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New Forms Of Gpe In Reaction Diffusion Systems

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Abstract
In this research, we have introduced mathematical models which in turn represents the equation of thermal combustion. Where these formulas were formulated according to a particular pattern and conditions. Studying and is working in this field a great interest in interpreting the boundness of the printed phenomena. In this work, we attempt to construct and present mathematical equations with an integrative continuous research in this field and access to all the sources and previous studies and all the works that have been provided proviously.

1- Introduction

In this research, we presented many mathematical equations or mathematical models which is represent equations of combustion front and reaction diffusion systems. We discussed how to construct these equations from the general equation which is called general phase equation(GPE). The general phase equation has the form

$$\partial_t u = a_1\nabla^2 u + a_2(\nabla u)^2 + b_1\nabla^4 u + b_2\nabla^3 u\nabla u + b_3(\nabla^2 u)^2 + b_4\nabla^2 u(\nabla u)^2 + b_5(\nabla u)^4 + g_1\nabla^6 u + g_2\nabla^5 u\nabla u + g_3\nabla^4 u\nabla^2 u + g_4(\nabla^3 u)^2 + g_5\nabla^4\psi(\nabla u)^2 + g_6(\nabla^2 u)^3 + g_7\nabla^3 u\nabla^2 u\nabla u + g_8\nabla^3 u(\nabla u)^3 + g_9(\nabla^2 u)^2(\nabla u)^2 + g_{10}\nabla^2 u(\nabla u)^4 + g_{11}(\nabla u)^6 + e_1\nabla^8 u + \cdots, \qquad (1)$$

describes the phase of oscillations in reaction-diffusion systems. Here u is the phase of oscillations and a_i, b_i, g_i, e_i.. are constant coefficients. The right-hand side of above equation can be viewed as a power series in small parameter $\epsilon \sim 1/L$, where L is a large characteristic spatial scale of variations of u. When constructing an equation you must follow the important conditions when choosing the terms of the building of the mathematical model must be given each term an important role for the validity of the equation, so we choose the term represent nonlinear excitation is considered as the source of energy, and then we choose the term plays the role of dissipation. Then we choose the limit plays the role of balance of equation. These terms contain constants and the constants changed continuously until we get more accurate results when solved the equation numerically. There are many researchers who have worked in this field. They have built many mathematical models and conducted numerical experiments. Some of these models presented linear mathematical equations **Tanaka** and **Kuramoto** mentioned a variety of physical and biological systems linked to the general phase equation: Cellular slime moulds, oscillating yeast cells under glycolysis and the Belousov Zhabotinsky reaction dispersed in water -in-oil. **Johannes Martinus Burgers(1895-1981**) is given velocity u and viscosity coefficient, the general form of Burgers equation is

$$\frac{\partial u}{\partial t} = a_1\frac{\partial^2 u}{\partial x^2} - a_2\frac{\partial u}{\partial x} \qquad (2)$$

If $a_1 > 0$ the respective terms of (GPE) are dissipative due to which u must be decreasing with time. Therefore higher order derivative should be very small and become negligible. Hence GPE lead to equation termed as Burgers equation. If $a_1 < 0$ or $a_2 > 0$ the respective terms of (GPE) become antidissipative so that pertur-bations of a at field u=constant grow. We can say that the terms provides the excitation

Kuramoto-Sivashinsky introduced the equation of 4^{nd} order

$$\nabla_t u = -\eta\nabla^2 u + (\nabla u)^2 - \nabla^4 u. \qquad (3)$$

In the case $a_1 = -\eta < 0$ (very small by assumption) and by the order of magnitude $a_2 = 1;\ b1 = -1$ and (GPE) reduces to the **Kuramoto-Sivashinsky** equation contains the linear anti- diffusion /Excitation term $-\eta\nabla^2 u$ be . It is

counterbalanced by dissipative term $-\nabla^4$ u and later the 6^{nd} order model developed by **Nikolaevskii(1989)** for seismic waves,

$$\nabla\nabla_t u = (\nabla u)^2 + \zeta\nabla^2 u + \nabla^4 u + \nabla^6 u, \qquad (4)$$

where $\zeta > 0$

Nikolaevskii equation also involves linear excitation $\nabla^4 u$. The excitation terms in these equations are linear and represent the internal source of energy. As consequence, the state of u = constant turns out unstable to infinitesimal perturbations and is globally destroyed and others provided nonlinear mathematical equation. **Strunin (1999)** showed that the equation

$$\frac{dU}{dt} = \frac{d^6U}{dx^6} - \left(\frac{dU}{dx}\right)^2 \frac{d^2U}{dx^2} + \left(\frac{dU}{dx}\right)^4 \qquad (5)$$

can be derived from a more general phase equation. It directly applies to the reaction-diffusion systems, but it is also phenomenological connected to the propagation of reaction fronts, which interest us in our project. Strunin(2009) derived a form of phase equation based on nonlinear excitation analytically. It is interested to investigate whether a truncation with nonlinear excitation is providing an equation with diverse dynamics and its applications. Unlike linear one, a nonlinear excitation can support localized dissipative structures sometimes referred to as au-solitudes and also complex regimes resulting from their interactions. The auto solitons is a type of a wave encountered in many physical, chemical and biological systems. Consider coefficient ≯8 L F, ≯9 L sand Ç5 = 1.

The GPE reduces to

ˇ Q L F ˇ6 Q :ˇQ;6 E :ˇQ;8 E ˇ: QÆ :x;

where F ˇ6 Q :ˇQ;6 an excitation term. This term controls the distortion of the front due to the reaction. Strunin derived equation (1) from the complex Ginzburg-Landau equation relevant to nonlocal reaction -diffusion systems. **Mayada Gassab Mohammed(2015)** introduced new form of (GPE). It has the form

$$\partial_t u = b_3(\nabla^2 u)^2 + b_5(\nabla u)^4 + g_1\nabla^6 u. \qquad (7)$$

If we look to (7), we will note that the dissipation term and the balance term is the same as the terms in (1) [5]

The difference here is in the expatiation term which is $b_3(\nabla^2 u)^2$ in our choice. Equation (7) is new model for the combustion front and it has the same mechanism of the equations (1). It works with its three terms as one unit to show the spinning wave for the combustion front.

2 Other possible forms of the GPE equation

2.1 Form A

Nonlinear excitation in this equation is possible through the term $b_4\nabla^2 u(\nabla u)^2$. Refer $b_4 = \varepsilon$ (assumed small) and $\varepsilon < 0$, suppose that the coefficients a_1, a_2, b_1, b_3, b_5, are all small enough so that the related terms can be ignored. Driven by the nonlinear excitation a perturbation of the flat state will develop and, if we want to achieve balance, we need to appropriately choose a higher-order term in u. A suitable term is $b_2\nabla^3 u\nabla u$, which is the same term as in equation (1). The dissipation can be obtained via the term $g_1\nabla^6 u$ as it is of higher order in $\nabla \sim 1/L$. Its action will be to smooth out the steep sections of u-field and prevent the formation of singularities. Denoting characteristic scale of the phase variations by $U > 0$ we evaluate in absolute value

$$\nabla^3 u\nabla u \sim U^2/L^4\,,\quad \nabla^2 u(\nabla u)^2 \sim U^3/L^4 \quad \text{and} \quad \nabla^6 u \sim U/L^6. \qquad (8)$$

The balance between the three terms (with $b_2 = 1$, $b_4 = \varepsilon$, $g_1 = 1$ by the order of magnitude) occurs when

$$U^2/L^4 \sim \varepsilon U^3/L^4 \sim U/L^6. \qquad (9)$$

The scales of the dissipative structures, will be determined by these two equations as follows

$$U \sim 1/\varepsilon\,,\quad L \sim \sqrt{\varepsilon}\,. \qquad (10)$$

The following smallness conditions are adopted:

$$a_1 = o(\varepsilon^{-2}),\quad a_2 = o(\varepsilon^{-1}),\quad b_1 = o(\varepsilon^{-1}),\quad b_3 = o(\varepsilon^{0}),\quad b_5 = o(\varepsilon^{2}). \qquad (11)$$

These conditions make the related terms of the general form of the phase equation ignored compared with the balance terms (9). For the latter, taking (10), we have

$$b_4\nabla^2 u(\nabla u)^2 \sim \varepsilon\, U^3/L^4 \sim \varepsilon^{-4} \qquad (12)$$

Using (10) and (11), we obtain

$$a_1\nabla^2 u \sim a_1\, U/L^2 \sim o(\varepsilon^{-4})$$

$$a_2(\nabla u)^2 \sim a_2\, U^2/L^2 \sim o(\varepsilon^{-4})$$

$$b_1\nabla^4 u \sim b_1\ U/L^4 \sim o(\varepsilon^{-\varepsilon}) \qquad (13)$$

$$b_3(\nabla^2 u)^2 \sim\ b_2\ U^2/L^4 \sim o(\varepsilon^{-4})$$

$$b_5(\nabla u)^4 \sim b_5\, U^4/L^4 \sim o(\varepsilon^{-4})$$

Now, look at the rest of the phase equation starting from the 6^{th} order term, $g_2\nabla^5 u\nabla u$. All nonlinear terms are ignored relative to the balance terms because of higher order in $U \sim \frac{1}{\varepsilon}$ and all the linear terms are ignored because of the higher order in $\nabla^2 \sim\ 1/L^2 \sim \frac{1}{\varepsilon}$ Therefore, the phase equation is reduced to

$$\partial_t u =\ b_2\nabla^3 u\nabla u + b_4\nabla^2 u(\nabla u)^2 + g_1\nabla^6 u\,. \qquad (14)$$

2.2 Form B

Nonlinear excitation in this equation is possible via the term $b_2\nabla^3 u\nabla u$. Denote b_2=ε (assumed small) and suppose that the coefficients a_1, a_2, b_1, b_3, b_4 are all small enough so that the respective terms can be ignored. Driven by the nonlinear excitation a perturbation of the flat state will grow and, if we want to achieve balance, we need to appropriately choose a higher-order term in u. A suitable term is $b_5(\nabla u)^4$, which is the same term as in equation (1). The dissipation can be obtained via the term $g_1\nabla^6 u$ as it is of higher order in $\nabla \sim 1/L$. Its action will be to smooth out the steep sections of u-field and prevent the formation of singularities. Denoting characteristic scale of the phase variations by U > 0 we evaluate absolute value

$$\nabla^3 u\nabla u \sim U^2/L^4\ , \quad (\nabla u)^4 \sim U^4/L^4 \quad \text{and } \nabla^6 u \sim U/L^6. \qquad (15)$$

The balance between the three terms (with $b_2\ = \varepsilon,\ b_5 = 1,\ g_1 = 1$ by the order of magnitude) occurs when

$$\varepsilon U^2/L^4 \sim\ U^4/L^4 \sim\ U/L^6. \qquad (16)$$

These two equations determine the scales of the dissipative structures,

$$U \sim \sqrt{\varepsilon}, \quad L \sim \varepsilon^{-3/4}. \qquad (17)$$

The following smallness conditions are adopted:

$$a_1 = o(\varepsilon^3),\ \ a_2 = o\left(\varepsilon^{5/2}\right),\ \ b_1 = o\left(\varepsilon^{3/2}\right),\ \ b_3 = o(\varepsilon),\ b_4 = o\left(\varepsilon^{1/2}\right) \qquad (18)$$

These conditions make the respective terms of the general form of the phase equation negligible compared with the balance terms (16). For the latter, taking (17), we have

$$b_2\nabla^3 u\nabla u \sim b_2\, U^2/L^4 \sim \varepsilon^5. \qquad (19)$$

Using (17) and (18), we obtain

≒5 ˇ6 Q 1 ≒5 7 / .6 1 K: 9,;

≒6 :ˇQ;6 1 ≒6 76 / .6 1 K: 9 ;Æ

≩5 ˇ8 Q 1 ≩5 7 / .8 1 K: 9 ;Æ :tr;

≯7 :ˇ6 Q;6 1 ≯7 76 / .8 1 K: 9,;

≩8 :ˇ6 Q;:ˇQ;6 1 ≩8 77 / .8 1 K: 9.;

Now, look at the rest of the phase equation (1) starting from the x order term, Ç6 ˇ9 QˇQ All nonlinear terms are negligible relative to the balance terms because of higher order in 7 1 ¯ and all the linear terms are negligible because of the higher order in ˇ6 1 s / .6 1 7/6. Therefore, the phase equation is reduced to

$$\partial_t u = b_2 \; \breve{3}u \;\; \breve{u} + b_5(\;\; \breve{u})^4 + g_1 \;\; \breve{6}u. \qquad :t\text{ß}$$

2.3 Form C

If we suppose that ≩6 L and ≩8 P rÆ≩8 L ¯ then we will have the nonlinear excitation term ≩6 ˇ7 QˇQ and nonlinear dissipation ≩8 ˇ6 Q:ˇQ;6 The latter can be treated as a diffusion term, ˇ6 Q with nonlinear coefficient :ˇQ;6 Suppose also that the coefficients a1, a2, b1, b3 are all small enough so that the respective terms can be ignored. Driven by the nonlinear excitation a perturbation of the at state will grow and, if we want to achieve balance, we need to choose another term to create steep sections of the field u. A suitable term is $b_5(\;\breve{u})^4$, as in form A. The dissipation term will be the same as in form A which is $g_1 \;\, ^6u$. We evaluate

ˇ7 QˇQ 1 76 / .8 Æ ˇ6 Q:ˇ ;6 1 77 / .8 Æ :ˇQ;8 1 78 / .8 =J @ ˇ: Q 1 7 / .: :tt;

The balance between the four terms,

76 .8 1 ¯ 77 .8 1 78 .8 1 7 .6 Æ :tu;

determines the scales of the dissipative structures

7 1 ¯ ,

. 1 ?7 8 :tv;

The following smallness conditions are adopted:

≒5 L K: 7 ;Æ ≒6 L Kk 9/6 ŒÆ ≩5 L Kk 7/6 ŒÆ ≯7 L K: ; :tw;

Let us show that these conditions make the respective terms of (1) negligible compared to the balance terms (22). For the latter, taking (23),

$$b_2\, \nabla^3 u \nabla \mathrm{u} \sim \varepsilon\, U^2/L^4 \sim \varepsilon^5,$$

$$b_4 \nabla^2 u\, (\nabla u)^2 \sim \sqrt{\varepsilon}\;\, U^3/L^4 \sim \varepsilon^5. \qquad (26)$$

By comparison, using (23) and (25),

$$a_1 \nabla^2 u \sim a_1\, U/L^2 \sim o(\varepsilon^5),$$

$$a_2 (\nabla u)^2 \sim a_2\, U^2/L^2 \sim o(\varepsilon^5),$$

$$b_1 \nabla^4 u \sim b_1\;\, U/L^4 \sim o(\varepsilon^5)\,, \qquad (27)$$

$$b_3 (\nabla^2 u)^2 \sim b_3\;\, U^2/L^4 \sim o(\varepsilon^5).$$

Looking at the rest of equation (1) starting from the 6^{th} order term, $g_2 \nabla^5 u \nabla u$, we note that all the linear terms are negligible as a result of the higher order in $\nabla^2 \sim 1/L^2 \sim \varepsilon^{3/2}$, and all nonlinear terms are negligible relative to the balance terms because of higher order in $U \sim \sqrt{\varepsilon}$. Therefore, the phase equation is reduced to

$$\partial_t u = b_2 \nabla^3 u \nabla u + b_4 \nabla^2 u (\nabla u)^2 + b_5 (\nabla u)^4 + g_1 \nabla^6 u\,. \quad b_4 > 0 \qquad (28)$$

2.4 Form D

Now suppose that ≽6 L and ≽8 O rÆ≽8 L F ¯. In this case we have two nonlinear excitation terms ≽6 ˇ7 Q ˇ Q and ≽8 ˇ6 Q : ˇ Q ;6 and of the same order provided the balance for the form C holds, namely (23). Here ≽8 ˇ6 Q : ˇ Q ;6 is nonlinear excitation as it can be treated as an anti-diffusion, F ˇ6 Q with the nonlinear coefficient : ˇ Q ;6 Suppose also that the coefficients a1, a2, b1, b3 are small enough so that the respective terms can be ignored. The terms for dissipation and for creating balance will be the same as in the previous forms which are g_1 6 u and b_5(ˇu)^4 respectively. Denoting the characteristic scale of the phase variations by U > 0, we evaluate in absolute value

ˇ7 Q ˇ 1 6 8, : ˇ6 Q ; : ˇ Q ;6 1 7 7 . 8 Æ : ˇ Q ;8 1 7 8 / . 8 and ˇ: Q 1 7 / . : :t{;

The balance between the four terms,

7 6 . 8 1 ¯ 7 7 . 8 1 7 8 . 8 1 7 . 6 :ur;

determines the scales of the dissipative structures

$$U \sim \sqrt{\varepsilon}\,,\; L \sim \varepsilon^{-3/4}\,. \qquad (31)$$

The following smallness conditions are adopted:

$$a_1 = o(\varepsilon^3)\,, \quad a_2 = o\left(\ \varepsilon^{5/2}\right), \;\; b_1 = o\left(\varepsilon^{3/2}\right), \;\; b_3 = o(\varepsilon). \qquad (32)$$

Let us show that these conditions make the respective terms of (1) negligible compared to the balance terms (29). For the latter, taking (30),

$$b_2\, \nabla^3 u \nabla \mathrm{u} \sim \varepsilon\, U^2/L^4 \sim \varepsilon^5,$$

$$b_4 \nabla^2 u\, (\nabla u)^2 \sim \sqrt{\varepsilon}\;\, U^3/L^4 \sim \varepsilon^5. \qquad (33)$$

By comparison, using (30) and (32),

$$a_1 \nabla^2 u \sim a_1\, U/L^2 \;\; \sim o(\varepsilon^5),$$

$$a_2 (\nabla u)^2 \sim a_2\, U^2/L^2 \sim o(\varepsilon^5),$$

$$b_1 \nabla^4 u \sim b_1\;\, U/L^4 \;\sim o(\varepsilon^5), \qquad (34)$$

$$b_3 (\nabla^2 u)^2 \sim b_3\;\, U^2/L^4 \;\sim o(\varepsilon^5).$$

The rest of equation (1) starting from the 6^{th} order term, $g_2 \nabla^5 u \nabla u$ will be negligible as a result of higher order in ∇^2or U. Therefore, the phase equation has the form (28) but with b4 < 0.

$$\partial_t u = \; b_2 \nabla^3 u \nabla u - b_4 \nabla^2 u (\nabla u)^2 + b_5 (\nabla u)^4 + \; g_1 \nabla^6 u\,. \qquad (35)$$

2.5 Form E

Nonlinear excitation in this equation is possible through the term $b_4 \nabla^2 u (\nabla u)^2$. Refer $b_4 = \varepsilon$ (assumed small) and $\varepsilon < 0$, suppose that the coefficients a_1, a_2, b_1, b_2, b_5, are all small enough so that the related terms can be ignored. Driven by the nonlinear excitation a perturbation of the flat state will develop and, if we want to achieve balance, we need to appropriately choose a higher-order term in *u*. A suitable term is >7 :˘6 Q;6 which is the same term as in equation (1). The dissipation can be obtained via the term G5 ˘: Qas it is of higher order in ˘1 s .. Its action will be to smooth out the steep sections of *u*-field and prevent the formation of singularities. Denoting characteristic scale of the phase variations by $U > 0$ we evaluate in absolute value

:˘6 Q;6 1 76 .8 Æ ˘6 Q: Q;6 1 76 .8 and ˘: Q 1 7 .: (ux;

The balance between the three terms (with $b_3 = 1$, $b_4 = \varepsilon$, $g_1 = 1$ by the order of magnitude) occurs when

76 .8 77 .8 7 .: :uy;

The scales of the dissipative structures, will be determined by These two equations as follows

$$U \sim \frac{1}{\varepsilon} \ , \ L \sim \sqrt{\varepsilon} \, . \tag{38}$$

The following smallness conditions are adopted:

$$a_1 = o(\varepsilon^{-2}), \quad a_2 = o(\varepsilon^{-1}), \quad b_1 = o(\varepsilon^{-1}), \quad b_2 = o(\varepsilon^{0}), \quad b_5 = o(\varepsilon^{2}). \tag{39}$$

These conditions make the related terms of the general form of the phase equation ignored compared with the balance terms (37). For the latter, taking (38), we have

$$b_4 \nabla^2 u \, (\nabla u)^2 \sim \varepsilon \ U^3/L^4 \sim \varepsilon^{-4}. \tag{40}$$

Using (38) and (39), we obtain

$$a_1 \nabla^2 u \sim a_1 \, U/L^2 \ \sim o(\varepsilon^{-4}),$$

$$a_2 (\nabla u)^2 \sim a_2 \, U^2/L^2 \sim o(\varepsilon^{-4}),$$

$$b_1 \nabla^4 u \sim b_1 \ U/L^4 \ \sim o(\varepsilon^{-4}) \, , \tag{41}$$

$$b_2 (\nabla^2 u)^2 \sim \ b_2 \ U^2/L^4 \sim o(\varepsilon^{-4}),$$

$$b_5 (\nabla u)^4 \sim b_5 \, U^4/L^4 \sim o(\varepsilon^{-4}).$$

Now, look at the rest of the phase equation starting from the 6^{th} order term, $g_2 \nabla^5 u \nabla u$. All nonlinear terms are ignored relative to the balance terms because of higher order in $U \sim \frac{1}{\varepsilon}$ and all the linear terms are ignored because of the higher order in $\nabla^2 \sim \ 1/L^2 \ \sim \frac{1}{\varepsilon}$ Therefore, the phase equation is reduced to

$$\partial_t u = \ b_3 (\nabla^2 u)^2 + b_4 \nabla^2 u (\nabla u)^2 + g_1 \nabla^6 u \, . \tag{42}$$

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