \max\{ \overline{\mu_{Af}}(\sigma x), \overline{\mu_{Af}}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (1)$$

$$\overline{\mu_{Af}}(\sigma xy) = f(\overline{\mu_A}(\sigma xy))$$

$$\leq \min\{ f(\overline{\mu_A}(\sigma x)), f(\overline{\mu_A}(\sigma y))\}$$

$$= \min\{ \overline{\mu_{Af}}(\sigma x), \overline{\mu_{Af}}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (2)$$

$$\overline{\lambda_{Af}}(\sigma x - \sigma y) = f(\overline{\lambda_A}(\sigma x - \sigma y))$$

$$\geq \min\{ f(\overline{\lambda_A}(\sigma x)), f(\overline{\lambda_A}(\sigma y))\}$$

$$= \min\{ \overline{\lambda_{Af}}(\sigma x), \overline{\lambda_{Af}}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (3)$$

$$\overline{\lambda_{Af}}(\sigma xy) = f(\overline{\lambda_A}(\sigma xy))$$

$$\geq \max\{ f(\overline{\lambda_A}(\sigma x)), f(\overline{\lambda_A}(\sigma y))\}$$

$$= \max\{ \overline{\lambda_{Af}}(\sigma x), \overline{\lambda_{Af}}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (4)$$

From (1),(2),(3) and ( 4) we identify that $A_f$ is ($\sigma$ -I A F I ).

**Theorem 2.5**

If $\{A_i | i \in I\}$ is a family of ($\sigma$ -IAFI) on R ,then $\prod_{i \in I} A_i$ is ($\sigma$ -IAFI) of R .

Proof

For all $x, y \in R$ we have

$$\prod_{i \in I} A_i = \left\{ (x, \prod_{i \in I} \overline{\mu_{A_i}}(x), \prod_{i \in I} \overline{\lambda_{A_i}}(x)),\ x \in R \right\}$$

$$\prod_{i \in I} \overline{\mu}_{A_i}(\sigma x - \sigma y) \leq \sup_{i \in I} (\max\{\overline{\mu}_{A_i}(\sigma x), \overline{\mu}_{A_i}(\sigma y)\})$$

$$= \max\{\sup_{i \in I} \overline{\mu}_{A_i}(\sigma x), \sup_{ii \in I} \overline{\mu}_{A_i}(\sigma y)\}$$

$$= \max\{\prod_{i \in I} \overline{\mu}_{A_i}(\sigma x), \prod_{i \in I} \overline{\mu}_{A_i}(\sigma y)\} \ldots\ldots\ldots\ldots\ldots . \quad (1)$$

$$\prod_{i \in I} \overline{\mu}_{A_i}(\sigma xy) = \inf_{i \in I} \{\overline{\mu}_{A_i}(\sigma xy)\}$$

$$\leq \inf_{i \in I} \min\{\overline{\mu}_{A_i}(\sigma x), \overline{\mu}_{A_i}(\sigma y)\}$$

$$= \min\{\inf_{i \in I} \overline{\mu}_{A_i}(\sigma x), \inf_{i \in I} \overline{\mu}_{A_i}(\sigma y)\}$$

$$= \min\{\prod_{i \in I} \overline{\mu}_{A_i}(\sigma x), \prod_{i \in I} \overline{\mu}_{A_i}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (2)$$

$$\prod_{i \in I} \overline{\lambda}_{A_i}(\sigma x - \sigma y) = \inf_{i \in I} \{\overline{\lambda}_{A_i}(\sigma x - \sigma y)\}$$

$$\geq \inf_{i \in I} (\min\{\overline{\lambda}_{A_i}(\sigma x), \overline{\lambda}_{A_i}(\sigma y)\})$$

$$= \min\{\inf_{i \in I} \overline{\lambda}_{A_i}(\sigma x), \inf_{i \in I} \overline{\lambda}_{A_i}(\sigma y)\}$$

$$= \min\{\prod_{i \in I} \overline{\lambda}_{A_i}(\sigma x), \prod_{i \in I} \overline{\lambda}_{A_i}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (3)$$

$$\prod_{i \in I} \overline{\lambda}_{A_i}(\sigma xy) \geq \sup_{i \in I} (\max\{\overline{\lambda}_{A_i}(\sigma x), \overline{\lambda}_{A_i}(\sigma y)\})$$

$$= \max\{\sup_{i \in I} \overline{\lambda}_{A_i}(\sigma x), \sup_{i \in I} \overline{\lambda}_{A_i}(\sigma y)\}$$

$$= \max\{\prod_{i \in I} \overline{\lambda}_{A_i}(\sigma x), \prod_{i \in I} \overline{\lambda}_{A_i}(\sigma y)\} \ldots\ldots\ldots\ldots \quad (4)$$

$$\prod_{i \in I} A_i = \left\{ (x, \prod_{i \in I} \overline{\mu_{A_i}}(x), \prod_{i \in I} \overline{\lambda_{A_i}}(x)),\ x \in R \right\}$$

is ($\sigma$ -I A F I).

**Corollary 2.6**

Let A= $\left\{ (x, \overline{\mu_A}(x), \overline{\lambda_A}(x)),\ x \in R \right\}$ be the ($\sigma$ -IAFI ) of a ring R . Then

1. if $\overline{\mu_A}(\sigma x) \neq \overline{\mu_A}(\sigma y)$ then $\overline{\mu_A}(\sigma x - \sigma y) = \max\{\overline{\mu_A}(\sigma x), \overline{\mu_A}(\sigma y)\}$

2. if $\overline{\lambda_A}(\sigma x) \neq \overline{\lambda_A}(\sigma y)$ then $\overline{\lambda_A}(\sigma x - \sigma y) = \min\{\overline{\lambda_A}(\sigma x), \overline{\lambda_A}(\sigma y)\},\ \forall x, y \in R$

**Theorem 2.7**

Let A= $\{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)), \ x \in R\}$ be the (IFS) of a ring R , then A is the ($\sigma$ -IAFI ) of R if and only if $A^c$ is the ($\sigma$ -IAFI ) of R.

<u>Proof</u>

($\rightarrow$ )

Let A be the ($\sigma$ -IAFI ) of a ring R ,then

1. $\overline{\mu}_A(\sigma x - \sigma y) \le \max\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$

2. $\overline{\mu}_A(\sigma xy) \le \min\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$

3. $\overline{\lambda}_A(\sigma x - \sigma y) \ge \min\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\}$

4. $\overline{\lambda}_A(\sigma xy) \ge \max\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\}, \ \forall x, y \in R$

The proof of (1) is as follows:

$A^c = \{(x, \overline{\mu}^c_A(x), \overline{\lambda}^c_A(x)), \ x \in R\}$ is (IFS) of a ring R, and we have

$\overline{\mu}_A(\sigma x - \sigma y) \le \max\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$ i.e

$\overline{\lambda}^c_A(\sigma x - \sigma y) \le \max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\}$

$\rightarrow \max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\} \ge \overline{\lambda}^c_A(\sigma x - \sigma y)$

$i.e \ \max\{1 - \overline{\mu}^c_A(\sigma x), 1 - \overline{\mu}^c_A(\sigma y)\} \ge 1 - \overline{\mu}^c_A(\sigma x - \sigma y)$

$\rightarrow 1 - \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\} \ge 1 - \overline{\mu}^c_A(\sigma x - \sigma y)$

$i.e \ , \ \overline{\mu}^c_A(\sigma x - \sigma y) \ge \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\}.$

The proof of (2) is as follows:

$\overline{\mu}_A(\sigma xy) \le \max\{\overline{\mu}_A(\sigma x), \overline{\mu}_A(\sigma y)\}$

$\overline{\lambda}^c_A(\sigma xy) \le \max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\}$ thus

$\max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\} \ge \overline{\lambda}^c_A(\sigma xy)$

$i.e \ \max\{1 - \overline{\mu}^c_A(\sigma x), 1 - \overline{\mu}^c_A(\sigma y)\} \ge 1 - \overline{\mu}^c_A(\sigma xy)$

$\rightarrow 1 - \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\} \ge 1 - \overline{\mu}^c_A(\sigma xy)$

$i.e \ , \ \overline{\mu}^c_A(\sigma xy) \ge \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\}.$

The proof of (3) is as follows:

$\overline{\lambda}_A(\sigma x - \sigma y) \ge \min\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\}$

$\overline{\mu}^c_A(\sigma x - \sigma y) \ge \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\}$

$\rightarrow \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\} \le \overline{\mu}^c_A(\sigma x - \sigma y)$

$i.e \ \min\{1 - \overline{\lambda}^c_A(\sigma x), 1 - \overline{\lambda}^c_A(\sigma y)\} \le 1 - \overline{\lambda}^c_A(\sigma x - \sigma y)$

$\rightarrow 1 - \max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\} \le 1 - \overline{\lambda}^c_A(\sigma x - \sigma y)$

$i.e \ , \ \overline{\lambda}^c_A(\sigma x - \sigma y) \le \max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\}.$

The proof of (4) is as follows:

$\overline{\lambda}_A(\sigma xy) \ge \min\{\overline{\lambda}_A(\sigma x), \overline{\lambda}_A(\sigma y)\}$ i.e

$\overline{\mu}^c_A(\sigma xy) \ge \min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\}$ .thus

$\min\{\overline{\mu}^c_A(\sigma x), \overline{\mu}^c_A(\sigma y)\} \le \overline{\mu}^c_A(\sigma xy)$

$i.e \ \min\{1 - \overline{\lambda}^c_A(\sigma x), 1 - \overline{\lambda}^c_A(\sigma y)\} \le 1 - \overline{\lambda}^c_A(\sigma xy)$

$\rightarrow 1 - \max\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\} \le 1 - \overline{\lambda}^c_A(\sigma xy)$

$i.e \ \min\{\overline{\lambda}^c_A(\sigma x), \overline{\lambda}^c_A(\sigma y)\} \le \overline{\lambda}^c_A(\sigma xy).$

From (1),(2),(3) and (4) we determine $A^c$ is ($\sigma$ -I A F I ).

$\leftarrow$ )

Clear .

**Proposition 2.8**

Let A= $\{(x, \overline{\mu}_A(x), \overline{\lambda}_A(x)), \ x \in R\}$ be the (IFS) of a ring R .Then A is the ($\sigma$ -IAFI ) of R if and only if , $A^c$ is (IFI) of R .

<u>Proof</u>

Let A be ($\sigma$ -IAFI ) of R .Then

1. $\overline{\mu_A}(\sigma x - \sigma y) \le \max\{\overline{\mu_A}(\sigma x), \overline{\mu_A}(\sigma y)\}$

2. $\overline{\mu_A}(\sigma xy) \le \min\{\overline{\mu_A}(\sigma x), \overline{\mu_A}(\sigma y)\}$

3. $\overline{\lambda_A}(\sigma x - \sigma y) \ge \min\{\overline{\lambda_A}(\sigma x), \overline{\lambda_A}(\sigma y)\}$

4. $\overline{\lambda_A}(\sigma xy) \ge \max\{\overline{\lambda_A}(\sigma x), \overline{\lambda_A}(\sigma y)\},\ \forall x, y \in R$

Holds to show that

$A^c = \{(x, \overline{\mu_A^c}(x), \overline{\lambda_A^c}(x)),\ x \in R\}$ is (IFI) of R

from1 we have

$\overline{\mu_A}(\sigma x - \sigma y) \le \max\{\overline{\mu_A}(\sigma x), \overline{\mu_A}(\sigma y)\}$ i.e

$\overline{\lambda_A^c}(\sigma x - \sigma y) \le \max\{\overline{\lambda_A^c}(\sigma x), \overline{\lambda_A^c}(\sigma y)\}$

$\rightarrow \max\{\overline{\lambda_A^c}(\sigma x), \overline{\lambda_A^c}(\sigma y)\} \ge \overline{\lambda_A^c}(\sigma x - \sigma y)$

$i.e\ \max\{1 - \overline{\mu_A^c}(\sigma x), 1 - \overline{\mu_A^c}(\sigma y)\} \ge 1 - \overline{\mu_A^c}(\sigma x - \sigma y)$

$\rightarrow 1 - \min\{\overline{\mu_A^c}(\sigma x), \overline{\mu_A^c}(\sigma y)\} \ge 1 - \overline{\mu_A^c}(\sigma x - \sigma y)$

$i.e\ ,\ \overline{\mu_A^c}(\sigma x - \sigma y) \ge \min\{\overline{\mu_A^c}(\sigma x), \overline{\mu_A^c}(\sigma y)\}.$

Similarly we can show that

$\overline{\lambda_A^c}(\sigma x - \sigma y) \le \max\{\overline{\lambda_A^c}(\sigma x), \overline{\lambda_A^c}(\sigma y)\}$ .

Also from 2 we have

$\overline{\mu_A}(\sigma xy) \le \min\{\overline{\mu_A}(\sigma x), \overline{\mu_A}(\sigma y)\}$ i.e

$\overline{\lambda_A^c}(\sigma xy) \le \min\{\overline{\lambda_A^c}(\sigma x), \overline{\lambda_A^c}(\sigma y)\}$ thus

$\min\{\overline{\lambda_A^c}(\sigma x), \overline{\lambda_A^c}(\sigma y)\} \ge \overline{\lambda_A^c}(\sigma xy)$

$i.e\ \min\{1 - \overline{\mu_A^c}(\sigma x), 1 - \overline{\mu_A^c}(\sigma y)\} \ge 1 - \overline{\mu_A^c}(\sigma xy)$

$\rightarrow 1 - \max\{\overline{\mu_A^c}(\sigma x), \overline{\mu_A^c}(\sigma y)\} \ge 1 - \overline{\mu_A^c}(\sigma xy)$

$i.e\ ,\ \overline{\mu_A^c}(\sigma xy) \ge \max\{\overline{\mu_A^c}(\sigma x), \overline{\mu_A^c}(\sigma y)\}.$

Similarly we can then determine that

$\overline{\lambda_A^c}(\sigma xy) \ge \max\{\overline{\lambda_A^c}(\sigma x), \overline{\lambda_A^c}(\sigma y)\}$ .

From (1),(2),(3) and (4) we identify $A^c$ is the (IFI) of R .When $A^c$ is the (IFI) of R we can also show that A is the ($\sigma$ -IAFI ) of R .

**Theorem 2.9**

The union of two ($\sigma$ -IAFIs) of a ring R is also ($\sigma$ -IAFI ) of R .

<u>Proof</u>

Let A,B be two ($\sigma$ -IAFI s ) of R , $A^c$, $B^c$ are (IFIs) of R by Theorem (2.8) , $A^c \cap B^c$ is the (IFI) of R $\rightarrow$ $(A \cup B)^c$ is (IFI) of R $\rightarrow$ $(A \cup B)$ is the ($\sigma$ -IAFI ) of R .

**Theorem 2.10**

The intersection of two ( $\sigma$ -IAFIs ) of a ring R ($\sigma$ -IAFI ) of R .

Proof

Let A,B be two ($\sigma$ -I A F Is ) of R , $A^c$, $B^c$ are (IFI) Of R by Theorem (2.9) , $A^c \cup B^c$ is (IFI) of R $\rightarrow$ $(A \cap B)^c$ is (IFI) of R $\rightarrow$ $(A \cap B)$ is ($\sigma$ -IAFI ) of R .

**Theorem 2.11**

Let $f : (R, +, .) \rightarrow (R', +', .')$ be a mapping from a ring R into a ring $R'$ we consider the following rules :

1. if B is a ($\sigma$ -IAFI ) of $R'$, then $f^{-1}(B)$ is ($\sigma$ -IAFI ) of R .
2. if A is a ($\sigma$ -IAFI ) of R, then $f(A)$ is ($\sigma$ -IAFI ) of $R'$.

<u>Proof</u>

1. Let B be the ($\sigma$ -IAFI ) of $R'$ .Then $B^c$ is the (IFI) of $R'$ by Theorem (2.8) and so $f^{-1}(B^c)$ is (IFI) of R by Theorem (1.5 ),i.e., $(f^{-1}(B))^c$ is the (IFI) of R , hence $f^{-1}(B)$ is ($\sigma$ -IAFI ) of R .
2. Let A is ($\sigma$ -IAFI ) of R .Then $A^c$ is (IFI) of R and $f(A^c)$ is (IFI) of $R'$ by Theorem (1.5) $f(A^c) = (f(A))^c$, so that $(f(A))^c$ is (IFI) of $R'$ and hence f(A) is ($\sigma$ -IAFI ) of $R'$ .

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