

THE SHEARED-SLAB METHOD (SSM):
A MODIFICATION ON THE CLASSICAL SLAB METHOD
(CSM)
FOR EXTRUSION ANALYSIS THROUGH CONICAL DIES⁺

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Abstract

This paper presents, for the first time, a very useful improvement on the classical slab method for direct-extrusion and drawing of solid rods through conical dies, in which an allowance for transverse shear effect is formulated successfully. The present asymptotic approximation for shear linearity within the billet-die domain, results in good agreement of the relative pressure ratio with those acquired by professional experiment, and in some cases it seems better than that achieved by the excellent upper-bound approaches (UBA) of spherical velocity field, while the averaging of the shear makes the formal expression of the pressure ratio very similar to that obtained by the lengthy UBA of right-trapezoidal velocity field. It can now confidently assign the localized inner surface pressure at any position of the die by the new developed governing equation of the acting forces for design and analysis purposes in relevant field.

المستخلص

يعرض هذا البحث، للمرة الاولى، تطويرات مفيدة في طريقة "الشريحة" الكلاسيكية للبيثق المباشر وكذلك سحب القضبان المصمتة خلال القوالب المخروطية، إذ تم رياضياً إدخال تأثير إجهاد القص المستعرض بنجاح كامل وبشكلين مبسطين لهذا الإجهاد. أعطى الشكل الأول (إجهاد القص الخطي) نتائج للضغط أو السحب النسبي جيدة الاتفاق مع تلك المستحصلة من التجارب المختبرية الموثوقة في تشكيل المعادن، كما وجد أنه، في حالات معينة من زوايا القالب ونسب التقلص في المساحة، كان أفضل نسبياً من طريقة الحد الأعلى ذات مجال السرعة الكروية، بينما أعطى الشكل الثاني (إجهاد القص المتوسط) تعبيراً لمعادلة الضغط النسبي مع زاوية القالب مشابه لذلك المستنبط بطريقة الحد الأعلى ذات مجال السرعة المنحرفة القائمة مع فرق السهولة والبساطة في الحل العددي. وهكذا أصبح الآن وبموثوقية عالية من تحديد قيم الإجهادات والضغطات لسطح القالب الداخلي وفي أية نقطة اعتباطية منه باستخدام المعادلة المطورة الحاكمة للتوازن لأغراض تصميم القالب و إجراء التحليل الميكانيكي لحالات مناظرة من التشكيل.

NOTATIONS

A	Die cross section area.	s	Sector arc length.
c	Cross sector chord length.	x	Die central coordinate.

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f_s	Integral term.	α	Die semi-angle.
k	Billet yield shear stress.	θ	Transverse coordinate.
L	Axial length.	σ_x	Localized forming stress.
m	Friction factor.	τ_f	Inner surface friction stress.
P_r	Slab radial pressure.	τ_s	Billet transverse shear stress.
R	Die cross section radius.	Δ	Difference operator.

1. Introduction

The first elementary theory of wire drawing and extrusion of strips and rods, was very understood since many past decades. The statements “*slug – equilibrium*”, “*local stress-evaluation*” or simply “*slab method*”, occasionally refer to the concept bases of that theory. The older theoretical works by *Sachs et.al* [1], *Avitzur* [2] and *Green*[3] stand as typical treatments on relevant subject. The best benefits of this method comes from its simplicity and ease of derivation and the capability of estimating of the locale-wise forming stress through the die bearing area. No shear stresses are being involved in the content of the method except that which is responsible for the friction at the slug-die interference surface. The method is based upon a classic engineering theory of plasticity, where no kinematic relationships of strains and strain rates are maintained as well as it makes use of the simple criteria of failure by *Tresca*[4]. These fruitful features of the *Classic Slab Method* (CSM) makes sorrowfully no fair with the unacceptable results gave for certain interval of the conical die angles. The older experimental work, in this field, was that independently conducted by *Wistreich*[5], who tried with several tests of wire drawing for different reduction of area (namely 10%, 20%, 30% and 40%) and nine distinct values of the die semi-angle within the range (2^0 - 12^0). Later on (after less than ten years), *Avitzur* in his paper[6] and then textbook[7], confirmed the analytical discrepancies of the CSM results, of relative drawing or direct-extrusion stresses, with *Wistreich*’s test results. The comparison show good agreement of results for very small conical die angles, but with very bad deviations for respective large angles.

The best achievement of the CSM, for conical die of almost 12^0 semi-angle, approached 13.6% relative decrease in the maximum stress for 40% reduction of area, while it increased to 62.5% for smaller reduction of area (namely 10%)- a matter which marked obviously the weakness of the classical method. The seventies of last century attended the employment of the powerful technique of *Upper-Bound Approach* (UBA) as an active tool of solution for many kinds of metal forming processes, both analytically (for simple die shapes) or numerically for general die profiles with the help of recent advances of the finite-element method (see for example, Refs. [8,9,10,11]). In contrary to the CSM the new UBA predicts the relative stress values overestimating the actual experimental ones, but generally the increase in results are much acceptable than the corresponding decrease given by the classical method. UBA is comparably much complicated in formality and computations. It adopts the hypothesis of strain rate established from a point-wise velocity field of the plastic deformed metal and a proposed constitutive relationships between the direct deviator stresses and shear stresses with the corresponding strain rates. The perfect-plastic ingots assumed to obey *Von-Mises* failure criteria[4], and the total power of deformation is carefully estimated, depending upon the expected (or actually suggested) velocity field of the plastic flow within the bearing volume and in

all its forms; the internal, shear and friction. The result then would be greater than or equal to the external ram power from which an upper margin of the relative forming stress can be determined. This is, in short, the alternative UB procedure in its simplest fashion. In references[12,13] a comprehensive theoretical treatments of UBA were successfully made for metal flow through conical die, with different proposals of kinematic velocity fields converging to specific pole.

The discontinuity velocity surface was taken individually as spherical, trapezoidal and triangular in shape, from which plots of the maximum relative stress, as varied with the die semi-angle, were built-up for certain parameters of reduction of area and friction factor. The plots show an existence of “optimum” forming stress at critical value of die angle generally independent upon the proposed shape of the discontinuity velocity surface. This argument of “optimization” can never be found in the *Lower-Bound Approach* (LBA) [7] or the CSM because of the mentioned lacks in the theory contents of such methods. The CSM did not deal at all with the localized deformation of the metal and the kind of stresses, acting on the boundaries of the “rigid” slab, are absolutely direct only. Therefore, in his opinion, the present author thinks that ignoring of the transverse shear stress represents the main cause of the CSM “failure”.

This paper tries to invigorate the work of slab method through imposing the missing shear stress into the direct-stress system on the balanced slab without losing of method originality. The “new” slab would be then “sheared” transversely and the equilibrium condition of the normal forces will be extended further. The present sheared-slab method (SSM), like any new concept in related field, is faced with some sort of difficulty concerning the mathematical expression of the inserting new item (the shear stress). Fortunately, this has been overcome through symptomatic approximation of the average stress through the ingot section surface and the die entrance-exist ends. An “extended” differential equation of stress-equilibrium is generated and the final maximum stress is mathematically tolerated by the present averaging limits of the shear stress. The new SSM has appreciably succeeded in raising the CSM from its fall, and the flow-stress behavior with the die angle, for the first time in relevant slab technique, is coming in acceptable agreement with those cleverly obtained by *Avitzur Right-trapezoidal Upper-Bound* approach (ARUB) for the complete range of die angle (refer to [12] using $B=90^\circ$ in equation (2-23)), whereas for die angles within the critical range of optimization, the present SSM gives the results very close to those of *Avitzur Spherical Upper-Bound* approach (ASUB). By this, the recent limit analysis, carried out by the efficient UBA shall not really overlook the old competitor CSM, which now is recreated with a new advanced fashion.

1. Theoretical Analysis

2.1. The Present Correction for Shear:

In conventional drawing process, the CSM formulation is very identical to that in the direct-extrusion process, with exception of using sign convention of the applied stresses and their end conditions. Anyhow, the following derivation is strictly applied to solid rod extrusion process. Fig.(1) illustrates the main processing zones in the container, die and land regions with the respective geometrical and mechanical notations very familiarly found in the CSM. The essential forming process is considered to take place in the die where a finitesimal slab disc of length dx is under static equilibrium of acting stresses (The direct

longitudinal stress σ_x , normal stress σ_r , the die inner surface pressure p_r and the associated friction stress τ_f with addition of the suggested shear stress τ_s). Fig.(2) shows an elemental slug, of the formed billet, in a magnified view where a small sector of the slug of swept angle $d\theta$ is supposed to be under equilibrium of forces, in two directions; the central and radial ones and as following:

$$\sigma_x(dA_{\theta 1}) + p_r(dA_s \sin \alpha) + \tau_f(dA_s \cos \alpha) = (\sigma_x + d\sigma_x)(dA_{\theta 2}) \quad \text{.....(1)}$$

$$p_r(dA_c \cos \alpha) + \tau_s(dA_{\theta 1}) = \sigma_r(dA_r) + \tau_f(dA_c \sin \alpha) + (\tau_s + d\tau_s)(dA_{\theta 2}) \quad \text{.....(2)}$$

Using the geometrical relationships of the segmental areas gives:

$$dA_{\theta 1} = \frac{1}{2} R^2 d\theta, \quad dA_c = ds \cdot dc, \quad dA_{\theta 2} = \frac{1}{2} (R + dR)^2 d\theta, \quad dA_r = dc \cdot dx \quad \text{.....(3)}$$

with the chord, arc and trigonometric definitions:

$$dc = R \cdot d\theta, \quad ds = dR/\sin \alpha, \quad dx = dc/\cos \alpha \quad \text{.....(4)}$$

then eq.(1) would be properly simplified into:

$$\frac{R}{2} \frac{d\sigma_x}{dR} + \sigma_x = p_r + \tau_f \tan \alpha \quad \text{.....(5)}$$

whereas Eq.(2) would be appeared as (ignoring the higher terms $d(\cdot) \cdot d(\cdot)$):

$$p_r = \sigma_r + \tau_f \tan \alpha + \tau_s \tan \alpha + \frac{R}{2} \frac{d\tau_s}{dR} \tan \alpha \quad \text{.....(6)}$$

Applying *Tresca*-yield criteria:

$$|\sigma_r - \sigma_x| = \sigma_Y \quad \text{.....(7)}$$

into Eq.(6), with suitable arranging of τ_s -terms, gives:

$$p_r = \sigma_Y + \sigma_x + (\tan \alpha) \tau_f + (\tan \alpha) \frac{1}{2R} \frac{d(\tau_s R^2)}{dR} \quad \text{.....(8)}$$

as an alternative form of Eq.(6). The last derivative term, in above equation, represents the present correction element for shear. The substitution of Eq.(8), into Eq.(5) yields a “*piece-wise difference*” equation of σ_x in the form:

$$d\sigma_x = \left\{ \sigma_Y + \tau_f (\cot \alpha + \tan \alpha) \right\} \frac{2dR}{R} + \tan \alpha \frac{d(\tau_s R^2)}{R^2} \quad \text{.....(9)}$$

The direct integrating of above equation requires a pre-admissibility of the shear stresses τ_f and τ_s . For a material-die interface conditions between slipping ($\tau_f=0$) and sticking ($\tau_f=k$), one can use:

$$\tau_f = mk \quad , \quad 0 \leq m \leq 1 \quad , \quad k = \frac{\sigma_Y}{\sqrt{3}} \quad \text{.....(10)}$$

where m is simply the friction factor and k is the yield shear stress. Thus, eq.(9) can now be written as:

$$d\left(\frac{\sigma_x}{\sigma_Y}\right)_D = \left\{1 + \frac{m}{\sqrt{3}}(\cot \alpha + \tan \alpha)\right\} \left(\frac{2dR}{R}\right) + \tan \alpha \frac{d\left(\frac{\tau_s}{\sigma_Y} R^2\right)}{R^2} \quad \text{.....(11)}$$

which represents the modified governing differential equation of σ_x within the die region (as indicated by suffix “D”). In the region of the cylindrical container (of length L_c) or the land (of length L_l), where $\cot \alpha \cdot dR = dx$ and $\tan \alpha = 0$, the differential equation alters easily to:

$$d\left(\frac{\sigma_x}{\sigma_Y}\right)_C = \frac{2m}{\sqrt{3}} \left(\frac{dx}{R}\right) \quad \text{.....(12)}$$

The suffix “C” denotes the mentioned cylindrical region. For the moment, when no mathematical expression of τ_s is pre-known as function of R (or x), the final integration of Eqs.(11,12) between the successive regions of land, die and container, leads to the present expression of the relative pressure ratio required to achieve the forming process as:

$$\left(\frac{\Delta \sigma_x}{\sigma_Y}\right) = \left\{1 + \frac{m}{\sqrt{3}}(\cot \alpha + \tan \alpha)\right\} 2 \ln \left(\frac{R_0}{R_f}\right) + \frac{2m}{\sqrt{3}} \left(\frac{L_c}{R_0} + \frac{L_l}{R_f}\right) + \tan \alpha \cdot f_s$$

where :

$$f_s = \int_{R=R_f}^{R=R_0} \frac{1}{R^2} d\left(\frac{\tau_s}{\sigma_Y} R^2\right) \quad \text{.....(13)}$$

where $\Delta \sigma_x = (\sigma_{ram} - \sigma_{exit})$ for extrusion, and $(\sigma_{draw} - \sigma_{pull})$ for drawing, noting that the integral term f_s is the new correction parameter for shear in the present SSM.

1.2 Approximate evaluation of f_s :

The down-written thoughts stand as preliminary step to derive an “approximate” formula for τ_s in terms of R , at given sector of the billet section as shown by Fig.(3) where the point-wise value of the stress is τ_s^* . It will be

agreed that at the die entrance (i.e. $R=R_0$) $\tau_s^*=0$ at the section centroid ($r=0$) and $\tau_s^*=k$ at the circumference ($r=R_0$). A linear approximation of the stress in terms of r can be then easily imposed as:

$$\tau_s^* = \frac{k}{R_0} r \quad 0 \leq r \leq R_0 \quad \text{.....(14)}$$

If the averaged value of this stress, throughout the sector area $A=\int r dr d\theta$, is denoted by τ_{s0} , then:

$$\tau_{s0} = \frac{\int \tau_s^* dA}{A} = \frac{\int_{r=0}^{r=R_0} \frac{k}{R_0} r \cdot r \cdot dr \cdot d\theta}{\int_{r=0}^{r=R_0} r \cdot dr \cdot d\theta} = \frac{2}{3} k \quad \text{.....(15)}$$

Supposing now that at the die exit (i.e. $R=R_f$) τ_s is vanishing, then a further linear approximation of τ_{s0} along the die length (from $R=R_0$ to $R=R_f$) can be established as:

$$\frac{\tau_s - \tau_{s0}}{R - R_0} = \frac{0 - \tau_{s0}}{R_f - R_0} \quad \text{from which}$$

$$\tau_s = \frac{2}{3} k \left(\frac{R - R_f}{R_0 - R_f} \right) \quad \text{.....(16)}$$

and hence, the integral term f_s , in eq.(14), would be evaluated as:

$$f_s = \frac{2}{3\sqrt{3}} \left[3 - \frac{2}{\left(\frac{R_0}{R_f} - 1 \right)} \ln \left(\frac{R_0}{R_f} \right) \right] \quad \text{.....(17)}$$

Thinking of that $1 < (R_0/R_f) < \infty$, then Eq.(17) implies:

$$0 < \frac{\ln\left(\frac{R_0}{R_f}\right)}{\left(\frac{R_0}{R_f} - 1\right)} < 1 \quad \text{from which :}$$

$$\frac{2}{3\sqrt{3}} < f_s < \frac{2}{\sqrt{3}} \quad \text{.....(18)}$$

which in turn yields to $\bar{f}_s$ as the averaged magnitude of f_s , in the form of:

$$\bar{f}_s = \frac{4}{3\sqrt{3}} \quad \text{.....(19)}$$

By this, the present final explicit form, of the relative pressure ratio, in Eq.(13), may be now written in two aspects:

(a) For linear shear :

$$\left(\frac{\Delta\sigma_x}{\sigma_Y}\right) = \left\{1 + \frac{m}{\sqrt{3}}(\cot\alpha + \tan\alpha)\right\} 2\ln\left(\frac{R_0}{R_f}\right) + \frac{2m}{\sqrt{3}}\left(\frac{L_c}{R_0} + \frac{L_l}{R_f}\right) + \frac{2\tan\alpha}{3\sqrt{3}}\left(3 - \frac{2\ln(R_0/R_f)}{(R_0/R_f) - 1}\right) \quad \text{.....(20)}$$

(b) For average shear :

$$\left(\frac{\Delta\sigma_x}{\sigma_Y}\right) = \left\{1 + \frac{m}{\sqrt{3}}(\cot\alpha + \tan\alpha)\right\} 2\ln\left(\frac{R_0}{R_f}\right) + \frac{2m}{\sqrt{3}}\left(\frac{L_c}{R_0} + \frac{L_l}{R_f}\right) + \frac{4\tan\alpha}{3\sqrt{3}} \quad \text{.....(21)}$$

3. Numerical and Experimental Results

In order to distinguish between the mentioned four approaches (The CSM, ARUB, ASUB and the present SSM) to quantify the relative pressure ratio as varied with the die semi-angle, Table(1) was built up showing the formal characteristic of $(\Delta\sigma_x/\sigma_Y)$ expression stated by each approach. Fig.(4) illustrates the main variation between the present two expressions of the pressure ratio concluded by the SSM, for arbitrary range of the die semi-angle, fixed friction factor m and three chosen values of the area reduction r . For sake of comparison of the final overall results of the approaches, Figs.(5,6,7,8) show the variation in $(\Delta\sigma_x/\sigma_Y)$ with α as predicted by the CSM, ARUB, ASUB and the present SSM (for average shear) respectively. Different area reduction values were chosen in these plots and for constant friction factor, as can be observed their. The final two Figs.(9,10) give the results of all approaches in one time, for another value of the area reduction and friction factor, with the die semi-angle varying within two ranges respectively for sake of clarification. At last, Table(2) displays the experimental and numerical findings of the four methods, for practical conditions of m , r and α being adapted from Refs.[5,6 & 12].

4. Discussion and Comparison

The general expression of $(\Delta\sigma_x/\sigma_Y)$, in Table(1), according to particular method of analysis, is evidently composed of three parts; the $(2\ln(R_0/R_f))$ one which comes from the internal power of deformation, the $(2m\ln(R_0/R_f))$ one which

is responsible for the friction power of deformation and the last remaining part which refers to the transverse shear effect (entirely ignored in the CSM). Due to the existence of $(\tan\alpha)$ in this part of shear, then it can be expected that at high levels of α -values, the shear will evidently play the major role in assessing the result rising of $(\Delta\sigma_x/\sigma_y)$, as can be seen in Figs.(5,6,7,8,9,10). In contrary to this, the CSM gives the results very much lower because of the $(\cot\alpha)$ which is definitely a falling function in α . In Fig.(4), the present SSM, for linear shear, estimates the pressure ratios less than that of the average shear, especially at high die semi-angles, however, no much significant variations are noticed at large value of area reduction. It can be claimed that up to 10 degrees of α , the two formulae of the SSM make acceptable coincidence. A little consideration into the formality and output results, of the ARUB and the SSM for average shear, is indicated by Table(1) and Figs.(6,8), shows that the two approaches are nearly similar, nevertheless, the SSM predicts slight lower values of the pressure ratio. In comparison of the whole results of the approaches together, Fig.(9) show that, for very small values of α (up to 2°), the different methods mostly coincide in results they give, but they diverges anywhere else. The CSM estimates the worse deviations of the pressure ratio, from the remaining three methods, at higher values of α even in the first range of $(0^\circ-30^\circ)$. The present SSM predicts acceptable findings (close to that of the ASUB) in this range of the angle. Almost 2.5% relative percentage difference of the SSM from ASUB results at $\alpha=30^\circ$, is achieved when $r=30\%$ and $m=0.044$. The relative difference is further increased to higher levels in the increasing of the die semi-angle as indicated by Fig.(10). In spite of that, the present SSM results is still better than that of the ARUB. The question of which approach predicts the closer results to the experimental observations, is quite answered by Table(2). In this table, seven test readings were acquired for similar number of the die semi-angles. It is very evidence that the SSM(linear shear) estimates the results more confidently, with absolute maximum percentage error less than 6.8% from experiment readings, whereas the ASUB, ARUB and the CSM come next with error less than 10.4%, 10.8% and 23% respectively. The benefit of the SSM lies in its simplicity of analysis and computations compared with the lengthy and complicated one of the UBA, besides it serves very efficiently to compute the local direct and shear stresses in the die region, if one thinks in adopting Eqs.(11,12) numerically, a matter preserves to be investigated in next coming works.

5. Conclusion

In brief, three essential remarks can be concluded from the current work as follows:

1. The present SSM has extremely succeeded in raising the fall of the CSM. The result of maximum error of more than 20% is improved now to the level of 7% only by the new method.
2. The present major role of the transverse shear stress is very great at relative higher values of the die semi-angles (more than 30°).
3. In accordance with the UPA, the new SSM gives its results keeping pace with the ARUB approach. The simplicity of the new approach is very obvious here compared with the complicated other ones.

Table(1): Summary of individual expression of the relative pressure ratio as stated by different four approaches in literature.

Approach		$\left(\frac{\Delta \sigma_x}{\sigma_Y} \right) =$
CSM		$\left(1 + \frac{m}{\sqrt{3}} \cot \alpha \right) 2 \ln \left(\frac{R_0}{R_f} \right)$
ARUB		$\left(F(\alpha)^* + \frac{m}{\sqrt{3}} (\cot \alpha + \tan \alpha) \right) 2 \ln \left(\frac{R_0}{R_f} \right) + \frac{4 \tan \alpha}{3\sqrt{3}}$
ASUB		$\left(f(\alpha)^{**} + \frac{m}{\sqrt{3}} \cot \alpha \right) 2 \ln \left(\frac{R_0}{R_f} \right) + \frac{2}{\sqrt{3}} \left(\frac{\alpha}{\sin^2 \alpha} - \cot \alpha \right)$
Present SSM	Linear shear	$\left(1 + \frac{m}{\sqrt{3}} (\cot \alpha + \tan \alpha) \right) 2 \ln \left(\frac{R_0}{R_f} \right) + \frac{2 \tan \alpha}{3\sqrt{3}} \left(3 - \frac{2 \ln(R_0 / R_f)}{(R_0 / R_f) - 1} \right)$
	Average shear	$\left(1 + \frac{m}{\sqrt{3}} (\cot \alpha + \tan \alpha) \right) 2 \ln \left(\frac{R_0}{R_f} \right) + \frac{4 \tan \alpha}{3\sqrt{3}}$

$$(*) F(\alpha) = \frac{1}{9 \tan^2 \alpha} \left[\left(1 + \frac{3}{\cos^2 \alpha} \right)^{3/2} - 8 \right]$$

$$(**) f(\alpha) = \frac{1}{\sin^2 \alpha} \left[1 - \cos \alpha \sqrt{1 - \frac{11 \sin^2 \alpha}{12}} + \frac{1}{\sqrt{132}} \ln \frac{1 + \sqrt{\frac{11}{12}}}{\sqrt{11/12} \cos \alpha + \sqrt{1 - \frac{11 \sin^2 \alpha}{12}}} \right]$$

Table(2): Comparison of the test and numerical results of the pressure ratio with the die semi-angle for variety of mentioned approaches (taking $L_c=L_f=0$, $m=0.044$ and $r=30\%$).

Die semi-angle (deg.)	$(\Delta \sigma_x / \sigma_Y)$ according to:					
	Experiment [5,12]	CSM	ARUB	ASUB	Present SSM	
					Linear shear	Averaged shear
2.33	0.560	0.579 (3.39%)	0.611 (9.11%)	0.611 (9.11%)	0.598 (6.79%)	0.611 (9.11%)
3.33	0.520	0.512 (-1.54%)	0.558 (7.31%)	0.557 (7.11%)	0.539 (3.65%)	0.558 (7.31%)
5.00	0.500	0.460 (-8.00%)	0.529 (5.80%)	0.528 (5.60%)	0.501 (-0.20%)	0.528 (5.60%)
6.50	0.475	0.436 (-8.21%)	0.526 (10.74%)	0.524 (10.32%)	0.489 (2.95%)	0.525 (10.53%)
8.00	0.485	0.421 (-13.20%)	0.532 (9.69%)	0.529 (9.07%)	0.486 (0.21%)	0.531 (9.48%)
10.00	0.500	0.408 (-18.40%)	0.547 (9.40%)	0.543 (8.60%)	0.489 (-2.20%)	0.545 (9.00%)
11.50	0.520	0.401 (-22.88%)	0.562 (8.08%)	0.557 (7.12%)	0.495 (-4.95%)	0.560 (7.69%)

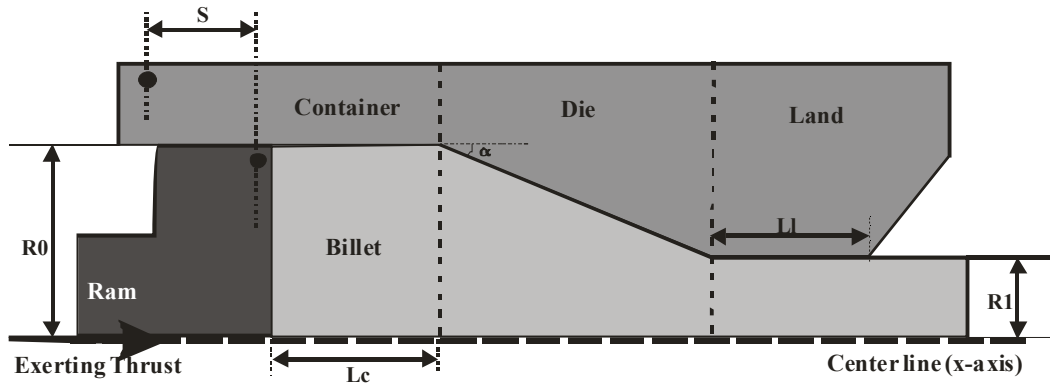


Fig.(1) Conventional notations of typical direct extrusion die “truncated cone”.

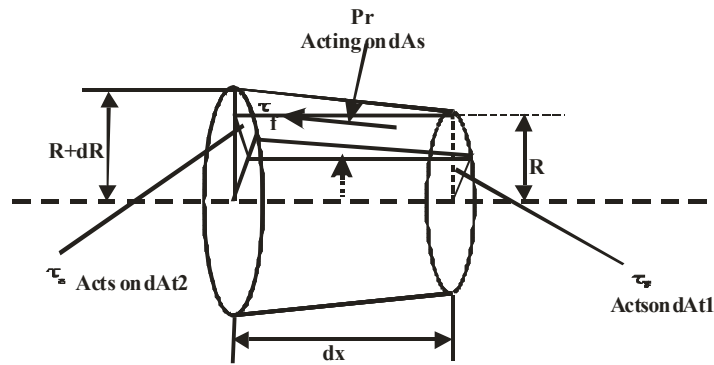


Fig.(2) A segmental element of the billet subjected to deformation stress system

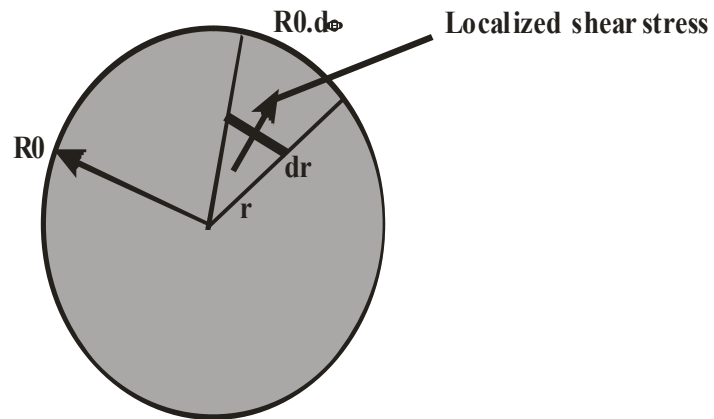
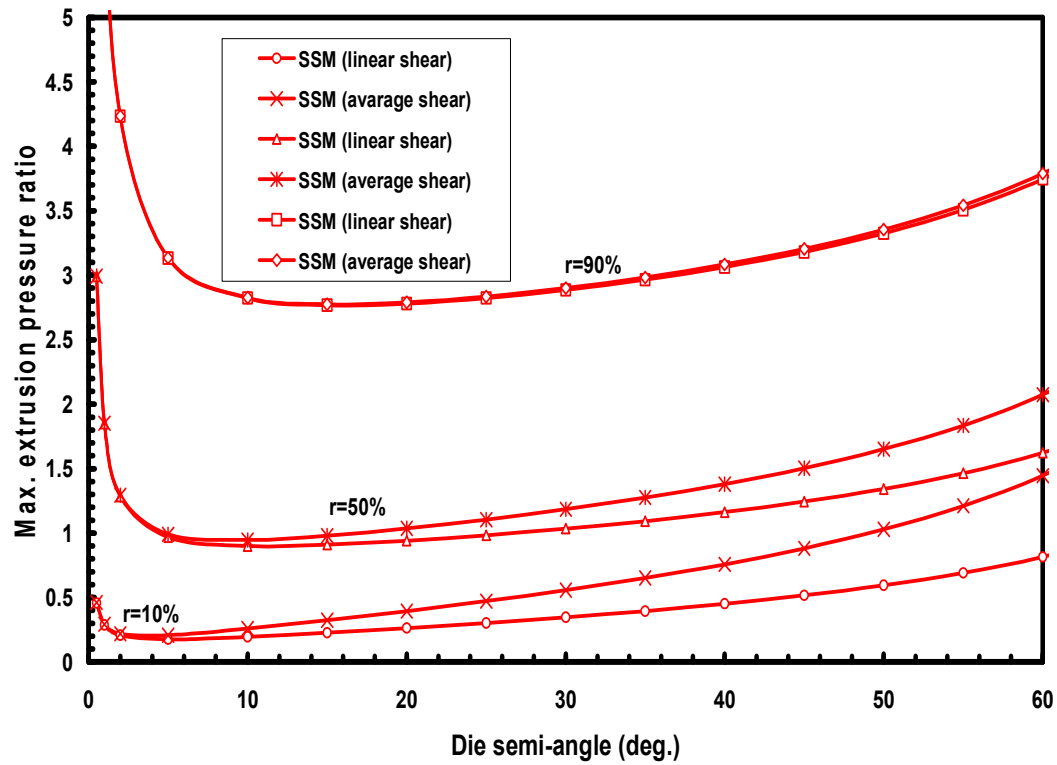


Fig.(3) A sectional sector of the billet at the entrance.



Fig(4). The difference between the present SSM(linear and averaged shear) results of the extrusion pressure ratio with die semi-angle, for typical values of friction factor and area reduction.

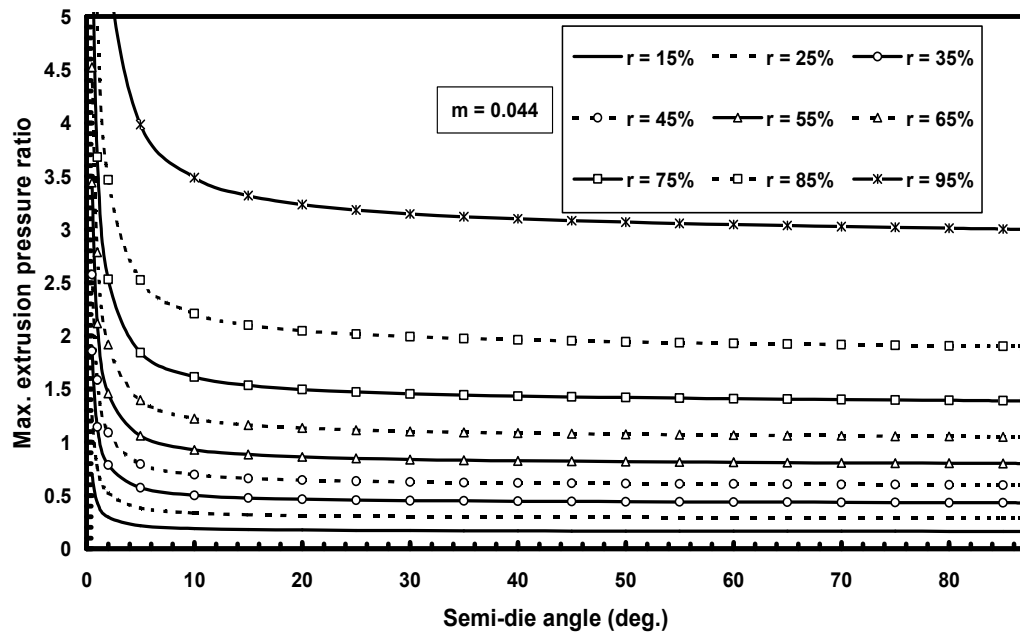


Fig.(5) The CSM results of the die inlet pressure ratio as varied with the semi-die angle for fixed friction factor and variety of area reductions.

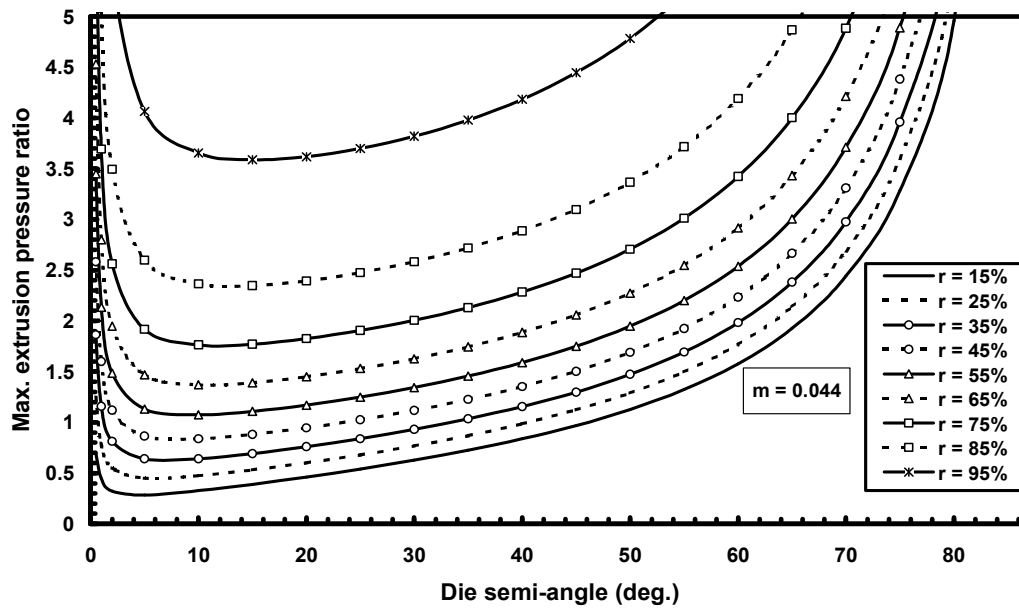


Fig.(6) The ARUB results of the die inlet pressure ratio as varied with the semi-die angle for fixed friction factor and variety of area reductions.

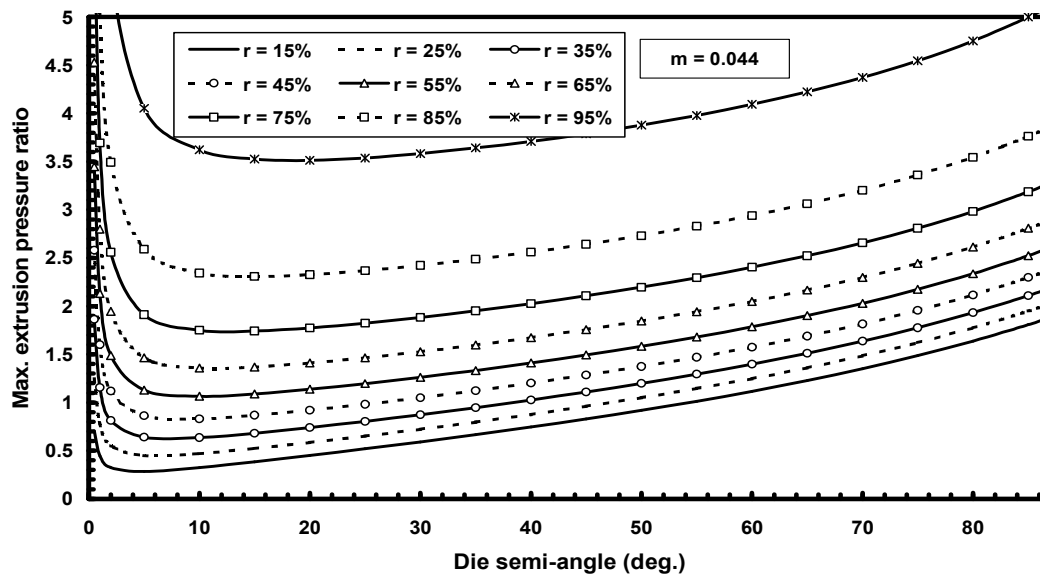


Fig.(7) The ASUB results of the die inlet pressure ratio as varied with the semi-die angle for fixed friction factor and variety of area reductions.

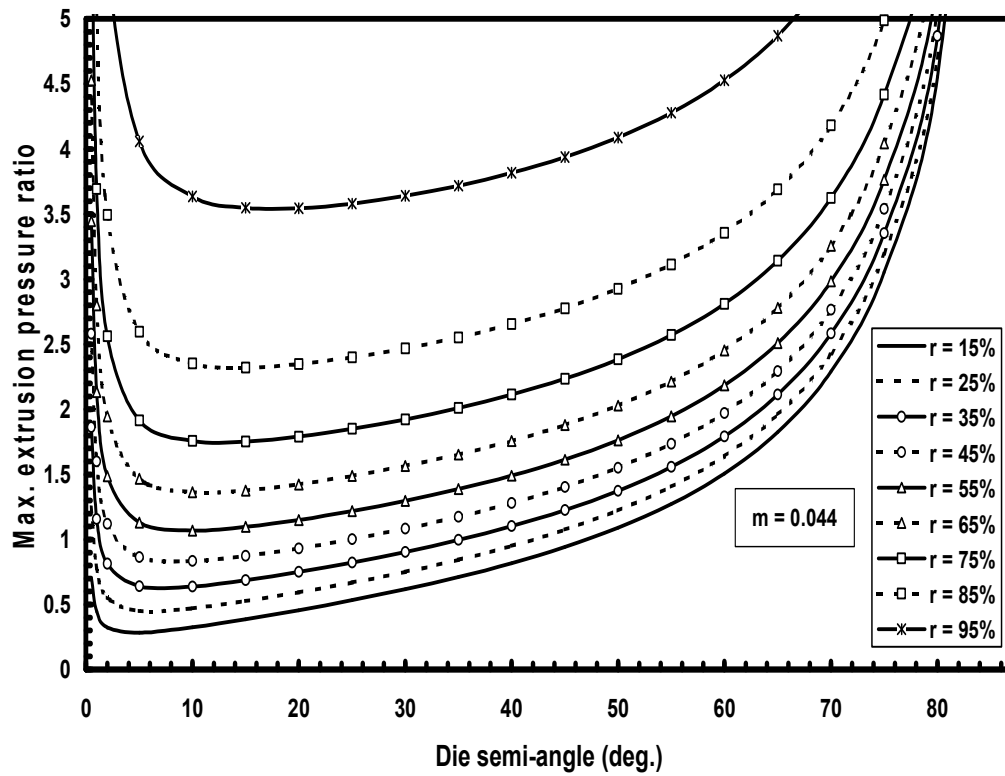


Fig.(8) The present SSM results of the die inlet pressure ratio as varied with the semi-die angle for fixed friction factor and variety of area reductions.

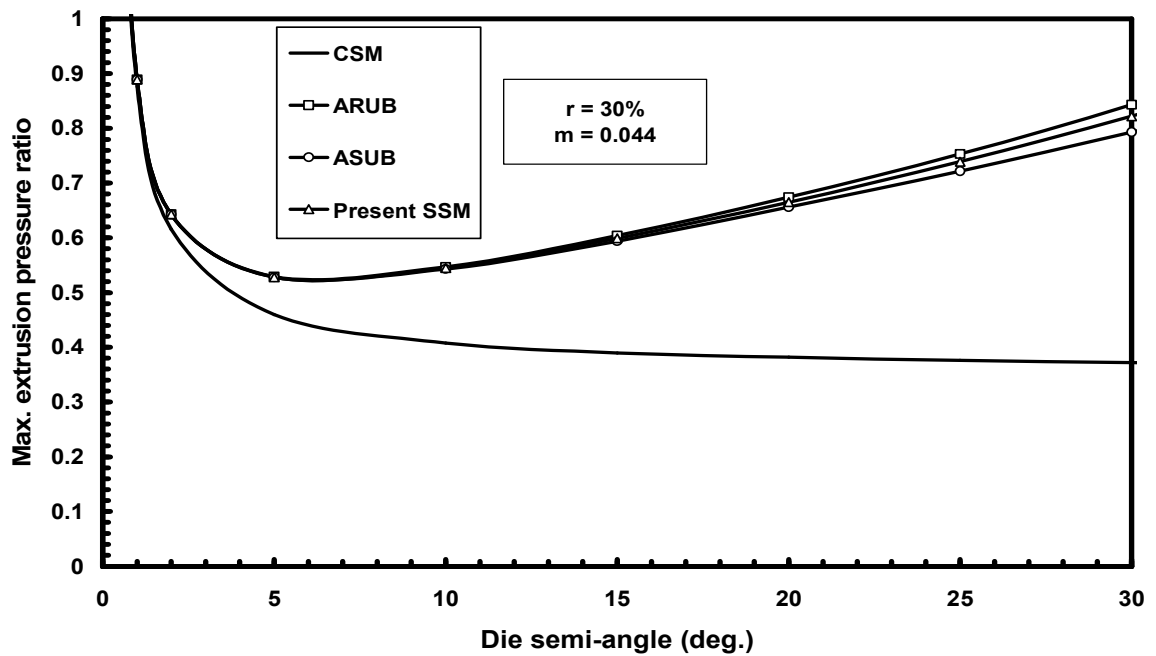


Fig.(9) Zooming of the theoretical results (of Fig.(5)) for the die-inlet pressure ratio as varied within a selected range of the semi-die angle (0-30 deg.).

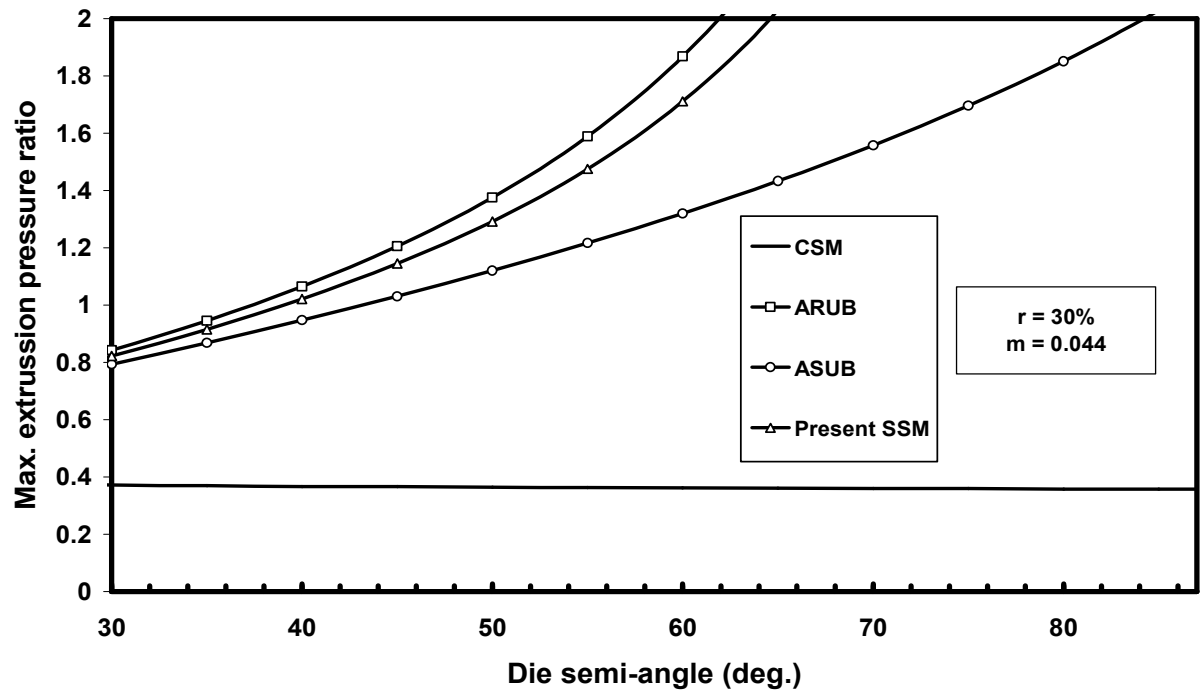


Fig.(10) Another zooming of the theoretical results (of Fig.(5)) for the die-inlet pressure ratio as varied within a range of the semi-die angle (>30 deg.).

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