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On Micro α- generalized closed and Micro *semi*- generalized closed in Micro -topological spaces

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ABSTRACT

We're going to study a new definitions in this paper that's are Micro α − generalized closed , Micro generalized α − closed, Micro generalized ꞩemi − closed and Micro ꞩemi − generalized closed. Also, we show the relationships between them in illustration diagram and gives some results and examples.

1. Introduction

One of an essential object in a topological space called closed sets so by using the 'Kuratowski closure axioms' or the axioms of it one can present the topology on sets. N. Levine[1], in (1970), introduce an important definition that provides for, A subset ß of (Ҳ, 𝔉) is named generalized closed if $Cl.$(ß) ⊆ Ʉ whenever ß ⊆ Ʉ for all Ʉ is open, P. Bhattacharya and B.K. Lahiri [2], in 1987 present semi-generalized closed sets in topology and in 1995 J. Dontchev [3] study the On generating ꞩemi -preopen sets. And it follows in 2002, J.Cao, M. Ganster and I. Reilly [4] described on generalized closed sets. Lellis Thivagar [5] given the concept of Ŋ*ano*-topology in 2013. Also, the authors Bhuvaneswari, Ezhilarasi [6] and Thanga Nachiyar [7] introduced on Ŋ*ano* genaeralized ꞩemi closed sets and Ŋ*ano* ꞩemi -generalized, on Ŋ*ano* A-generalized closed sets and Ŋ*ano* generalized A-closed sets in the spaces of Ŋ*ano*-topology, resp. in 2014. Many authors [8,9] study on Ŋ*ano* Topology and Micro topological spaces. Recently in 2020 the topic of Micro-α-open sets and Micro-α- continuous functions in Micro topological spaces was given by Jasim and Rasheed [10]. This prompts us to continue to develop and generalize this topic by presenting some important definitions such as Micro α − generalized closed , Micro generalized α − closed, Micro generalized ꞩemi − closed and Micro ꞩemi − generalized closed and noting some results on them in a diagram and gives some examples.

2. Preliminaries

In principle, we call some important basics and acquaintances.

Definition2.1[8]: Let Ҳ a non empty finite set things called the universal and $\mathfrak{R}$ is the equivalence relation on Ҳ named as the indiscernibility relation elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (Ҳ, $\mathfrak{R}$) is said to be approximation space. Let Ą ⊆ Ҳ ,

1. The Lower approximation of Ą with respect to $\mathfrak{R}$ the set of all things, which can be for certain classified as Ą with respect to $\mathfrak{R}$ and is defined by $L_{\mathfrak{R}}$(Ą)=$\cup_{x\in}$Ҳ $\{\mathfrak{R}(x): \mathfrak{R}(x) \subseteq$ Ą$\}$, *Where* $\mathfrak{R}(x)$ denotes the equivalence class determined by x.

2. The upper approximation of Ą with respect to $\mathfrak{R}$ is these of all things, which can be possibly classified as X with respect to $\mathfrak{R}$ and is defined by $U_{\mathfrak{R}}$(Ą) = $\cup_{x\in}$Ҳ$\{\mathfrak{R}(x): \mathfrak{R}(x) \cap$ Ą $\neq \emptyset\}$.

3.The boundary region of X with respect to $R$is the set of all things, which can be classified neither as Ą nor as not Ą with respect to $\mathfrak{K}$ and is defined by $B_{\mathfrak{K}}(Ą) = U_{\mathfrak{K}}(Ą) - L_{\mathfrak{K}}(Ą)$.

Definition 2.2[8]: Let Ҳ be the universal, $\mathfrak{K}$ be an equivalence relation on Ҳ and $\mathfrak{F}_{\mathfrak{K}}(Ą) = \{Ҳ, \emptyset, L_{\mathfrak{K}}(Ą), U_{\mathfrak{K}}(Ą), B_{\mathfrak{K}}(Ą)\}$ where $Ą \subseteq Ҳ$. Then $\mathfrak{F}_{\mathfrak{K}}(Ą)$ satisfies The following axioms.

1. Ҳ *and* $\emptyset \in \mathfrak{F}_{\mathfrak{K}}(Ą)$.
2. The union of the elements of any sub collection of $\mathfrak{F}_{\mathfrak{K}}(Ą)$ is in $\mathfrak{F}_{\mathfrak{K}}(Ą)$.
3. The intersection of the elements of any finite sub collection of $\mathfrak{F}_{\mathfrak{K}}(Ą)$ is in $\mathfrak{F}_{\mathfrak{K}}(Ą)$.

That is $T_R(X)$ forms a topology on Ҳ called as Nano topology on Ҳ with respect to Ą. We call $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą))$ as the Nano topological space .The elements of $\mathfrak{F}_{\mathfrak{K}}(Ą)$ are called Nano open sets .

Definition2.3[9]: Let $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą))$ be Ŋ*ano*-topological space then $\mu_{\mathfrak{K}}(Ą) = \{N_1 \cup (N_2 \cap \mu): N_1, N_2 \in \mathfrak{F}_{\mathfrak{K}}(Ą)\}$ is called 'Micro-topology' of $\mathfrak{F}_{\mathfrak{K}}(Ą)$ by μ where $\mu \notin \mathfrak{F}_{\mathfrak{K}}(Ą)$.

Definition2.4[9]: A Micro -topology $\mu_{\mathfrak{K}}(Ą)$ satisfies the next axioms:

a) Ҳ and $\phi \in \mu_{\mathfrak{K}}(Ą)$.
b) The union of the elements of each sub-collection of $\mu_{\mathfrak{K}}(Ą)$ is in $\mu_{\mathfrak{K}}(Ą)$.
c) The intersection of the elements of each finite sub collection of $\mu_{\mathfrak{K}}(Ą)$ is in $\mu_{\mathfrak{K}}(Ą)$.

Then $\mu_{\mathfrak{K}}(Ą)$ is called Micro topology on Ҳ with respect to Ą. The tripartite $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ is called 'Micro -topological space' and the elements of $\mu_{\mathfrak{K}}(Ą)$ are called 'Micro -open' sets and the complement of a 'Micro -open' set is called a 'Micro -closed' set.

Definition2.5[1]: A subset ß of $(Ҳ, \mathfrak{F})$ is called generalized closed (shortly g -closed) if $Cl.(ß) \subseteq Ʉ$ and for all $ß \subseteq Ʉ$ for all Ʉ is open in $(Ҳ, \mathfrak{F})$.

3. Micro $\alpha -$ GENERALIZED Closed AND Micro semi $-$ GENERALIZED Closed Sets

Here in this section we will provide definitions and examples and some results obtained

Definition3.1[10]: The Micro-α-closure form a set ß of a Micro topological space $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ is the intersection of all Micro-α-closed sets that contain ß and denoted by Mic. $\alpha - Cl.(ß)$.

Definition3.2: A subset ß of a space $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ is said to be Micro generalized $\alpha -$ closed (shortly Mic. g$\alpha -$ closed) if Mic. $\alpha - Cl.(ß) \subseteq Ʉ$ and for all $ß \subseteq Ʉ$ for all Ʉ is Micro $\alpha - open$. The complements of Micro generalized $\alpha -$ closed is called Micro generalized $\alpha -$ open.

Definition3.3: A subset ß of a space $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ is said to be Micro $\alpha -$ generalized closed (shortly Mic.α g $-$ closed) if Mic. $\alpha - Cl.(ß) \subseteq Ʉ$ and for all $ß \subseteq Ʉ$ for all Ʉ is Micro *open*. The complements of Micro α −generalized closed is called Micro α −generalized open.

Example3.4: Let's we have the universe set $Ҳ = \{1,2,3,4\}$ with the equivalence relation $Ҳ/\mathfrak{K} = \{\{1\},\{3\},\{2,4\}\}$ and $Ą = \{2,4\}$ then the *Nano − topology* is $\mathfrak{F}_{\mathfrak{K}}(Ą) = \{\phi, Ҳ, \{2,4\}\}$. Let $\mu = \{2\}$,The Micro − topology is $\mu_{\mathfrak{K}}(Ą) = \{\phi, Ҳ, \{2\}, \{2,4\}\}$. Sets of closed Micro denoted $C\mu_{\mathfrak{K}}(Ą) = \{\phi, Ҳ, \{1,3,4\}, \{1,3\}\}$. We can deduce the collection of all Micro α-open as:

$\alpha O(Ҳ) = \{\phi, Ҳ, \{2\}, \{1,2\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$

And Micro α-closed as:

$M\alpha C(Ҳ) = \{\phi, Ҳ, \{1,3,4\}, \{3,4\}, \{1,4\}, \{1,3\}, \{4\}, \{3\}, \{1\}\}$

Now, we will take the sets of all power of Ҳ and we check which one of them are M.g.closed , M.αg.closed and M.gα.*closed* as in the table:

Table 1

P(Ҳ)	M.*Cl.*(Ą)	M.α*Cl.*(Ą)	M.g.closed	M.αg.	M.gα.
{1}	{1,3}	{1}	T	T	T
{2}	Ҳ	Ҳ	F	F	F
{3}	{1,3}	{3}	T	T	T
{4}	{1,3,4}	{4}	F	T	T
{1,2}	Ҳ	Ҳ	T	T	F
{1,3}	{1,3}	{1,3}	T	T	T
{1,4}	{1,3,4}	{1,4}	T	T	T
{2,3}	Ҳ	Ҳ	T	T	F
{2,4}	Ҳ	Ҳ	F	F	F
{3,4}	{1,3,4}	{3,4}	T	T	T
{1,2,3}	Ҳ	Ҳ	T	T	F
{1,2,4}	Ҳ	Ҳ	T	T	F
{1,3,4}	{1,3,4}	{1,3,4}	T	T	T
{2,3,4}	Ҳ	Ҳ	T	T	F
ϕ	ϕ	ϕ	T	T	T
Ҳ	Ҳ	Ҳ	T	T	T

Here we consider three collections:

The collection of

$$MgC(Ҳ) = \left\{\begin{matrix}\phi, Ҳ, \{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\}, \\ \{1,2,4\},\{1,3,4\},\{2,3,4\}\end{matrix}\right\}$$

The collection of

$$M\alpha gC(Ҳ) = \left\{\begin{matrix}\phi, Ҳ, \{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\} \\ ,\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\}\end{matrix}\right\}$$

The collection of

$Mg\alpha C(Ҳ) = \{\phi, Ҳ, \{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\}$

Result3.5: In a space $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ we have,

1) Each M.g.closed is M.αg.closed.
2) Each M.g.α closed is M.αg.closed.

Proof:

1) Assume that $ß \in MgC(Ҳ)$ in a space $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ then Mic.$Cl.(ß) \subseteq Ʉ$ for $ß \subseteq Ʉ$ and for each Ʉ is Micro *open*. Since Mic.$\alpha - Cl.(ß) \subseteq$ Mic.$Cl.(ß) \subseteq Ʉ$ for each Ʉ is Micro *open*, So $ß \in M\alpha gC(Ҳ)$.

2) Assume that $ß \in Mg\alpha C(Ҳ)$ in a space $(Ҳ, \mathfrak{F}_{\mathfrak{K}}(Ą), \mu_{\mathfrak{K}}(Ą))$ then Mic.$\alpha - Cl.(ß) \subseteq Ʉ$ for $ß \subseteq Ʉ$ for each Ʉ is *Micro* $\alpha - open$. Since 'every Micro *open* is *Micro* $\alpha - open$' then Mic.$\alpha - Cl.(ß) \subseteq Ʉ$ for $ß \subseteq Ʉ$ and for each Ʉ is Micro *open*, So $ß \in Mg\alpha C(Ҳ)$.

Remarks3.6: From the above result:

1) Not all ⍰. αg. closed is ⍰. g. closed like in the previous example the set ß = {4} ∈ ⍰αgC(Ⱪ) but ß ∉ ⍰gC(Ⱪ).

2) Also, not all ⍰. αg. closed must be ⍰. g. α closed as in the same example. The set Ŋ = {1,2,3} ∈ ⍰αgC(Ⱪ) but not belong to ⍰gαC(Ⱪ).

3) Both ⍰gC(Ⱪ) and ⍰gαC(Ⱪ) are independent as : The set $\mathcal{H}$ = {1,2} ∈ ⍰gC(Ⱪ) but not in ⍰gαC(Ⱪ) and ß = {4} ∈ ⍰gαC(Ⱪ) but not in ⍰gC(Ⱪ).

Proposition3.7: If each one of Ą and ß are ⍰ic. αg. −closed then Ą ∪ ß will be ⍰ic. αg. −closed.

Proof:

Suppose that Ą and ß are ⍰ic. αg. −closed then we have ⍰ic. α − *Cl.* (Ą) ⊆ Ʉ for Ą ⊆ Ʉ for all Ʉ is ⍰icro α − *open* and ⍰ic. α − *Cl.* (ß) ⊆ Ʉ and for all ß ⊆ Ʉ for all Ʉ is ⍰icro *open*. Now, since Ą ⊆ Ʉ and ß ⊆ Ʉ , so Ą ∪ ß ⊆ Ʉ s.t. Ʉ is ⍰icro *open*, Then ⍰ic. α − *Cl.* (Ą ∪ ß) = ⍰ic. α − *Cl.* (Ą) ∪ ⍰ic. α − *Cl.* (ß) ⊆ Ʉ hence, ⍰ic. α − *Cl.* (Ą ∪ ß) ⊆ Ʉ *whenever* Ą ∪ ß ⊆ Ʉ for all Ʉ is ⍰icro *open* implies that Ą ∪ ß be ⍰ic. αg. −closed.

Remark3.8: The intersection of two ⍰ic. αg. −closed in a space (Ⱪ, $\mathfrak{F}_{\mathfrak{R}}$(Ą), ꞕ$_{\mathfrak{R}}$(Ą)) is not ⍰ic. αg. −closed , since in example (3.4) {2,3,4}, {1,2,4} are ⍰ic. αg. −closed but {2,4} is not.

Definition3.9: The ⍰icro-ꞩemi-closure form a set ß of a ⍰icro topological space (Ⱪ, $\mathfrak{F}_{\mathfrak{R}}$(Ą), ꞕ$_{\mathfrak{R}}$(Ą)) is the intersection of all ⍰icro-ꞩemi-closed sets that contain ß and denoted by ⍰ic. ꞩ − *Cl.* (ß).

Definition3.10: A subset ß in a space (Ⱪ, $\mathfrak{F}_{\mathfrak{R}}$(Ą), ꞕ$_{\mathfrak{R}}$(Ą)) is said to be ⍰icro generalized ꞩemi − closed (shortly ⍰ic. gꞩ − closed) if ⍰ic. ꞩ − *Cl.* (ß) ⊆ Ʉ for ß ⊆ Ʉ and for all Ʉ is ⍰icro *open*. The complements of ⍰icro generalized ꞩemi − closed is called ⍰icro generalized ꞩemi − open.

Definition3.11: A subset ß of a space (Ⱪ, $\mathfrak{F}_{\mathfrak{R}}$(Ą), ꞕ$_{\mathfrak{R}}$(Ą)) is said to be ⍰icro ꞩemi − generalized closed (shortly ⍰ic.ꞩ g − closed) if ⍰ic. ꞩ − *Cl.* (ß) ⊆ Ʉ for ß ⊆ Ʉ for all Ʉ is ⍰icro ꞩemi *open*. The complements of ⍰icro ꞩemi −generalized closed is called ⍰icro ꞩemi −generalized open.

Example3.12: In example (3.4) let ꞕ = {3}, The ⍰icro − topology is ꞕ$_{\mathfrak{R}}$(Ą) = {ϕ, Ⱪ, {3}, {2,4}, {2,3,4}}. The Cꞕ$_{\mathfrak{R}}$(Ą) = {ϕ, Ⱪ, {1,2,4}, {1,3}{1}}

We can write the collection of all ⍰icro ꞩemi open as:

⍰ꞩO(Ⱪ) = {ϕ, Ⱪ, {3}, {1,3}, {2,3}, {2,4}, {1,2,4}, {2,3,4}}

And the collection of ⍰icro ꞩemi closed as: ⍰ꞩC(Ⱪ) = {ϕ, Ⱪ, {1,2,4}, {2,4}, {1,3}, {3}, {1}}

Consider the table:

Table 2

P(Ⱪ)	⍰. ꞩ*Cl.* (Ą)	⍰. g. ꞩ	⍰. ꞩg.
{1}	{1}	T	T
{2}	{2,4}	T	T
{3}	{3}	T	T
{4}	{2,4}	T	T
{1,2}	{1,2,4}	T	T
{1,3}	{1,3}	T	T
{1,4}	{1,2,4}	T	T
{2,3}	Ⱪ	F	F
{2,4}	{2,4}	T	T
{3,4}	Ⱪ	F	F
{1,2,3}	Ⱪ	T	T
{1,2,4}	{1,2,4}	T	T
{1,3,4}	Ⱪ	T	T
{2,3,4}	Ⱪ	F	F
ϕ	ϕ	T	T
Ⱪ	Ⱪ	T	T

In this table, the collection of ⍰gꞩC(Ⱪ) and ⍰ꞩgC(Ⱪ) are the same.

Example3.13: We take the universe set Ⱪ = {1,2,3,4} with the equivalence relation Ⱪ/$\mathfrak{R}$ = {{2}, {4}, {1,3}} and Ą = {1,2} then the *Nano − topology* is $\mathfrak{F}_{\mathfrak{R}}$(Ą) = {$\phi$, Ⱪ, {2}, {1,3}, {1,2,3}}. Let ꞕ = {1}, The ⍰icro − topology is ꞕ$_{\mathfrak{R}}$(Ą) = {ϕ, Ⱪ, {1}, {2}, {1,2}, {1,3}, {1,2,3}}. Sets of closed ⍰icro

Cꞕ$_{\mathfrak{R}}$(Ą) = {ϕ, Ⱪ, {2,3,4}, {1,3,4}, {3,4}, {2,4}, {4}}.

The collection of ⍰icro α-open is: ⍰αO(Ⱪ) = {ϕ, Ⱪ, {1}, {2}, {1,2}, {1,3}, {1,2,3}, {1,2,4}}. And ⍰icro α-closed as:

⍰αC(Ⱪ)
= {ϕ, Ⱪ, {2,3,4}, {1,3,4}, {3,4}, {2,4}, {4}, {3}}

The collection of ⍰icro ꞩemi open is:

⍰ꞩO(Ⱪ)
= {ϕ, Ⱪ, {1}, {2}, {1,2}, {1,3}, {1,4}, {2,4}, {1,2,3}, {1,3,4}, {1,2,4}}

And the collection of ⍰icro ꞩemi closed as:

ꞩC(Ⱪ) = {ϕ, Ⱪ, {2,3,4}, {1,3,4}, {3,4}, {2,4}, {2,3}, {1,3}, {4}, {2}, {3}}

Now, we will take the sets of all power of Ⱪ and we check which one of them are ⍰. αg. closed , ⍰. gα. *closed* , ⍰. ꞩg. closed and ⍰. g. ꞩ closed as in the table:

Table 3

P(Ⱪ)	⍰. ꞩ*Cl.* (Ą)	⍰. ꞩg.	⍰. g. ꞩ	⍰. α*Cl.* (Ą)	⍰. gα.	⍰. αg.
{1}	{1,3}	F	F	{1,3,4}	F	F
{2}	{2}	T	T	{2,4}	F	F
{3}	{3}	T	T	{3}	T	T
{4}	{4}	T	T	{4}	T	T
{1,2}	Ⱪ	F	F	Ⱪ	F	F
{1,3}	{1,3}	T	T	{1,3,4}	F	F
{1,4}	{1,3,4}	F	T	{1,3,4}	F	T
{2,3}	{2,3}	T	T	{2,3,4}	F	F
{2,4}	{2,4}	T	T	{2,4}	T	T
{3,4}	{3,4}	T	T	{3,4}	T	T
{1,2,3}	Ⱪ	F	F	Ⱪ	F	F
{1,2,4}	Ⱪ	F	T	Ⱪ	F	T
{1,3,4}	{1,3,4}	T	T	{1,3,4}	T	T
{2,3,4}	{2,3,4}	T	T	{2,3,4}	T	T
ϕ	ϕ	T	T	ϕ	T	T
Ⱪ	Ⱪ	T	T	Ⱪ	T	T

Here we consider four collections: The collection of ⍰icro semi generalized closed

⍰ꞩgC(X) $= \begin{Bmatrix} \phi, X, \{2\}, \{3\}, \{4\}, \{1,3\}, \{2,3\}, \{2,4\}, \{3,4\}, \\ \{1,3,4\}, \{2,3,4\} \end{Bmatrix}$

The collection of ⍰icro generalized semi closed

⍰gꞩC(X) $= \begin{Bmatrix} \phi, X, \{2\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \\ \{1,2,4\}, \{1,3,4\}, \{2,3,4\} \end{Bmatrix}$

The collection of

⍰gαC(X) $= \{\phi, X, \{3\}, \{4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}$

The collection of

⍰αgC(X) $= \begin{Bmatrix} \phi, X, \{3\}, \{4\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,4\} \\ , \{1,3,4\}, \{2,3,4\} \end{Bmatrix}$

Result3.14: In a space $(X, \mathfrak{F}_{\mathfrak{R}}(A), ɱ_{\mathfrak{R}}(A))$ we have,

1) Each ⍰. g. α closed is ⍰. g. ꞩ closed.
2) Each ⍰. αg. closed is ⍰. g. ꞩ closed.
3) Each ⍰. ꞩg. closed is ⍰. g. ꞩ closed.

Proof:

1) Assume that ß ∈ ⍰gαC(X) in a space $(X, \mathfrak{F}_{\mathfrak{R}}(A), ɱ_{\mathfrak{R}}(A))$ then ⍰ic. α*Cl*. (ß) ⊆ Ʉ for ß ⊆ Ʉ for each Ʉ is ⍰icro α − *open*. Since 'every ⍰icro *open* is ⍰*icro* α − *open*' and ⍰ic. ꞩ − *Cl*. (ß) ⊆ ⍰ic. α*Cl*. (ß) ⊆ Ʉ for each Ʉ is ⍰icro *open*, So ß ∈ ⍰gꞩC(X).

2) Assume that ß ∈ ⍰αgC(X) in a space $(X, \mathfrak{F}_{\mathfrak{R}}(A), ɱ_{\mathfrak{R}}(A))$ then ⍰ic. α − *Cl*. (ß) ⊆ Ʉ for ß ⊆ Ʉ for each Ʉ is ⍰*icro open*. Since ⍰ic. ꞩ − *Cl*. (ß) ⊆ ⍰ic. α*Cl*. (ß) ⊆ Ʉ then ⍰ic. ꞩ − *Cl*. (ß) ⊆ Ʉ for ß ⊆ Ʉ for each Ʉ is ⍰icro *open*, So ß ∈ ⍰gꞩC(X).

3) Assume that ß ∈ ⍰ꞩgC(X) in a space $(X, \mathfrak{F}_{\mathfrak{R}}(A), ɱ_{\mathfrak{R}}(A))$ then ⍰ic. ꞩ*Cl*. (ß) ⊆ Ʉ for ß ⊆ Ʉ for each Ʉ is ⍰icro ꞩ − *open*. Since 'every ⍰icro *open* is ⍰*icro* ꞩ − *open*' and ⍰ic. ꞩ − *Cl*. (ß) ⊆ Ʉ for ß ⊆ Ʉ for each Ʉ is ⍰icro *open*, So ß ∈ ⍰gꞩC(X).

Remarks3.15: From the above result:

1) Not all ⍰. gꞩ. closed be ⍰. g. α closed like in the previous example the set ß = {2} ∈ ⍰gꞩC(X) but ß ∉ ⍰gαC(X).
2) Also, not all ⍰. gꞩ. closed must be ⍰. αg. closed as in the same example. The set Ņ = {2,3} ∈ ⍰gꞩC(X) but not belong to ⍰αgC(X).
3) Also, not all ⍰. gꞩ. closed must be ⍰. ꞩg. closed as in the same example. The set Ņ = {1,2,4} ∈ ⍰gꞩC(X) but not belong to ⍰ꞩgC(X).
4) Both ⍰ꞩgC(X) and ⍰αgC(X) are independent as : The set $\mathcal{H}$ = {2} ∈ ⍰ꞩgC(X) but not in ⍰αgC(X) and ß = {1,4} ∈ ⍰αgC(X) but not in ⍰ꞩgC(X).

Finally, we obtain the following diagram from the previous results

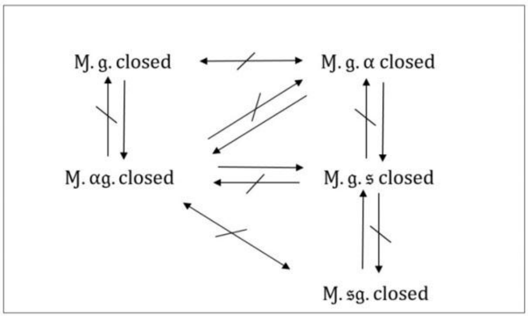

Diagram 1

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حول مايكرو ألفا المعممة المغلقة وشبه المعممة المغلقة في الفضاءات التبولوجية الدقيقة

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الملخص

سنقوم بعمل تعريفات جديدة في هذه الورقة وهي مايكرو ألفا معممة مغلقة ، مايكرو معممة ألفا مغلقة ، مايكرو معممة شبه مغلقة و مايكرو شبه معممة مغلقة. كما نعرض العلاقات بينهما في مخطط توضيحي ونعطي بعض النتائج والأمثلة.