A NEW APPLICATION OF THEORY OF MATRICES IN VECTOR ANALYSIS AND ELECROMAGNETIC THEORY[+]

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Abstract

This papar aims ،mainly ،to introduce the new concept of ‘‘RYAD and ellustrating its mathematical structure . Many new concepts are Introduced like the concepts of ’’effect ’’ ؛ ’’effect matrix ’’ ؛ ’’ effect diagram ’’؛’’effect coordinates system ’’؛’’ RYADIC transformation of coordination’’؛’’RYADIC equation’’؛’’ RYADIC matrix multiplication … ’’؛etc.

المستخلص

يهدف البحث الى إدخال مفهوم جديد هو " الرياد " ووصف البناء الرياضي له كما يهدف الى إدخـــال عدة مفاهيم أخرى جديدة ، مثلاُ " المصفوفة التاثيريه " ، مخطط التاثيرات ، " نظام الإحداثيات التاثيريـــه ،" التحويل الريادي للإحداثيات ،" المعادلة الريادية الاساسيه "و"الضرب الريادي للمصفوفات"،...الخ .

Mathematical Analysis / “ RYAD “ as a Mathematical Concept

Let $f(x_i)$ and $g(x_i)$ be two continuous functions, where x_i are some continuosindependent variables,and where $i = 1, 2, 3, \ldots\ldots, I$. Assume that there exists some finite positive number m of mathematical operations ; $p_1, p_2, p_3, \ldots, p_m$, which can be applied to $f(x_i)$ or $g(x_i)$ in the form of $p_j[f(x_i)]$ or $p_j[g(x_i)]$, where $j = 1, 2, 3, \ldots\ldots, m$. Assume that there exists another finite positive number n of other mathematical operations; $q_1, q_2, q_3, \ldots\ldots, q_n$ which can be also applied to $f(x_i)$ in the form of $q_k[g(x_i)]$, where $k = 1, 2, 3, \ldots\ldots, n$. Assume, also, that the operations p_j $(j = 1, 2, 3, \ldots\ldots, m)$ and q_k $(k = 1, 2, 3, \ldots\ldots, n)$ are arranged in rectangular (matrix) form such that the operations p_j represent the rows of the matrix while the operations q_k represent its columns as shown in the following arrangement.

No. of columns = n

No. of Rows = m

$$\begin{array}{c|cccc|c}
 & q_1 & q_2 & q_3 \ldots\ldots & q_n & \\
\hline
p_1 & x_{11} & x_{i2} & x_{i3}\ldots\ldots & x_{1n} & 1 \\
p_2 & x_{21} & x_{22} & x_{23}\ldots\ldots & x_{2n} & 2 \\
p_3 & x_{31} & x_{32} & x_{33}\ldots\ldots & x_{3n} & 3 \\
\vdots & \ldots\ldots & & & & \\
. & \ldots\ldots & \ldots\ldots & \ldots\ldots & . & \\
. & \ldots\ldots & \ldots\ldots & \ldots\ldots & . &
\end{array} \quad (1)$$

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$$p_m \quad x_{m1} \quad x_{m2} \quad x_{m3} \ldots\ldots x_{mn}^{m}$$

The next step is to imagine, that there exists some graph formed from the two sets of operations p_j and q_k such that each operator p_j is connected with each q_k. This oriented connection between p_j to q_k [(p_j , p_k)] or from q_k may be interrupted as some " effect " between the two. This effect is oriented, and the corresponding rectangular graph (matrix) of effects in called the " effect graph " or " effect matrix ", correspondingly. It is convenient to symbolize effect between p_j and q_k by the notation

$\overset{\cap}{(p_j , q_k)}$, where the symbol $\cap$ refers to the presence of the effect. Therefore, the above arrangement can be represented as follows:

$$\begin{array}{c|ccccc}
\overset{\cap}{(o , o)} & q_1 & q_2 & q_3 & \ldots\ldots & q_n \\
\hline
p_1 & \overset{\cap}{(p_1, q_1)} & \overset{\cap}{(p_1 , q_2)} & \overset{\cap}{(p_1 , q_3)} & \ldots\ldots & \overset{\cap}{(p_1 , q_n)} \\
p_2 & \overset{\cap}{(p_2 , q_1)} & \overset{\cap}{(p_2 , q_2)} & \overset{\cap}{(p_2 , q_3)} & \ldots\ldots & \overset{\cap}{(p_2 , q_n)} \\
p_3 & \overset{\cap}{(p_3 , q_1)} & \overset{\cap}{(p_3 , q_2)} & \overset{\cap}{(p_3 , q_3)} & \ldots\ldots & \overset{\cap}{(p_3 , q_n)} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
p_m & \overset{\cap}{(p_m , q_1)} & \overset{\cap}{(p_m , q_2)} & \overset{\cap}{(p_m , q_3)} & \ldots\ldots & \overset{\cap}{(p_m , q_n)}
\end{array} \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Assume that there exists a rectangular matrix of coefficients, $\overline{M}$, with an order (moon) in the following form:

$$\overline{A} \triangleq \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \ldots & a_{3n} \\ . & . & . & . & . \\ a_{m1} & a_{m2} & a_{m3} & \ldots & a_{mn} \end{bmatrix}$$

Let us symbolize the effect matrix by the symbol $\overline{M}$ with order (moon); i.e. ,

$$\overline{M} \triangleq \begin{bmatrix} \overset{\frown}{(p_1, q_1)} & \overset{\frown}{(p_1, q_2)} & \overset{\frown}{(p_1, q_3)} & \cdots\cdots & \overset{\frown}{(p_1, q_n)} \\ \overset{\frown}{(p_2, q_1)} & \overset{\frown}{(p_2, q_2)} & \overset{\frown}{(p_2, q_3)} & \cdots\cdots & \overset{\frown}{(p_2, q_n)} \\ \overset{\frown}{(p_3, q_1)} & \overset{\frown}{(p_3, q_2)} & \overset{\frown}{(p_3, q_3)} & \cdots\cdots & \overset{\frown}{(p_3, q_n)} \\ . & . & . & & . \\ . & . & . & & . \\ \overset{\frown}{(p_m, q_1)} & \overset{\frown}{(p_m, q_2)} & \overset{\frown}{(p_m, q_3)} & \cdots\cdots & \overset{\frown}{(p_m, q_n)} \end{bmatrix} \quad \ldots\ldots(3)$$

where $\overline{M}$ is an (moon) effect matrix. If the oriented effect (p_j , q_k) could be interpreted, mathematically, as some oriented mathematical operation, F, originating at p_j and ending at q_k (i.e. , from p_j to q_k), then this may be, simply, symbolized by

$$(p_j , q_k) \longrightarrow [p_j] \;\; F\, [q_k] . \qquad (4)$$

This operator F may be of any type what so ever (e.g., it may refer to addition, subtraction, multiplication, division…, etc.). The quantity $[p_j] \; F \; [q_k]$ may be scaled (or normalized), simply, by multiplication with the corresponding scaling (normalization) coefficient, a_{jk}: that is,

$$[p_j] \;\; F \; [q_k] \longrightarrow a_{jk} [p_j] \;\; F \; [q_k] . \qquad (5)$$

The above quantity in eqn. (5) is called the " scaled (normalized) F – RYAD ". Forming the matrix equation of (5), yields

$$a_{jk} [p_j] \;\; F \;\; [q_k] ; \quad \begin{matrix} j = i, 2, 3, \ldots, m, \\ k = i, 2, 3, \ldots, n \end{matrix} \longrightarrow \overline{A} \,.\, \overline{M}_F . \qquad (6)$$

where $\overline{M}_F$ refers to the (moon) F – RYADic matrix, and the symbol " . " refers to a special type of matrix multiplication in which each element of the matrix A is multiplied with the corresponding element in the matrix $\overline{M}$ and, summing up all of these products. This special type of matrix multiplication is called the RYADic matrix multiplication.

The quantity $\overline{A}\ \overline{M}\ _F$ is an operator which is applied only to the function f' (x_i) to yield.

$$[\overline{A} \cdot \overline{M}]\ f\ (x_i) \qquad (7)$$

Assume the eqn. (7) is located in some coordinates system, *S*, called the effect "effect coordinates system". This effect coordinates system is something like the usually X – Y coordinates system in which the X – axis represents the self (auto) effect between

any two quantities [e.g., ($\overset{\frown}{u}$, u)] and its Y – axis represents the mutual (cross) effect

between any two quantities [e.g., ($\overset{\frown}{u}$, v) or ($\overset{\frown}{v}$, u)]. Let this effect coordinates system *S*, be rotated by an angle of θ to get another effect coordinates system S'. In this case eqn. (7) will be changed to:

$$[\overline{A} \cdot \overline{M}]'\ f'(x_i) = [\overline{A} \cdot \overline{M}]'\ f'(x_i), \qquad (8)$$

where(F')is other an mathematical operation which could be of any type whatsoever [e.g.,addition,convolution, multiplication, ……, etc] and $f'(x_i)$ is nothing but g (x_i). In this case, eqn. (8) may then, be rewritten as:-

$$[\overline{B} \cdot \overline{N}\ _G]\ g(x_i), \qquad (9)$$

Where $\overline{B} = \overline{N}$; $\overline{M} = \overline{A}$; $F' =$ G and g (k_i) = f' (x_i) . (10)

The quantity $\overline{B}\ \overline{N}\ _G$ is called " scaled G – operational RYAD " operating on the second function g (x_i). The fundamental theory of RYADs states, that the " RYADic transformation of effect coordinates is invariant "; i.e.,

$$[\overline{A} \cdot \overline{M}\ _F]\ f'(x_i) = [\overline{B} \cdot \overline{N}\ _g]\ g(x_i) = \text{invariant}, \qquad (11)$$

Equation (11) is called " fundamental RYADic equation ". This equation finds a very wide variety of applications. The following topics apply this fundamental equation in the fields of vector analysis and electromagnetic.

First Application (A)
" RYADic approach in Vector Analysis "
" Basic operations on (between) vectors using the RYADic approach "

Let $\bar{A} = a_1 \bar{i} + a_2 \bar{j} + a_3 \bar{k}$; $\bar{B} = b_1 \bar{i} + b_2 \bar{j} + b_3 \bar{k}$ (12)

Or $\bar{A} = (a_1 \quad a_2 \quad a_3) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix}$; $\bar{B} = (b_1 \quad b_2 \quad b_3) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix}$. (13)

It can be rewritten also as:-

$\bar{A} = (\bar{i} \quad \bar{j} \quad \bar{k}) \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$; $\bar{B} = (\bar{i} \quad \bar{j} \quad \bar{k}) \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$. (14)

Then:-

$$\bar{A} \circ \bar{B} = (a_1 \quad a_2 \quad a_3) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix} \circ (\bar{i} \quad \bar{j} \quad \bar{k}) \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} =$$

$$= (a_1 \quad a_2 \quad a_3) \begin{pmatrix} \bar{i} \circ \bar{i} & \bar{i} \circ \bar{j} & \bar{i} \circ \bar{k} \\ \bar{j} \circ \bar{i} & \bar{j} \circ \bar{j} & \bar{j} \circ \bar{k} \\ \bar{k} \circ \bar{i} & \bar{k} \circ \bar{j} & \bar{k} \circ \bar{k} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (15)$$

OR

$$\bar{A} \circ \bar{B} = (\bar{i} \quad \bar{j} \quad \bar{k}) \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \circ (b_1 \quad b_2 \quad b_3) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix} =$$

$$= (\bar{i} \quad \bar{j} \quad \bar{k}) \begin{pmatrix} a_1 \circ b_1 & a_1 \circ b_2 & a_1 \circ b_3 \\ a_2 \circ b_1 & a_2 \circ b_2 & a_2 \circ b_3 \\ a_3 \circ b_1 & a_3 \circ b_2 & a_3 \circ b_3 \end{pmatrix} \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix} . \quad (16)$$

More generally,

$$= \left(\bar{i} \quad \bar{j} \quad \bar{k} \right) \begin{bmatrix} a_1 \circ b_1 & a_1 \circ b_2 & a_1 \circ b_3 \\ a_2 \circ b_1 & a_2 \circ b_2 & a_2 \circ b_3 \\ a_3 \circ b_1 & a_3 \circ b_2 & a_3 \circ b_3 \end{bmatrix} \begin{bmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{bmatrix} . \qquad (16)$$

More generally,

$$\bar{A} \circ \bar{B} = \begin{bmatrix} a_1 \circ b_1 & a_1 \circ b_2 & a_1 \circ b_3 \\ a_2 \circ b_1 & a_2 \circ b_2 & a_2 \circ b_3 \\ a_3 \circ b_1 & a_3 \circ b_2 & a_3 \circ b_3 \end{bmatrix} \circ \begin{bmatrix} \bar{i} \circ \bar{i} & \bar{i} \circ \bar{j} & \bar{i} \circ \bar{k} \\ \bar{j} \circ \bar{i} & \bar{j} \circ \bar{j} & \bar{j} \circ \bar{k} \\ \bar{k} \circ \bar{i} & \bar{k} \circ \bar{j} & \bar{k} \circ \bar{k} \end{bmatrix}$$

$$= \begin{bmatrix} a_1 \circ b_1 & a_1 \circ b_2 & a_1 \circ b_3 \\ a_2 \circ b_1 & a_2 \circ b_2 & a_2 \circ b_3 \\ a_3 \circ b_1 & a_3 \circ b_2 & a_3 \circ b_3 \end{bmatrix} \circ \begin{bmatrix} \bar{i} \circ \bar{i} & \bar{i} \circ \bar{j} & \bar{i} \circ \bar{k} \\ \bar{j} \circ \bar{i} & \bar{j} \circ \bar{j} & \bar{j} \circ \bar{k} \\ \bar{k} \circ \bar{i} & \bar{k} \circ \bar{j} & \bar{k} \circ \bar{k} \end{bmatrix} \qquad (17)$$

OR

$$\bar{A} \circ \bar{B} = (a_1 b_1)(\bar{i} \circ \bar{i}) + (a_2 b_1)(\bar{i} \circ \bar{j}) + (a_3 b_1)(\bar{i} \circ \bar{k}) + (a_2 b_1)(\bar{j} \circ \bar{i})$$
$$+ (a_2 b_1)(\bar{j} \circ \bar{j}) + (a_2 b_1)(\bar{j} \circ \bar{k}) + (a_3 b_1)(\bar{k} \circ \bar{i}) + (a_3 b_1)(\bar{k} \circ \bar{j})$$
$$+ (a_3 b_1)(\bar{k} \circ \bar{k}) . \qquad (18)$$

Examples:-

A – Let $\circ \longrightarrow .$ [dot scalar product of two vectors] , then

$\bar{i} \circ \bar{i} \rightarrow \bar{i} . \bar{i} = 1$; $\bar{i} \circ \bar{j} \rightarrow \bar{i} . \bar{j} = 0$; $\bar{i} \circ \bar{k} \rightarrow \bar{i} . \bar{k} = 0$;
$\bar{j} \circ \bar{i} \rightarrow \bar{j} . \bar{i} = 0$; $\bar{j} \circ \bar{j} \rightarrow \bar{j} . \bar{j} = 1$; $\bar{j} \circ \bar{k} \rightarrow \bar{j} . \bar{k} = 0$;
$\bar{k} \circ \bar{i} \rightarrow \bar{k} . \bar{i} = 0$; $\bar{k} \circ \bar{j} \rightarrow \bar{k} . \bar{j} = 0$; $\bar{k} \circ \bar{k} \rightarrow \bar{k} . \bar{k} = 1$;

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(19)

Thus

$$\bar{A} \circ \bar{B} = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_3 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{as expected .} \qquad (20)$$

B – Let $\circ \longrightarrow x$ [cross (vector) product of two vectors] , then

$\bar{i} \circ \bar{i} \rightarrow \bar{i} \, x \, \bar{i} = \bar{o}$; $\bar{i} \circ \bar{j} \rightarrow \bar{i} . \bar{j} = \bar{k}$; $\bar{i} \circ \bar{k} \rightarrow \bar{i} \, x \, \bar{k} = -\bar{j}$;
$\bar{j} \circ \bar{i} \rightarrow \bar{j} \, x \, \bar{i} = -\bar{k}$; $\bar{i} \circ \bar{j} \rightarrow \bar{i} . \bar{j} = \bar{o}$; $\bar{j} \circ \bar{k} \rightarrow \bar{j} \, x \, \bar{k} = \bar{i}$;
$\bar{k} \circ \bar{i} \rightarrow \bar{k} \, x \, \bar{i} = \bar{j}$; $\bar{k} \circ \bar{j} \rightarrow \bar{k} \, x \, \bar{j} = -\bar{i}$; $\bar{k} \circ \bar{k} \rightarrow \bar{k} \, x \, \bar{k} = \bar{o}$; (21)

Thus

$$\bar{A} \circ \bar{B} = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix} \circ \begin{bmatrix} \bar{o} & \bar{k} & -\bar{j} \\ -\bar{k} & \bar{o} & \bar{i} \\ \bar{j} & -\bar{i} & \bar{o} \end{bmatrix} =$$

$$= \bar{i} (a_2 b_3) - a_3 b_2) + \bar{j} (a_3 b_1 - a_1 b_3) + \bar{k} (a_1 b_2 - a_2 b_1) =$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \quad \text{as expected .} \qquad (22)$$

$$c - \bar{A} \, . \bar{A} = \begin{bmatrix} a_1^2 & a_1 a_2 & a_1 a_3 \\ a_2 a_1 & a_2^2 & a_2 a_3 \\ a_3 a_1 & a_3 a_2 & a_3^2 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= a_1^2 + a_2^2 + a_3^2 = |\bar{A}|^2 = A^2 \quad \text{as expected.} \tag{23}$$

d – $\bar{A} \times \bar{A} = \begin{pmatrix} a_1^2 & a_1 a_2 & a_1 a_3 \\ a_2 a_1 & a_2^2 & a_2 a_3 \\ a_3 a_1 & a_3 a_2 & a_3^2 \end{pmatrix} \cdot \begin{pmatrix} \bar{o} & \bar{k} & -\bar{j} \\ -\bar{k} & \bar{o} & \bar{i} \\ \bar{j} & -\bar{i} & \bar{o} \end{pmatrix} =$

$$= (a_2 a_3 - a_3 a_2)\,\bar{i} + \bar{j}\,(a_3 a_1 - a_1 a_3) + \bar{k}\,(a_1 a_2 - a_2 a_1) =$$

$$= \bar{o} + \bar{o} + \bar{o} = \bar{o} \quad \text{as expected.} \tag{24}$$

e – Given $\bar{A} = (a_1, a_2, a_3)$; $\bar{B} = (b_1, b_2, b_3)$; $\bar{C} = (c_1, c_2, c_3)$ then:-

$$[\bar{A}\,\bar{B}\,\bar{C}] \equiv \bar{A}\cdot\bar{B}\times\bar{C} = \tag{25}$$

$$= \left(\begin{matrix} \begin{pmatrix} a_1 b_1 c_1 & a_1 b_2 c_1 & a_1 b_3 c_1 \\ a_2 b_1 c_1 & a_2 b_2 c_1 & a_2 b_3 c_1 \\ a_3 b_1 c_1 & a_3 b_2 c_1 & a_3 b_3 c_1 \end{pmatrix} & \\ & \begin{pmatrix} a_1 b_1 c_2 & a_1 b_2 c_2 & a_1 b_3 c_2 \\ a_2 b_1 c_2 & a_2 b_2 c_2 & a_2 b_3 c_2 \end{pmatrix} \end{matrix} \right) \circ \longrightarrow$$

$$
\begin{array}{ccc}
a_3\, b_1 c_2 & a_3\, b_2\, c_2 & a_3\, b_3\, c_2
\end{array}
$$

$$
\left(\begin{array}{cc|c}
a_1\, b_1 c_3 & a_1\, b_2\, c_3 & a_1\, b_3\, c_3 \\
a_2\, b_1 c_3 & a_2\, b_2\, c_3 & a_2\, b_3\, c_3 \\
a_3\, b_1 c_3 & a_3\, b_2\, c_3 & a_3\, b_3\, c_3
\end{array}\right) \qquad \ldots\ldots\ldots\ldots\ldots(26)
$$

$$
\longrightarrow\ \mathrm{o} \left(
\begin{array}{c}
\left(\begin{array}{c|c|c}
\bar{i}\,.\,\bar{i} \times \bar{i} & \bar{i}\,.\,\bar{j} \times \bar{i} & \bar{i}\,.\,\bar{k} \times \bar{i} \\
\bar{j}\,.\,\bar{i} \times \bar{i} & \bar{j}\,.\,\bar{j} \times \bar{i} & \bar{j}\,.\,\bar{k} \times \bar{i} \\
\bar{k}\,.\,\bar{i} \times \bar{i} & \bar{k}\,.\,\bar{j} \times \bar{i} & \bar{k}\,.\,\bar{k} \times \bar{i}
\end{array}\right) \\
\left(\begin{array}{c|c|c}
\bar{i}\,.\,\bar{i} \times \bar{j} & \bar{i}\,.\,\bar{j} \times \bar{j} & \bar{i}\,.\,\bar{k} \times \bar{j} \\
\bar{j}\,.\,\bar{i} \times \bar{j} & \bar{j}\,.\,\bar{j} \times \bar{j} & \bar{j}\,.\,\bar{k} \times \bar{j} \\
\bar{k}\,.\,\bar{i} \times \bar{j} & \bar{k}\,.\,\bar{j} \times \bar{j} & \bar{k}\,.\,\bar{k} \times \bar{j}
\end{array}\right) \\
\left(\begin{array}{c|c|c}
\bar{i}\,.\,\bar{i} \times \bar{k} & \bar{i}\,.\,\bar{j} \times \bar{k} & \bar{i}\,.\,\bar{k} \times \bar{k} \\
\bar{j}\,.\,\bar{i} \times \bar{k} & \bar{j}\,.\,\bar{j} \times \bar{k} & \bar{j}\,.\,\bar{k} \times \bar{k} \\
\bar{k}\,.\,\bar{i} \times \bar{k} & \bar{k}\,.\,\bar{j} \times \bar{k} & \bar{k}\,.\,\bar{k} \times \bar{k}
\end{array}\right)
\end{array}
\right) \mathrm{o}
$$

OR

$$\bar{A}.\bar{B} \times \bar{C} = (a_2\, b_3 - a_3\, b_2)\, c_1 + (a_3\, b_1 - a_1\, b_3)\, c_2 + (a_1\, b_2 - a_2\, b_1)\, c_3\,. \qquad (27)$$

but
$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\, b_2\, c_3 - a_1\, b_3\, c_2 - a_2\, b_1 c_3 + a_2\, b_3\, c_1 + a_3\, b_1 c_2 - a_3\, b_2\, c_1$$
$$= (a_2\, b_3 - a_3\, b_2)\, c_1 + (a_3\, b_1 - a_1\, b_3)\, c_2 + (a_1\, b_2 - a_2\, b_1)\, c_3 \qquad (28)$$

Therefore:-
$$\bar{A}\,.\,\bar{B} \times \bar{C} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad \text{as expected}\,. \qquad (29)$$

It can be, also, proved that $\bar{A} \times \bar{B}\,,\ \bar{C} = \bar{A}\,, \bar{B} \times \bar{C} = [\ \bar{A} \quad \bar{B} \quad \bar{C}\ ]$ by exactly the same manner explained above.

f – More generally

given $\bar{A} = (a_1 \quad a_2 \quad a_3 \quad \ldots\ldots\ldots \quad a_n \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \\ \vdots \end{pmatrix}$; (30)

$\bar{B} = (b_1 \quad b_2 \quad b_3 \quad \ldots\ldots \quad b_n) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \\ \vdots \end{pmatrix}$; $\bar{C} = (c_1 \quad c_2 \quad c_3 \ \ldots\ c_n) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \\ \vdots \end{pmatrix}$ (31)

$\bar{D} = (d_1 \quad d_2 \quad d_3 \quad \ldots\ldots \quad d_n) \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \\ \vdots \end{pmatrix}$; etc ……… then

$\bar{A} \circ \bar{B} \circ \bar{C} \circ \bar{D} \ldots\ldots\ldots \quad , \quad =$

$$\bar{D} = (\, d_1 \quad d_2 \quad d_3 \quad \ldots\ldots \quad d_n\,)\begin{bmatrix} \vdots \\ \bar{i} \\ \bar{j} \\ \bar{k} \\ \vdots \end{bmatrix} \quad ; \qquad \text{etc} \;\ldots\ldots\; \text{then}$$

$$\bar{A} \circ \bar{B} \circ \bar{C} \circ \bar{D} \ldots\ldots \quad , \qquad =$$

Where $\circ \longrightarrow$. or x

$$= \begin{pmatrix}
iiii \ldots & ijii \ldots & ikii \ldots & iiij \ldots & ijij \ldots & ikij \ldots & iiik \ldots & ijik \ldots & ikik \ldots \\
jiii \ldots & jjii \ldots & jkii \ldots & jiij \ldots & jjij \ldots & jkij \ldots & jiik \ldots & jjik \ldots & jkik \ldots \\
kiii \ldots & kjii \ldots & kkii \ldots & kiij \ldots & kjij \ldots & kkij \ldots & kiik \ldots & kjik \ldots & kkik \ldots \\
iiji \ldots & ijji \ldots & ikji \ldots & iiji \ldots & ijji \ldots & ikji \ldots & iijk \ldots & ijjk \ldots & ikjk \ldots \\
jiji \ldots & jjji \ldots & jkji \ldots & jiji \ldots & jjji \ldots & jkji \ldots & jijk \ldots & jjjk \ldots & jkjk \ldots \\
kiji \ldots & kjji \ldots & kkji \ldots & kijj \ldots & kjjj \ldots & kkjj \ldots & kijk \ldots & kjjk \ldots & kkjk \ldots \\
iiki \ldots & ijki \ldots & ikki \ldots & iikj \ldots & ijkj \ldots & ikkj \ldots & iikk \ldots & ijkk \ldots & ikkk \ldots \\
jiki \ldots & jjki \ldots & jkki \ldots & jikj \ldots & jjkj \ldots & jkkj \ldots & jikk \ldots & jjkk \ldots & jkkk \ldots \\
kiki \ldots & kjki \ldots & kkki \ldots & kikj \ldots & kjkj \ldots & kkkj \ldots & kikk \ldots & kjkk \ldots & kkkk \ldots
\end{pmatrix} \circ \longrightarrow$$

$$\longrightarrow \circ \begin{pmatrix}
a_1 b_1 c_1 d_1 \,. & a_1 b_2 c_1 d_1 \,. & a_1 b_1 c_1 d_2 \,. & a_1 b_1 c_1 d_2 \,. & a_1 b_2 c_1 d_2 \,. & a_1 b_2 c_1 d_2 & \\
a_2 b_1 c_1 d_1 \,. & a_2 b_2 c_1 d_1 \,. & a_2 b_1 c_1 d_2 \,. & a_2 b_1 c_1 d_2 \,. & a_2 b_3 c_1 d_2 \,. & a_2 b_2 c_1 d_2 & \\
a_3 b_1 c_1 d_1 \,. & a_3 b_2 c_1 d_1 \,. & a_3 b_1 c_1 d_2 \,. & a_3 b_1 c_1 d_2 \,. & a_3 b_3 c_1 d_2 \,. & a_3 b_2 c_1 d_2 & \\
a1\, b_1 c_2 d_1 \,. & a_1 b_2 c_2 d_1 \,. & a_1 b_1 c_2 d_2 \,. & a_1 b_1 c_2 d_2 \,. & a_1 b_2 c_2 d_2 \,. & a_1 b_2 c_2 d_2 & \\
a_2 b_1 c_2 d_1 \,. & a_2 b_2 c_2 d_1 \,. & a_2 b_1 c_2 d_2 \,. & a_2 b_1 c_2 d_2 \,. & a_2 b_3 c_2 d_2 \,. & a_2 b_2 c_2 d_2 & \\
a_3 b_1 c_2 d_1 \,. & a_3 b_2 c_2 d_1 \,. & a_3 b_1 c_2 d_2 \,. & a_3 b_1 c_2 d_2 \,. & a_3 b_3 c_2 d_2 \,. & a_3 b_2 c_2 d_2 & \\
a_1 b_1 c_3 d_1 \,. & a_1 b_2 c_3 d_1 \,. & a_1 b_1 c_3 d_2 \,. & a_1 b_1 c_3 d_2 \,. & a_1 b_2 c_3 d_2 \,. & a_1 b_2 c_3 d_2 & \\
a_2 b_1 c_3 d_1 \,. & a_2 b_2 c_3 d_1 \,. & a_2 b_1 c_3 d_2 \,. & a_2 b_1 c_3 d_2 \,. & a_2 b_3 c_3 d_2 \,. & a_2 b_2 c_3 d_2 & \\
a_3 b_1 c_3 d_1 \,. & a_3 b_2 c_3 d_1 \,. & a_3 b_1 c_3 d_2 \,. & a_3 b_1 c_3 d_2 \,. & a_3 b_3 c_3 d_2 \,. & a_3 b_2 c_3 d_2 & \\
 & & & & a_1 b_1 c_1 d_3 \,. & a_1 b_2 c_1 d_3 \,. & a_1 b_3 c_1 d_3 \;. \\
 & & & & a_2 b_1 c_1 d_3 \,. & a_2 b_2 c_1 d_3 \,. & a_2 b_3 c_1 d_3 \\
 & & & & a_3 b_1 c_1 d_3 \,. & a_3 b_2 c_1 d_3 \,. & a_3 b_3 c_1 d_3 \\
 & & & & a1\, b_1 c_2 d_3 \,. & a_1 b_2 c_2 d_3 \,. & a_1 b_3 c_2 d_3 \\
 & & & & a_2 b_1 c_2 d_3 \,. & a_2 b_2 c_2 d_3 \,. & a_2 b_3 c_2 d_3 \\
 & & & & a_3 b_1 c_2 d_3 \,. & a_3 b_2 c_2 d_3 \,. & a_3 b_3 c_2 d_3 \\
 & & & & a_1 b_1 c_3 d_3 \,. & a1\, b_2 c_3 d_3 \,. & a_1 b_3 c_3 d_3 \\
 & & & & a_2 b_1 c_3 d_3 \,. & a_2 b_2 c_3 d_3 \,. & a_2 b_3 c_3 d_3 \\
 & & & & a_3 b_1 c_3 d_3 \,. & a_3 b_2 c_3 d_3 \,. & a_3 b_3 c_3 d_3
\end{pmatrix}$$

(32–b)

g – Let $\bar{A} = \bar{A}_x\, \bar{a}_x + \bar{A}_y\, \bar{a}_y + \bar{A}_z\, \bar{a}_z$, and $\bar{\nabla} \triangleq \dfrac{\partial}{\partial x}\, \bar{a}_x + \dfrac{\partial}{\partial y}\, \bar{a}_y + \dfrac{\partial}{\partial z}\, \bar{a}_z$, then (33) g – Let $\overline{A} = \overline{A}_x\,\overline{a}_x + \overline{A}_y\,\overline{a}_y + \overline{A}_z\,\overline{a}_z$, and $\overline{\nabla} \triangleq \dfrac{\partial}{\partial x}\,\overline{a}_x + \dfrac{\partial}{\partial y}\,\overline{a}_y + \dfrac{\partial}{\partial z}\,\overline{a}_z$, then (33)

$$\overline{\nabla} \circ \overline{A} = \begin{array}{cc} & \begin{array}{cccc} \circ & A_x & A_y & A_z \end{array} \\ & \\ \left(\begin{array}{c} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{array} \right. & \left. \boxed{\phantom{\begin{array}{ccc} \dfrac{\partial A_x}{\partial x} & \dfrac{\partial A_y}{\partial x} & \dfrac{\partial A_z}{\partial x} \end{array}}} \right) \end{array} \circ \begin{pmatrix} \overline{i} \circ \overline{i} & \overline{i} \circ \overline{j} & \overline{i} \circ \overline{k} \\ \overline{j} \circ \overline{i} & \overline{j} \circ \overline{j} & \overline{j} \circ \overline{k} \\ \overline{k} \circ \overline{I} & \overline{k} \circ \overline{j} & \overline{k} \circ \overline{k} \end{pmatrix} =$$

$$= \begin{pmatrix} \dfrac{\partial A_x}{\partial x} & \dfrac{\partial A_y}{\partial} & \dfrac{\partial A_z}{\partial} \\ \dfrac{\partial A_x}{\partial} & \dfrac{\partial A_y}{\partial} & \dfrac{\partial A_z}{\partial} \\ \dfrac{\partial A_x}{\partial z} & \dfrac{\partial A_y}{\partial z} & \dfrac{\partial A_z}{\partial z} \end{pmatrix} \begin{pmatrix} \overline{i} \circ \overline{i} & \overline{i} \circ \overline{j} & \overline{i} \circ \overline{k} \\ \overline{i} \circ \overline{i} & \overline{i} \circ \overline{j} & \overline{i} \circ \overline{k} \\ \overline{i} \circ \overline{i} & \overline{i} \circ \overline{j} & \overline{i} \circ \overline{k} \end{pmatrix}. \qquad (34)$$

Examples:-

A – If $\circ \longrightarrow \bullet$, then (35)

$$\overline{\nabla} \,.\, \overline{A} = \begin{pmatrix} \dfrac{\partial A_x}{\partial x} & \dfrac{\partial A_y}{\partial x} & \dfrac{\partial A_z}{\partial x} \\ \dfrac{\partial A_x}{\partial y} & \dfrac{\partial A_y}{\partial y} & \dfrac{\partial A_z}{\partial y} \\ \dfrac{\partial A_x}{\partial z} & \dfrac{\partial A_y}{\partial z} & \dfrac{\partial A_z}{\partial z} \end{pmatrix} \circ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \longrightarrow \text{Divergence of } \overline{A} \text{ (scalar).} \qquad (36)$$

B – If $\circ \longrightarrow x$, then:- (37)

$$\bar{\nabla} \times \bar{A} = \begin{pmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_y}{\partial x} & \frac{\partial A_z}{\partial x} \\ \frac{\partial A_x}{\partial y} & \frac{\partial A_y}{\partial y} & \frac{\partial A_z}{\partial y} \\ \frac{\partial A_x}{\partial z} & \frac{\partial A_y}{\partial z} & \frac{\partial A_z}{\partial z} \end{pmatrix} \circ \begin{pmatrix} \bar{o} & \bar{k} & -\bar{j} \\ -\bar{k} & \bar{i} & \bar{i} \\ \bar{j} & -\bar{i} & \bar{o} \end{pmatrix} =$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \bar{i} - \bar{j} \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \bar{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right). \qquad (38)$$

but,

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \bar{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) - \bar{j} \left(\frac{\partial A_z}{\partial x} - \frac{\delta A_x}{\delta z} \right) + \bar{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right). \qquad (39)$$

Thus

$$\bar{\nabla} \times \bar{A} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad \text{as expected.} \qquad (40)$$

C – Let $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$, and suppose that

$$(\bar{a}_x \overset{\cap}{,} \bar{a}_x) = \bar{a}_x \; ; \quad (\bar{a}_x \overset{\cap}{,} \bar{a}_y) = \bar{o} \; ; \quad (\bar{a}_x \overset{\cap}{,} \bar{a}_z) = \bar{o}$$

$$(\bar{a}_y \overset{\cap}{,} \bar{a}_x) = \bar{o} \; ; \quad (\bar{a}_y \overset{\cap}{,} \bar{a}_y) = \bar{a}_y \; ; \quad (\bar{a}_y \overset{\cap}{,} \bar{a}_z) = \bar{o}$$

$$(\bar{a}_z \overset{\cap}{,} \bar{a}_x) = \bar{o} \; ; \quad (\bar{a}_y \overset{\cap}{,} \bar{a}_y) = \bar{o} \; ; \quad (\bar{a}_z \overset{\cap}{,} \bar{a}_z) = \bar{a}_z \text{, then} \qquad (41)$$

$$\bar{\nabla} \times \bar{A} = \begin{pmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_y}{\partial x} & \frac{\partial A_z}{\partial x} \\ \frac{\partial A_x}{\partial y} & \frac{\partial A_y}{\partial y} & \frac{\partial A_z}{\partial y} \\ \frac{\partial A_x}{\partial z} & \frac{\partial A_y}{\partial z} & \frac{\partial A_z}{\partial z} \end{pmatrix} \circ \begin{pmatrix} \bar{a}_x & \bar{o} & \bar{o} \\ \bar{o} & \bar{a}_y & \bar{o} \\ \bar{o} & \bar{o} & \bar{a}_z \end{pmatrix} =$$

$$= \frac{\partial A_x}{\partial x}\,\bar{a}_x + \frac{\partial A_x}{\partial y}\,\bar{a}_y + \frac{\partial A_z}{\partial z}\,\bar{a}_z \, . \qquad (42)$$

If $A_x = A_y = A_z = A$, then (43)

$$\bar{\nabla} \circ \bar{A} = \frac{\partial A}{\partial x}\,\bar{a}_x + \frac{\partial A}{\partial y}\,\bar{a}_y + \frac{\partial A}{\partial z}\,\bar{a}_z \longrightarrow \text{gradient (vector quantity)} \equiv \qquad (44)$$

$$= \bar{\nabla} A \, .$$

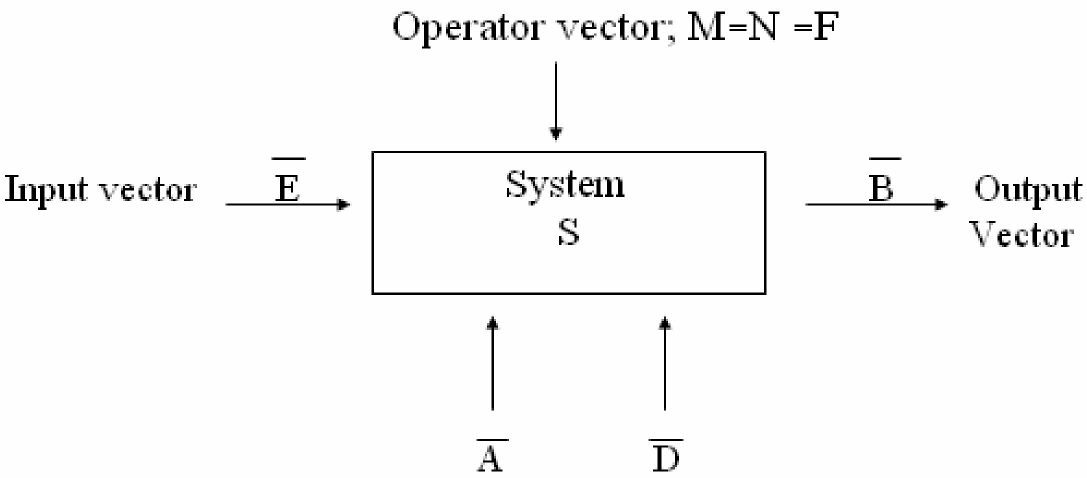
A is called the vector equivalent of the scalar A.

D – If o⟶•, then

$$\bar{\nabla} \, . \, \bar{\nabla} = \left(\begin{array}{c|ccc} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \hline \frac{\partial}{\partial x} & & & \\ \frac{\partial}{\partial y} & & & \\ \frac{\partial}{\partial z} & & & \end{array} \right) \circ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \equiv \bar{\nabla}^2 \quad \text{(Laplacian opeator)} \qquad (45)$$

APPENDIX



Final Results and Conclusions

1-The new concept of ‘‘ RYAD’’is introduced, illustrated and interpretated.

2- The fundamental basis of ‘’RYADIC’’ theory ‘‘ is stated as followes;’’RYADIC equations [Laws] are invariant under RYADIC transformation of effect coordinates’’.

3-An application of the RYADIC concept is illustrated. This application is from the field of vector analysis .

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