THREE-DIMENSIONAL CONFORMAL TRANSFORMATION+

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Abstract

A three-dimensional conformal transformation involves converting from one three-dimensional system to other systems. In the transformation, true shape is retained. The aim of this paper is using this type of coordinate transformation to connect the World Geodetic System (WGS84) with local coordinates system, because it gives the following facilities:

- **Infinite.**
- **Accurate results**
- **Minimum cost**
- **Need minimum Ground Control Points (GCPS).**

Thus, the researchers have designed a program (TRANS.m) is wlitter in (MATLAB) language in order to achieve this task (3D conformal transformation at high speed and accuracy.

المستخلص

يستخدم التحويل الثلاثي المحاور في التحويلات الجارية بين الأنظمة الثلاثية المحاور التي يتطلب العمل فيها التحويل من نظام ثلاثي المحاور إلى نظام ثلاثي آخر. ان هذا الأسلوب من التحويل يحافظ على الشكل الحقيقي للنظام ألإحداثي تضمن البحث استخدام هذا النوع من التحويل من اجل ربط النظام العالمي (WGS84) الذي يعد المرجع المستخدم في أجهزة الـــ (GPS)مع النظام المحلي المستخدم لتميزه بأنة غير محدد وينتج عنه نتائج دقيقة وبأقل كلفة ممكنة ويحتاج إلى عدد قليل من نقاط الضبط الأرضي، تم تصميم نظام برمجي (TRANS.m) كتب بلغة (MATLAB)من اجل القيام بهذه المهمة (التحويل بين النظامين) بسرعة ودقة عاليتين .

Introduction:

In the last few year (WGS84) was designed to be a global system nowadays, This system become very important because of many facilities, (infinite, accurate results, minimum cost and need minimum Ground Control Points (GCPS)).

In this paper the local coordinates system (base on local spheroid (Clark 1880)) is connected with GPS coordinates system (based on WGS84 spheroid), because it is always encountered in surveying work especially with advanced surveying Global Position System (GPS).

+ **Received on 1/12/2004 , Accepted on 8/2/2006**
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For this reason, the parameters of transformation of coordinates must be connecting both systems. These parameters are found by the observation equation (least square). These complex computations required a solution with assisted of computer. A program (software) to solve this computation in rapid and accurate way is established in this paper (the flow chart of this program is illustrated in Appendix A).

D Transformation:

The purpose of a transformation is to transform coordinates from one 3D system to other systems (see Fig 1). Several different transformation approaches exist.

The one that is used will depend on the result required. The basic field procedure for determination parameters is the same no matter which approach is taken.

Firstly, coordinates must be available in both coordinate systems (i.e. in WGS84 and in the local system) for at least three (and preferably four) common points. The more common points, which are included in the transformation, the more opportunity which has for redundancy and error checking.

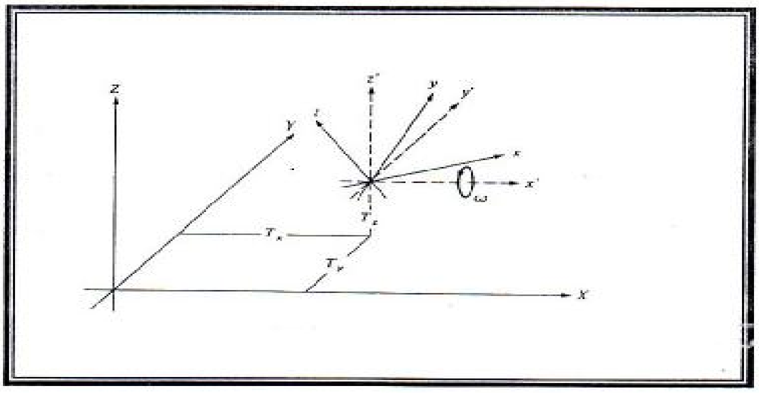


Fig. (1) The two 3D-systems (X,Y,Z), (x,y,z) and the parameters of transformation between them [1]

The transformation parameters can be calculated using one of the transformation approaches. It is important to note that the transformation will only apply to points in the area bounded by the common points (see Fig. 2). Points outside of this area should not be transformed using the calculated parameters but should. Form part of new transformation area.

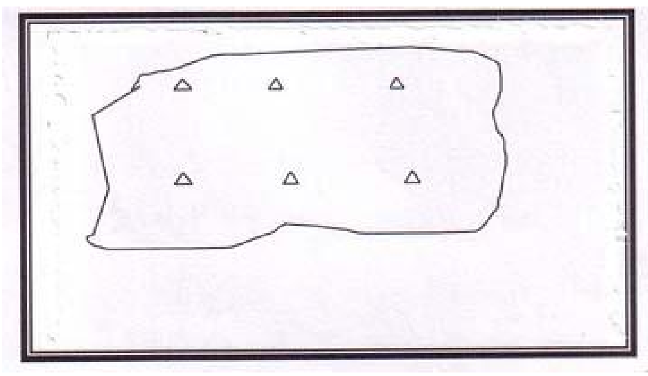


Fig. (2) The best distribution of GCPS on the study area**.**

<u>Geodetic Aspects</u>:

Since GPS has become increasingly popular as a surveying and navigation instrument, surveyors and navigators required a basic understanding of how GPS positions relate to standard mapping system.

Determining a position with GPS achieves a fundamental goal of geodesy, the determination of absolute position with uniform accuracy at all points on the earth's surface [2]. Using classical geodetic and surveying techniques, determination of position is always relative to the starting points of the survey, with accuracy achieved being dependent on the distance from this point. GPS therefore, offers a significant advantage over conventional techniques.

The science of geodesy is basic to GPS, and conversely, GPS has become a major tool in geodesy. This is evident if we look at the following aims of geodesy :

1. Establishment and maintenance of national and global three-dimensional geodetic control networks on land, recognizing the time-varying nature of these networks due to the plate movement.
2. Measurement and representation of geodynamic phenomena (polar motion, earth tides, and crystal motion).
3. Determination of the gravity field of the earth including temporal variations [3].

The GPS Coordinate System:

Although the earth may appear to be a uniform sphere viewed from space, the surface is far from uniform. Due to the fact that GPS has to give coordinates at any point on the earth's surface, it uses geodetic coordinate system based on ellipsoid. An ellipsoid (also a known as a spheroid) is a sphere that has been flattened or squashed.

An ellipsoid is chosen that most accurately approximate the shape of the earth. This ellipsoid has no physical surface but is mathematically defined surface [4].
There are actually many different ellipsoids or mathematical definitions of the earth's surface. The ellipsoid used by GPS is known as WGS84 or (World Geodetic System 1984).

A point on the surface of the earth can be defined by using latitude, longitude and ellipsoid height. An alternative method for defining the position of point is the Cartesian coordinate system, using distance in the X,Y and Z-axis from the origin or center of the earth. This is the method primarily used by GPS for defining the location of point in space.

Local Coordinate System:

Just as with GPS coordinates, local coordinates or coordinates used in a practical country's maps are based on a local ellipsoid, designed to match the geode (Fig 3) in the area. Usually, these coordinates will have been projected on to a plane surface to provide grid coordinates.

The ellipsoid used in most local coordinate systems throughout the world was first defined many years ago, before the advent of space could not be applied to other areas of the earth. Hence, each country defines a mapping system reference frame based on a local ellipsoid [5].

When using GPS, the coordinates of calculated positions are based on the WGS84 ellipsoid. Existing coordinates are usually in a local coordinate system and therefore the GPS coordinates have to be transformed into this local system.

In order to transform a coordinate from one system to another, the origins and axes of the ellipsoid must be known relative to each other. From this transformation, the shift in space X,Y and Z from one origin to the other can be determined, followed by any rotation about the X,Y and Z-axis and any change in scale between the two ellipsoids.

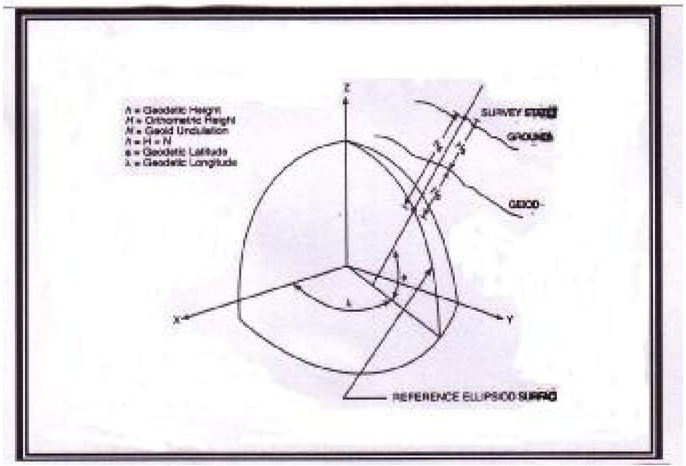


Fig. (3) The Local Coordinates System [5].

Theoretical Analysis:

Rotation:

The three-dimensional conformal transformation is one of the most commonly used transformations in many applications such as analytical photogrammetry and geodetic surveying. It is a process, which preserves three-dimensional shape.

Consider the right-handed systems shown in the diagrams of Fig(4).

An XYZ-system is shown, and in (a) points with values in another XYZ-system are to be transformed to it. The transformation involves the use of some or all of seven independent transformation parameters: three rotations (w, j, k), three translations (X_c, Y_c, Z_c), and a scale-change (S). The number used will depend on the application. For example, if the origins of two systems coincide the translations will be zero (0), and if there is no scale-change the scale-factor (S) will equal unity.

The diagrams show the effect of positive rotations (w, j, k) taken in that order, and about the xyz-axes respectively. A positive rotation is defined as a clockwise rotation when viewed from the origin in direction of positive axes. The final composite rotation matrix (R) is derived by considering the individual rotations in sequence. If the sequence is (w, j, k) then the xyz-axes are the once-removed and twice-removed axes respectively. Each individual rotation matrix represents a transformation in two dimensions.

$R = R_z . R_y . R_x$ …. (1)

The formulae that represent the nine elements of R are applicable to all valuess of (w, j, k) between (+180 to –180), they relate to specific sequence of rotations, and will be different from another sequence.

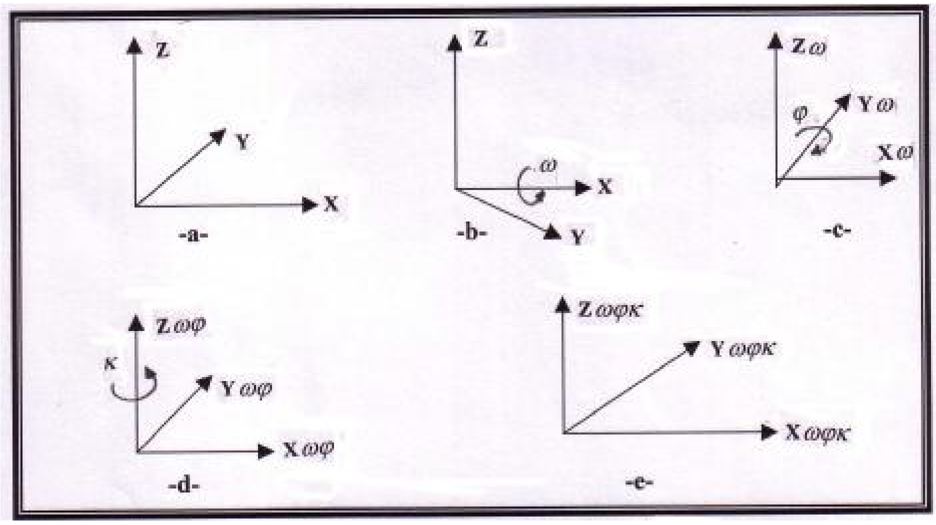


Fig. (4) Sequence of rotations (w, j, k) : a- initial position, b-w -rotation angle, c- after w -rotation, d-after j -rotation, e-after k -rotation [6].

When the individual rotation matrices in eq.(1) are combined together to form one composite rotation matrix the result is [7]:

$$R = \begin{bmatrix} \cos j \cos\kappa & \sin\omega \sin j \cos\kappa + \cos\omega \sin\kappa & -\cos\omega \sin j \cos\kappa + \sin\omega \sin\kappa \\ -\cos j \sin\kappa & -\sin\omega \sin j \sin\kappa + \cos\omega os\omega c & \cos\omega \sin j \cos\kappa + \sin\omega \sin\kappa \\ \sin j & -\sin\omega \cos j & \cos\omega \cos j \end{bmatrix}$$

And so:

$$\begin{bmatrix} x\omega j \kappa \\ y\omega j \kappa \\ z\omega j \kappa \end{bmatrix} = R. \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \ldots. \quad (2)$$

R is compiled from three orthogonal matrices, therefore it is itself orthogonal; and because it is orthogonal the inverse matrix M will equal its transpose. Thus

$M = R^{-1} = R^{T}$

And therefore

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^{T}. \begin{bmatrix} x\omega j \kappa \\ y\omega j \kappa \\ z\omega j \kappa \end{bmatrix} \quad \ldots. \quad (3)$$

Scaling and Translation:

To achieve the remaining parameters of three-dimensional conformal transformation, it is necessary to multiply the equations by a scale factor (S) and then add the translation (T_x,T_y,T_z) to the coordinates derived in equation (2) as follows [8]:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = S.\begin{bmatrix} x\omega j\ \kappa \\ y\omega j\ \kappa \\ z\omega j\ \kappa \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_Z \end{bmatrix} \quad \ldots. \quad (4)$$

Therefore, the total transformation, expressed in terms of original xyz-coordinates, may be written as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = S.R.\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_Z \end{bmatrix} \quad \ldots. \quad (5)$$

or

$X=S.R.x+X_T$

Suppose there are three points p,q and r whose coordinates are known in both systems. Then a total of nine equations may be written as follows [1]:

$$\left.\begin{aligned}
X_p &= s(m_{11}x_p + m_{21}y_p + m_{31}z_p) + T_x \\
Y_p &= s(m_{12}x_p + m_{22}y_p + m_{32}z_p) + T_y \\
Z_p &= s(m_{13}x_p + m_{23}y_p + m_{33}z_p) + T_z \\
X_q &= s(m_{11}x_q + m_{21}y_q + m_{31}z_q) + T_x \\
Y_q &= s(m_{12}x_q + m_{22}y_q + m_{32}z_q) + T_y \\
Z_q &= s(m_{13}x_q + m_{23}y_q + m_{33}z_q) + T_z \\
X_r &= s(m_{11}x_r + m_{21}y_r + m_{31}z_r) + T_x \\
Y_r &= s(m_{12}x_r + m_{22}y_r + m_{32}z_r) + T_y \\
Z_r &= s(m_{13}x_r + m_{23}y_r + m_{33}z_r) + T_z
\end{aligned}\right\} \quad \ldots. \quad (6)$$

Equations (6) above are non linear equation involving the seven unknowns $S, \omega, j, \kappa, T_{x,} T_y$ and T_z, to solve these equations, they are linearized using Taylor's theorem. In accordance with Taylor's theorem, the linearized form of the first three equations, which relate to point p is:

$$\left.\begin{aligned}
X_p &= (X_p)_o + \left(\frac{\partial X_p}{\partial S}\right)_o dS + \left(\frac{\partial X_p}{\partial \omega}\right)_o d\omega + \left(\frac{\partial X_p}{\partial j}\right)_o dj + \left(\frac{\partial X_p}{\partial \kappa}\right)_o d\kappa + \left(\frac{\partial X_p}{\partial T_x}\right)_o dT_x + \left(\frac{\partial X_p}{\partial T_y}\right)_o dT_y + \left(\frac{\partial X_p}{\partial T_z}\right)_o dT_z \\
Y_p &= (Y_p)_o + \left(\frac{\partial Y_p}{\partial S}\right)_o dS + \left(\frac{\partial Y_p}{\partial \omega}\right)_o d\omega + \left(\frac{\partial Y_p}{\partial j}\right)_o dj + \left(\frac{\partial Y_p}{\partial \kappa}\right)_o d\kappa + \left(\frac{\partial Y_p}{\partial T_x}\right)_o dT_x + \left(\frac{\partial Y_p}{\partial T_y}\right)_o dT_y + \left(\frac{\partial Y_p}{\partial T_z}\right)_o dT_z \\
Z_p &= (Z_p)_o + \left(\frac{\partial Z_p}{\partial S}\right)_o dS + \left(\frac{\partial Z_p}{\partial \omega}\right)_o d\omega + \left(\frac{\partial Z_p}{\partial j}\right)_o dj + \left(\frac{\partial Z_p}{\partial \kappa}\right)_o d\kappa + \left(\frac{\partial Z_p}{\partial T_x}\right)_o dT_x + \left(\frac{\partial Z_p}{\partial T_y}\right)_o dT_y + \left(\frac{\partial Z_p}{\partial T_z}\right)_o dT_z
\end{aligned}\right\} \ldots (7)$$

By simply changing subscripts, expressers similar to equations, are written for points q and, given a total of nine linearized equations $(X_p)_o$, $(Y_p)_o$ and $(Z_p)_o$ are the right hand sides of the first three equations. Eq. (6) evaluated at the initial approximations; $\left(\partial X_p / \partial S\right)_o, \left(\partial X_p / \partial w\right)_o$, etc., are partial derivatives with respect to the indicated unknowns evaluated at the initial approximations; and $dS, dw, dj, dk, dT_x, dT_y$ and dT_Z are corrections to the initial approximations. The unites of dw, dj, and dk are in radians.

Submitting letters for partial derivative coefficients, and adding residuals make this redundant system of equations consistent, all nine linear equations (7)are given below [1].

$$\left.\begin{aligned}
a_{11}dS+a_{12}d\omega+a_{13}dj+a_{14}d\kappa+a_{15}dT_x+a_{16}dT_y+a_{17}dT_z &= [X_p-(X_p)_o]+v_{xp} \\
a_{21}dS+a_{22}d\omega+a_{23}dj+a_{24}d\kappa+a_{25}dT_x+a_{26}dT_y+a_{27}dT_z &= [Y_p-(Y_p)_o]+v_{yp} \\
a_{31}dS+a_{32}d\omega+a_{33}dj+a_{34}d\kappa+a_{35}dT_x+a_{36}dT_y+a_{37}dT_z &= [Z_p-(Z_p)_o]+v_{zp} \\
a_{41}dS+a_{42}d\omega+a_{43}dj+a_{44}d\kappa+a_{45}dT_x+a_{46}dT_y+a_{47}dT_z &= [X_q-(X_q)_o]+v_{xq} \\
a_{51}dS+a_{52}d\omega+a_{53}dj+a_{54}d\kappa+a_{55}dT_x+a_{56}dT_y+a_{57}dT_z &= [Y_q-(Y_q)_o]+v_{yq} \\
a_{61}dS+a_{62}d\omega+a_{63}dj+a_{64}d\kappa+a_{65}dT_x+a_{66}dT_y+a_{67}dT_z &= [Z_q-(Z_q)_o]+v_{zq} \\
a_{71}dS+a_{72}d\omega+a_{73}dj+a_{74}d\kappa+a_{75}dT_x+a_{76}dT_y+a_{67}dT_z &= [X_r-(X_r)_o]+v_{xr} \\
a_{81}dS+a_{82}d\omega+a_{83}dj+a_{84}d\kappa+a_{85}dT_x+a_{86}dT_y+a_{87}dT_z &= [Y_r-(Y_r)_o]+v_{yr} \\
a_{91}dS+a_{92}d\omega+a_{93}dj+a_{94}d\kappa+a_{95}dT_x+a_{96}dT_y+a_{97}dT_z &= [Z_r-(Z_r)_o]+v_{zr}
\end{aligned}\right\} \ldots. \;(8)$$

Equations may be expressed in the matrix form as follows:

AX=L+V

where

$$A=\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} \\
a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97}
\end{bmatrix},\quad
X=\begin{bmatrix} ds \\ d\omega \\ dj \\ d\kappa \\ dT_x \\ dT_y \\ dT_z \end{bmatrix},\quad
L=\begin{bmatrix} X_p-(X_p)_o \\ Y_p-(Y_p)_o \\ Z_p-(Z_p)_o \\ X_q-(X_q)_o \\ Y_q-(Y_q)_o \\ Z_q-(Z_q)_o \\ X_r-(X_r)_o \\ Y_r-(Y_r)_o \\ Z_r-(Z_r)_o \end{bmatrix},\quad
V=\begin{bmatrix} vx_p \\ vy_p \\ vz_p \\ vx_q \\ vy_q \\ vz_q \\ vx_r \\ vy_r \\ vz_r \end{bmatrix}$$

The solution is iterated, due to use of Taylor's series unit negligibly small values are obtained for the corrections to initial approximations of unknown transformation parameters.

Once the transformation factors have been found, the transformation coordinates of all points whose coordinates are known only in the original system may be found by applying equation of this type.

Experimental Tests:

In this paper there are two experimental tests applied to connect the two systems (GPS coordinates system based on spheroid WGS84) and local coordinates system based on spheroid Clark 1880). These tests are:

The First Experimental Test:

This test was applied to the existing data*. The Cartesian coordinates (X,Y,Z) of Ground Control Points (GCPS) are known in two systems (Local coordinates system and GPS coordinates system**).

* The values of seven GCPS (Ground Control Points) are taken from reference [9].

** The GPS coordinates are observed using Surveying GPS instrument

The both systems are connected using (TRANS.m) program (see appendix A) to find the seven unknown parameters (S, w, j, k, T_x, T_y *and* T_z). Then the local Cartesian coordinates of three points are found from GPS coordinates system.

The experimental test can be summarized in the following tables:

1- Table (1) shows the Cartesian coordinates of four GCPS in two systems (Local and GPS coordinates systems).
2- Table (2) illustrates the values of seven unknown parameters of transformation (S, w, j, k, T_x, T_y *and* T_z).
3- Table (3) shows the ground coordinates of three cheek points in GPS coordinate system, the local coordinate system and the residual (v_x,v_y and v_z) of these points which was found from (TRANS.m) program.

The Second Experimental Tests:

In this test, the ten GCPS coordinates are observed using Navigation GPS instrument (GARMIN GPS 72 instrument) and Topographic Map has scale 1/100000. The study area (Fig (5)) of this test was selected in the southern of Baghdad called (Jaser Dialy).

Both systems (GPS coordinate system (UTM (E_G,N_G,h_G) based on spheroid WGS84 and Local coordinates system (UTM (E,N,h) based on spheroid Clark 1880) are connected using (TRANS.m) program for finding the seven unknown parameters (S, w, j, k, T_x, T_y *and* T_z).Then the local UTM coordinates of five points are found from GPS coordinate system.

This test can be summarized in the following tables:

4- Table (4) shows the UTM coordinates of five GCPS in two systems (Local and GPS coordinate systems).
5- Table (5) illustrates the values of seven unknown parameters of transformation (S, w, j, k, T_x, T_y *and* T_z).
6- Table (6) shows the ground coordinates of five cheek points in GPS coordinate system, the local coordinate system and the residual (v_x,v_y and v_z) of these points which was found from (TRANS.m) program.

Table (1) Cartesian coordinate of four GCPS in two systems (Local and GPS Coordinate System)

Point	WGS84			Local System		
	X (m)	Y (m)	Z (m)	X_g (m)	Y_g (m)	Z_g (m)
1	5083012.893	1855174.455	3365429.159	5069594.098	1850357.036	3379624.442
2	5082986.893	1855268.420	3365416.427	5069568.202	1850450.764	3379611.682
3	5082935.034	1855322.355	3365463.342	5069516.342	1850504.503	3379659.356
4	5082980.866	1855467.782	3365316.625	5069562.480	1850649.701	3379511.957

Table (2) The initial and final Value of seven unknown parameters of 3D-Transformation.

No	Parameters	Initial Value	Final Value

1	S (meter)	0.081	0.998254
2	w (rad)	0.000	-0.000291
3	j (rad)	0.000	-0.000915
4	k (rad)	-0.009	-0.000641
5	T_x (meter)	1.000	-2649.905
6	T_y (meter)	1.000	694.006
7	T_z (meter)	1.000	15963.086

Table (3) : The Ground coordinates of the three points in GPS coordinate System, Derived Local Coordinates System and Residual (V_X,V_Y,V_Z) of the Derived Coordinates

Point	GPS Coordinates System			Local Coordinates System					
	X (m)	Y (m)	Z (m)	X_g (m)	Y_g (m)	Z_g (m)	V_{Xg} (m)	V_{Yg} (m)	V_{Zg} (m)
1	5083112.067	1855479.029	3365113.623	5070617.103	1850998.135	3377338.154	-0.021	-0.119	0.055
2	508319.761	1855166.494	3365140.242	5069776.256	1850349.379	3379354.922	0.074	0.023	0.076
3	5080477.280	1856294.902	3368653.303	5067594.357	1849997.102	3382855.797	0.083	0.008	-0.198
RMSE*							± 0.063	± 0.067	± 0.126
TRMSE**							± 0.163 m		

* RMSE (Root Mean Square Error)= $\pm\sqrt{\dfrac{\Sigma v^2}{n}}$

** TRMSE (Total Root Mean Square Error) = $\pm\sqrt{RMSE_X^2 + RMSE_Y^2 + RMSE_Z^2}$

Table (4) UTM Coordinates of five GCPS in two systems (Local and GPS Coordinates System) (Second Test).

point	WGS84 (UTM)			Local System (UTM)		
	E_G (m)	N_G (m)	h_G (m)	E (m)	N (m)	h (m)
1	٤٥٤٢٢٦,٨٤	٣٦٧٦٥٣٧,٦	22.31	454601.46	3676284	29.72
2	٤٥٤٩٧٨,٠٩	٣٦٧٧٢٦٢,٩	36.42	455309.35	3676959.1	33.82
3	٤٥٤٥١٩,٣٤	٣٦٧٧٧٤١,٨	37.51	454910.32	3677394.4	40.25
4	٤٥٦٢٦١,١٦	٣٦٧٩٢٠٢,٦	20.15	456627.32	3678851.5	31.52
5	٤٥٥٧٤١,٣٩	٣٦٧٩٧٠٧,١	18.15	456071.95	3679255	30.18

Table (5) The initial and Final Value of Seven Unknown parameters of 3D-Transformation (Second Test).

No	Parameters	Initial Value	Final Value
1	S (meter)	0.936	0.976037
2	w (rad)	0.000	0.003437

3	j (rad)	0.000	0.000789
4	k (rad)	0.013	-0.013855
5	T_x (meter)	1.000	-38422.676
6	T_y (meter)	1.000	94330.091
7	T_z (meter)	1.000	-1920.775

Table (6) : The Ground Coordinates of the five Points in GPS Coordinates System, Derived Local Coordinates System and Residual (V_X,V_Y,V_Z) of the Derived Coordinates

Point	GPS Coordinates System			Local Coordinates System					
	E_G (m)	N_G (m)	h_G (m)	E (m)	N (m)	h (m)	V_E (m)	V_N (m)	V_h (m)
1	**454885.94**	**3680101.9**	**23.21**	**455220.78**	**3679546.7**	**33.32**	**12.234**	**-8.611**	**-1.929**
2	**456307.01**	**3679721.9**	**19.33**	**456613.67**	**3679427.0**	**32.13**	**5.086**	**4.745**	**0.772**
3	**457118.92**	**3682178.7**	**32.31**	**457325.68**	**3681856.1**	**32.53**	**-10.688**	**-12.181**	**1.532**
4	**457222.13**	**3682129.5**	**31.85**	**457564.95**	**3681865.2**	**33.11**	**9.460**	**-11.695**	**-0.342**
5	**455070.22**	**3681238.0**	**33.45**	**455351.60**	**3680930.4**	**28.12**	**11.324**	**15.431**	**2.573**
RMSE*							**± 10.074**	**± 11.135**	**± 1.637**
TRMSE**								**± 15.105**	

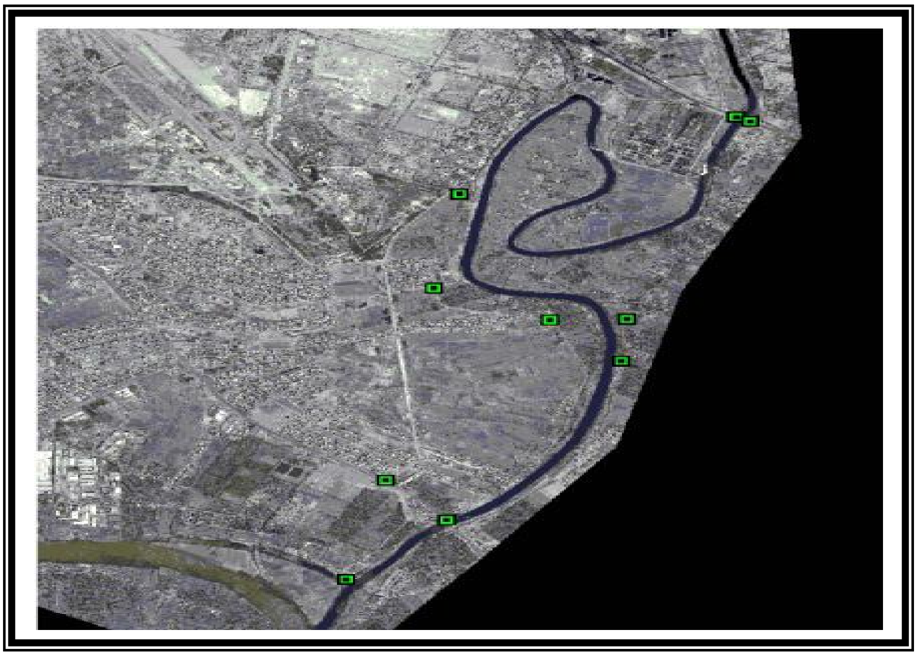


Fig. (5) GCPS in study area (Jeser Dialy).

Fig (6) Topographic Map of study area (Jeser Dialy).

Results Analysis:

From the two experimental test, the Total Root Mean Square Error (TRMSE) in the first test is small (relatively) (see Table (3)) and equal to ($\pm$0.163) m. This error (RMSE in the derived coordinates ($v_{X_g}, v_{Y_g}, v_{Z_g}$)) is due to the following points:

1- The high accuracy of GPS instrument (Surveying GPS Instrument) used to find the ground coordinates of points (in WGS84).
2- The high accuracy of ground control points (GCPS) (third order GCPS) which used in the transformation process (Local system).
3- The good distribution of the GCPS in the study area.

On the other hand, the Total Root Mean Square Error (TRMSE) in the second test is equal to ($\pm$ 15.105) m. and is due to the following points:

1- The low accuracy of GPS instrument (Navigation GPS Instrument) used to find the ground coordinates of points (in WGS84).
2- The low accuracy of the ground control points (GCPS) (selected from topographic map has scale 1/100000) used in the transformation process (Local system).
3- The irregular distribution of the (GCPS) in the study area.

Conclusions:

The conclusions from the obtained results for two tests can be summarized in the following points:

1- The accuracy of 3D-transformation system depends on the accuracy of GPS instrument and the accuracy of GCPS. In the first test the accuracy of the transformation (in the derive coordinates) was ($\pm$ 0.163) meter because the Surveying GPS instrument and high order (third order) GCPS were used. While, in the second test the low accuracy of the transformation (in the derive coordinates) was (± 15.105 m) because Navigation GPS instruments and GCPS derived from 1/100000 topographic map were used in this test.

2- The (TRANS.m) program can determine the parameters between any two systems and then transform each points that lies in one of them by using minimum number of GCPS.

3- The (TRANS.m) program which is built in this work is efficient and successful in performing this project.

4- The (TRANS.m) program has ability to transform the two coordinates systems with 3D and 2D.

Recommendation:

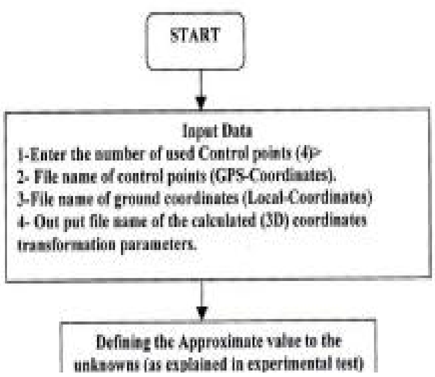
The recommendation related to this study can be summarizes in the following points:

1- Select the best distributed ground control points (GCPS) with a high order (first or second order) for study area.

2- Use accurate GPS instrument to find Cartesian or UTM coordinates of the points in the WGS84.

3- Transform all the national network system to WGS84.

Appeendix A



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