



Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: <http://tjps.tu.edu.iq/index.php/j>



On some generalized recent operators in topological Spaces

Siham I. Aziz¹, Nabila I. Aziz²

¹ General Directorate of Kirkuk Province Education, Ministry of Education, Kirkuk, Iraq

² Department of Physics, College of Education -Tuzkhurmatu , Tikrit University, Tikrit, Iraq

DOI: <http://dx.doi.org/10.25130/tjps.27.2022.058>

ARTICLE INFO.

Article history:

-Received: 21 / 2 / 2022

-Accepted: 14 / 6 / 2022

-Available online: / / 2022

Keywords: Λ regular generalized*, Λ regular generalized**, Λ generalized** regular

Corresponding Author:

Name: Siham I. Aziz

E-mail:

suham_70100@yahoo.com

nabila.be@tu.edu.iq

Tel:

ABSTRACT

In this paper we introduce a new classes of operators namely (Λ regular generalized*, Λ regular generalized**, Λ generalized** regular) (briefly, Λrg^* , Λrg^{**} , Λg^{**r} , respectively) in topological spaces and study some of their properties with some important theorems.

1. Introduction

In 1971 [1] defined semi-closure in topological space. In (1982) [2] studied the concept of closure operator. In (2003) [6] used the concepts of open sets, closure operator, and semi-closure to present and analyze several weak separation axioms. Later, [7] used the notions of pre-open sets and pre-closure operator to develop certain weak separation axioms. Some theorems of this nature theorem have been proved by some researchers: Let (X, τ) be a T.S. (topological space) and A subset of X , is the intersection of all closed (resp. semi-closed, pre-closed, semi-pre-closed, b-closed) sets of X containing A is called the closure of A the closure of A [5] (resp. semi-closure [1], pre-closure [3], semi-pre-closure [4], b-closure [5]) of A .

In (2018) [8] introduced $\pi g^* \beta$ -closure in a topology space. In the year (2020) [9] and [10] introduced Pre-weakly generalized closed sets and presented Soft – interior and soft – closure in soft topology.

In this paper we present a new class of different operators we knew him on regular generalized open sets namely (Λ regular generalized*, Λ regular generalized**, Λ generalized** regular) (briefly, Λrg^* , Λrg^{**} , Λg^{**r} , respectively) and study some of their properties with some important theorems.

2. Preliminaries

This section of the manual lists some of the required definitions and theorems.

Definition 2.1[11]: A subset A of a space X is called regular generalized*- open [resp. regular generalized** - open] set (briefly rg^* -o(x), rg^{**} -o(x)) if $\text{int}(A) \subseteq U$, whenever $U \subseteq A$ and U is regular [resp. pre regular] closed set $\subseteq X$.

Definition 2. 2[11]: A subset A of a space X is called generalized** regular open set (briefly, g^{**r} -o(x)) if $\text{Rint}(A) \subseteq U$, whenever $U \subseteq A$ and U is pre-regular closed set $\subseteq X$.

Definition 2.4[12]: Let (X, τ) is a topological space as well as the operator $\Lambda_{GR}(F) = \bigcap \{U : F \subseteq U, U \text{ is generalized regular open set } \subseteq X\}$

3. On Some generalized recent operators in topological spaces

Definition 3.1: The new operators are listed below. Allow K to take the initiative = $[rg^*, rg^{**}]$ sets. $\Lambda_K(A) = \bigcap \{U : A \subseteq U, U \text{ is } K \in (rg^*, rg^{**}) \text{ open } \subseteq X\}$

Example 3.2: $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a, c, e\}, \{a, b, c, e\}, \{a, b\}\}$

$\tau^c = \{\emptyset, X, \{d\}, \{b, d\}, \{c, d, e\}\}$

$R\text{-}o(x) = \{\emptyset, X\}$, $R\text{-}c(x) = \{X, \emptyset\}$.

PreR-

$o(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{a, e\}, \{a, d\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, e\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, e\}, \{a, c, d\}, \{b, c, d\}, \{b, c, e\}, \{c, d, e\}\}$

$\{b,d,e\}, \{a,d,e\}, \{a,b,c,d\}, \{a,b,c,e\}, \{a,c,d,e\}, \{b,c,d,e\}, \{a,b,d,e\}$

$\text{PreRc}(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a,b\}, \{a,c\}, \{a,e\}, \{a,d\}, \{b,c\}, \{b,d\}, \{b,e\}, \{c,e\}, \{c,d\}, \{d,e\}, \{a,b,d\}, \{a,b,e\}, \{a,c,e\}, \{a,c,d\}, \{b,c,d\}, \{b,c,e\}, \{c,d,e\}, \{b,d,e\}, \{a,d,e\}, \{a,b,c,d\}, \{a,b,c,e\}, \{a,c,d,e\}, \{b,c,d,e\}, \{a,b,d,e\}\}$

$A = \{a, b, c\}, \Lambda_K(A) = X$,

$K = (\text{rg}^*, \text{rg}^{**})\text{-open sets} = \{\emptyset, X, \{a\}, \{d\}, \{e\}, \{b, c\}, \{b, d\}, \{c, e\}, \{d, e\},$

$\{a, c, d\}, \{b, c, d\}, \{a, d, e\}, \{c, d, e\},$

$\{b, c, d, e\}\}$

Lemma 3.3: (X, τ) be a topological space, $\Lambda_K: q(x) \rightarrow q(x)$ is operator which satisfies the properties.

1. $\Lambda_K(\emptyset) = \emptyset, \Lambda_K(X) = X$

2. $\Lambda_K(A) \supseteq A$

3. $\Lambda_K \Lambda_K(A) = \Lambda_K(A)$

4. If $B \supseteq A, \Lambda_K(B) \supseteq \Lambda_K(A)$.

5. $\Lambda_K(A \cap B) \subseteq \Lambda_K(A) \cap \Lambda_K(B)$.

6. $\Lambda_K(A \cup B) \supseteq \Lambda_K(A) \cup \Lambda_K(B)$.

Proof: 2- $\Lambda_K(A) \supseteq A$, Since $A \subseteq K, K \in (\text{rg}^*, \text{rg}^{**})\text{open sets} \subseteq X$, by definition (3.1) we get $A \subseteq \Lambda_K(A)$

3- $\Lambda_K \Lambda_K(A) = \Lambda_K(A)$, $\Lambda_K(A) = \bigcap \{G: A \subseteq G, G \text{ is } K \in (\text{rg}^*, \text{rg}^{**})\text{open} \subseteq X\}$.

$\Lambda_K[\Lambda_K(A)] = \bigcap [\bigcap \{G: A \subseteq G, G \text{ is } K \in (\text{rg}^*, \text{rg}^{**})\text{open} \subseteq X\}]$

$= \bigcap \{G: A \subseteq G, G \text{ is } K \in (\text{rg}^*, \text{rg}^{**})\text{open} \subseteq X\} = \Lambda_K(A)$

4- If $B \supseteq A, \Lambda_K(B) \supseteq \Lambda_K(A)$, $\Lambda_K(A) = \bigcap \{G: A \subseteq G, G \text{ is } K \in (\text{rg}^*, \text{rg}^{**})\text{open} \subseteq X\}$.

Since $A \subseteq S$, then

$\bigcap \{G: D \subseteq G, G \text{ is } K \in (\text{rg}^*, \text{rg}^{**})\text{open} \subseteq X\} \subseteq \bigcap \{G: S \subseteq G, G \text{ is } K \in (\text{rg}^*, \text{rg}^{**})\text{open} \subseteq X\} \Rightarrow \Lambda_K(B) \supseteq \Lambda_K(A)$.

5- $\Lambda_K(A \cap B) \subseteq \Lambda_K(A) \cap \Lambda_K(B)$, since $A \cap B \subseteq A$ and $A \cap B \subseteq B$

then $\Lambda_K(A \cap B) \subseteq \Lambda_K(A)$ and $\Lambda_K(A \cap B) \subseteq \Lambda_K(B)$.

Hence $\Lambda_K(A \cap B) \subseteq \Lambda_K(A) \cap \Lambda_K(B)$.

6- $\Lambda_K(A \cup B) \supseteq \Lambda_K(A) \cup \Lambda_K(B)$.

Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$

$\Rightarrow \Lambda_K(A) \subseteq \Lambda_K(A \cup B), \Lambda_K(B) \subseteq \Lambda_K(A \cup B)$.

Hence $\Lambda_K(A \cup B) \supseteq \Lambda_K(A) \cup \Lambda_K(B)$.

Remark 3.5: The following example shows that the converse section (5-6) of lemma (3.3) is not valid.

Example 3.6: $X = \{a, b, c, d, e\}$

$\tau = \{\emptyset, X, \{e\}, \{a, b\}, \{d, e\}, \{a, b, e\}, \{c, d, e\}, \{a, b, c, d\}, \{a, b, d, e\}\}$, $\text{Pro}(x) = \{\emptyset, X, p(x)\}$, $U = \{a, b, c, d, e\}$

$(\text{rg}^*, \text{rg}^{**})\text{open sets} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$.

i. $A = \{b\}, B = \{a, c, d\}, \Lambda_K(A) = \{b, d\}, \Lambda_K(B) = \{a, c, d\}, \Lambda_K(A \cup B) = X$, but $\Lambda_K(A) \cup \Lambda_K(B) = \{a, b, c, d\}$. then $\Lambda_K(A \cup B) \neq \Lambda_K(A) \cup \Lambda_K(B)$.

ii. $A = \{c, d, e\}, B = \{a, b, c\}, \Lambda_K(A) = X, \Lambda_K(B) = X, \Lambda_K(A \cap B) = \{c\}$
 $\Lambda_K(A) \cap \Lambda_K(B) = X$, then $\Lambda_K(A \cap B) \neq \Lambda_K(A) \cap \Lambda_K(B)$

Lemma 3.7: (X, τ) be a topological space, $\Lambda_{g^{**r}}: L(x) \rightarrow L(x)$ is operator satisfy the conditions.

1- $\Lambda_{(g^{**r})}(\emptyset) = \emptyset, \Lambda_{(g^{**r})}(X) = X$

2- $\Lambda_{(g^{**r})}(S) \supseteq S$

3- $\Lambda_{(g^{**r})} \Lambda_{(g^{**r})}(A) = \Lambda_{(g^{**r})}(A)$

4- If $B \supseteq A, \Lambda_{(g^{**r})}(B) \supseteq \Lambda_{(g^{**r})}(A)$

5- $\Lambda_{(g^{**r})}(A \cap B) \subseteq \Lambda_{(g^{**r})}(A) \cap \Lambda_{(g^{**r})}(B)$

6- $\Lambda_{(g^{**r})}(A \cup B) \supseteq \Lambda_{(g^{**r})}(A) \cup \Lambda_{(g^{**r})}(B)$.

Proof: (2- 3-4-5-6) The same steps as the previous proof of lemma(3-3) .

Remark 3.8: The following example shows that the converse section (5-6) of lemma (3.7) is not valid.

Example 3.9: Let $X = \{a, b, c, d, e\}$ and the corresponding topology be

$\tau = \{\emptyset, X, \{d\}, \{c, e\}, \{c, d, e\}, \{a, b, c, e\}\}$

$\tau^c = \{\emptyset, X, \{d\}, \{a, b\}, \{a, b, d\}, \{a, b, c, e\}\}$

$R\text{-o}(x) = \{\emptyset, X, \{d\}, \{a, b, c, e\}\}$, $R\text{-c}(x) = \{X, \emptyset, \{a, b, c, e\}, \{d\}\}$.

$\text{PreRo}(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{a, e\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, e\}, \{c, d\}, \{d, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, e\}, \{a, c, d\}, \{b, c, d\}, \{b, c, e\}, \{c, d, e\}, \{b, d, e\}, \{a, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, c, d, e\}, \{b, c, d, e\}, \{a, b, d, e\}\}$

$\text{PreRc}(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{a, e\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, e\}, \{c, d\}, \{d, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, e\}, \{a, c, d\}, \{b, c, d\}, \{b, c, e\}, \{c, d, e\}, \{b, d, e\}, \{a, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, c, d, e\}, \{b, c, d, e\}, \{a, b, d, e\}\}$

$g^{**r}\text{-open}$

$\text{set} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{e\}, \{a, b\}, \{a, c\}, \{a, e\}, \{a, e\}, \{b, c\}, \{b, e\}$

$\{c, e\}, \{a, b, c\}, \{a, b, e\}, \{a, c, e\}, \{b, c, e\}\}$.

1- $A = \{e\}, B = \{a, c, d\}, \Lambda_K(A) = \{e\}, \Lambda_K(B) = \{a, c, d\}, \Lambda_K(A \cup B) = X$, but $\Lambda_K(A) \cup \Lambda_K(B) = \{a, c, d, e\}$, then $\Lambda_K(A \cup B) \neq \Lambda_K(A) \cup \Lambda_K(B)$.

2- $A = \{c, d, e\}, B = \{a, b, d\}, \Lambda_K(A) = X, \Lambda_K(B) = X, \Lambda_K(A \cap B) = \{d\}$

$\Lambda_K(A) \cap \Lambda_K(B) = X$, then $\Lambda_K(A \cap B) \neq \Lambda_K(A) \cap \Lambda_K(B)$

Theorem 3.10: $\Lambda_{(\text{rg}^*, \text{rg}^{**})}(A) = A \leftrightarrow A$ is a regular generalized* [resp. regular generalized**] open set.

Proof: Let A be a regular generalized* open [resp. regular generalized** open] set.

Since $A \subseteq \Lambda_{\text{rg}^*, \text{rg}^{**}}(A)$ and $\text{int}(A) \subseteq U$, U is regular [resp. pre regular] closed set then

$\Lambda_{\text{rg}^*, \text{rg}^{**}}(A) \subseteq A$, therefore $\Lambda_{\text{rg}^*, \text{rg}^{**}}(A) = A$.

Conversely: if $\Lambda_{\text{rg}^*, \text{rg}^{**}}(A) = A$. To prove A is a regular generalized* [resp. regular generalized**] open set

Since arbitrary intersection of regular generalized* [resp. regular generalized**] open set is regular generalized* [resp. regular generalized**] open set. Then A is a regular generalized* [resp. regular generalized**] open set.

Theorem 3.11: $\Lambda_{(g^{**r})}(A) = A \leftrightarrow A$ is generalized regular** open set.

Proof: Let A be a generalized regular** open set.

Since $A \subseteq \Lambda_{g^{**r}}(A)$, $\text{Rint}(A) \subseteq U$, U is Pre regular closed set. then $\Lambda_{g^{**r}}(A) \subseteq A$, therefore $\Lambda_{g^{**r}}(A) = A$.

Conversely: if $\Lambda_{g^{**r}}(A) = A$. To prove A is a generalized regular** open set

Since arbitrary intersection of generalized regular** open set is generalized regular** open set
Hence A is generalized regular** open set.

4. Conclusions

We introduced a new type of operators namely (Λ regular generalized*, Λ regular generalized**, Λ generalized** regular) (briefly, Λrg^* , Λrg^{**} , Λg^r ,

respectively) in topological spaces and study some of their properties with some important theorems. In the next study we can generalized to the Nano topological space and micro topological space. As well as the definition of operator Λ can generalized on (α , β and b) open sets.

References

- [1] Hildebrand, S.G. (1971). Semi-closure. *Texas J. Sci.*, **22**:99-112,
- [2] Dunham ,W. (1982). A new closure operator for non- T4 topologies. *Kyungpook Math. J.*, **22** :55-60,
- [3] Mashhour, A. S. , Hasanein ,I. A. and El-Deeb, S. N. (1983). Continuous and open mappings. *Acta Math. Hungar*, **41**: 213-218.
- [4] Andrijevic, D. (1986) .Semi-preopen sets. *Mat. Vesnik*, **38**: 24-32.
- [5] Andrijevic, D. (1996). On b-open sets. *Mat. Vesnik*, **48**: 56-64.
- [6] Caldas, M., Georgiou D. N., and Jafari, S. (2003). Characterizations of low separation axioms and closure operator. *Bol. Soc. Paran. Mat*, **2112(3)**: 1-14.
- [7] Caldas, M., Fukutake,T. , jafari, S. and Noiri,T. (2005). Some applications of -preopen sets in topological spaces. *Bulletin of the Institute of Mathematics Academia Sinica* , **33(3)**.
- [8] Devika, A. and Vani, R. (2018). On $\pi g^*\beta$ -Closed Sets in Topological Spaces. *J Appl Computat Math*, 7:3,DOI: 10.4172/2168-9679.1000413.
- [9] Bhardwaj, N. and Mir, A. (2020). Pre weakly generalized closed sets in topological spaces, *Journal of Physics: Conference Series* , IOP Publishing,1531 (2020) 012082,doi:10.1088/1742-6596/1531/1/012082
- [10] Savita R. , Ridam G. and Kusum D. (2020). On soft ω -interior and soft ω -closure in soft topological spaces. *Journal of Interdisciplinary Mathematics*,**6(23)**,
- [11] <https://doi.org/10.1080/09720502.2020.1815399>
- [12] Aziz, N. I. and Jasim, T. H. (2014). On generalized minimal open sets. A Thesis Submitted to the Council of the College of Education for Pure Science , , In Partial Fulfillment of the Requirement for the Degree of Master of Science in Mathematics. Tikrit University, Tikrit, Iraq: 92 pp.
- [13] Bhattacharya ,S. (2011). On generalized regular closed sets. *Contemp. Math. Sciences*, **6(3)** : 145-152.

على بعض العوامل المعممة الحديثة في الفضاءات التبولوجية

سهام ابراهيم عزيز¹ ، نائلة ابراهيم عزيز²

¹المديرية العامة لتربية محافظة كركوك ، وزارة التربية والتعليم ، كركوك ، العراق

²قسم الفيزياء ، كلية التربية ، طوزخورماتو ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث نقدم فئات جديدة لبعض العوامل اسميناها (Λ regular generalized, Λ regular generalized, Λ generalized regular) (باختصار ، Λrg^* ، Λrg^{**} ، Λg^r ، على التوالي) في الفضاء التبولوجي وأعطينا بعض الأمثلة الهامة ودرسنا بعض خصائصها وبعض النظريات .