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A New Proposed TSK rule base model for Fuzzy System

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Abstract

TSK model was the interested model that the researchers focused on because of the flexibility of this model in real problems , Although TSK model has main disadvantage : depends -at the beginning- on random parameters that may distribute the convergence of the solution.

In this paper, We discuss the disadvantages of TSK model; and try to cover it by a new proposed TSK rule base model that merges fuzzy sets properties and the nonlinear function in the rule. This means that rule output will depend not just on the random parameters, but on the outputs centers in every rule too. We used the proposed model to simulate pendulum real problem plant, and the experiment results showed increasing accuracy and decreasing training time in comparison with the traditional TSK rule base model.

Key words: rule base, TSK rule base model, fuzzy system, inverted pendulum.

Introduction

Since 1965, when Lotfi Zadeh [1] proposed a multi-valued theory (Fuzzy set theory), as a powerful control tool; many researches focused on the component of the fuzzy system, and tried to answer it's main question , “how dose the core of the fuzzy system think? “[1,2].

Using modeling ,good core for fuzzy system can be built to remove some of the complexity of real problems, and to simplify it as possible as it could, and by modeling; we may gain a more precise understanding of significant properties of real problems. [3]

Fuzzy Systems are rule base (core of fuzzy system) in the form IF-THEN rules, that save the way of reasoning and response of the Fuzzy System. Since then; many rule base models are introduced like Mamdani, Larsen, Tsukamoto and TSK (Takagi-Sugeno-Kang)[1,10], where every model has its advantages and disadvantages.

In this paper ,We will discuss the disadvantages of TSK rule base model and compare them via standard rule base model ,then try to suggest a new rule base model to improve the performance and the accuracy of overall Fuzzy system.

Pure Fuzzy Systems

The standard Fuzzy System (FS) where is also called Mamdani rule base model, contain four main components [1,4] :

- **Fuzzifier:** receive the crisp values as an inputs and convert them to a pair (x,μ (x)) as output.

  Where x is the input crisp value.

  μ (X) is the degree of membership x to the fuzzy sets in the universe of discourse of the inputs.
- **Rule base:** contains set of rules that determine the events and the corresponding results as an output (the brain of FS). The rules in standard FS come in the form :

$$R^{(l)}{:}IF\ \ x_1\ is\ F_1^l\ and \cdots and\ \ x_n\ is\ F_n^l\ then\ y^l\ is\ G^l \qquad (1)$$

Where and are fuzzy sets. X are the inputs; while x= ($x_1,x_2 \ldots x_n$).THEN part could contain more than one fuzzy set as an output; and the rules that match the inputs are called fired rules.

- **Inference engine** determine a mechanism in order to produce an output from collections of IF-THEN fuzzy rules that are fired.

- **Defuzzifier** receive fuzzy valued from the inference engine and convert it to a crisp value again.

Disadvantages of standard FS.

The main disadvantage in the standard FS are that the inputs and outputs are fuzzy sets, while most engineering real problems want to use real-valued variables [4, 5] and the hole of the universe of discourse instead of using term-valued to represent them.

The other disadvantage is the need to train fuzzy sets in the fuzzy system that may not determine good events to good results in the beginning.

TSK Fuzzy System.

A different fuzzy system model was introduced by Takagi and Sugeno in 1985 , and later by Sugeno and Kang while the main difference between standard and TSK fuzzy system is in the rule base Model[5,6]; The rules in TSK come in the form :

$$R^{(l)}\text{:}IF\ \ x_1\ is\ F_1^l\ and \cdots and\ \ x_n\ is\ F_n^l\ then\ \ \ y^l = f(g)$$

Where:

$$f(g) = p_0^l + p_1^l x_1 + \cdots + p_n^l x_n \tag{2}$$

Where p: p= (p_0,.., pn); are real value parameter. THEN part could contain more than one nonlinear function $f\ (g)$ as output. Output of TSK fuzzy system $y\ (x^v)$ is obtained by the weighted average of all fired rules y^l in the system:

$$y(x^v) = \frac{\sum_{l=1}^{m} w^l y^l}{\sum_{l=1}^{m} w^l}$$

$$where \qquad x^v = (x_1, x_2 \cdots x_n) \tag{3}$$

The weight w^l implies the truth value of the IF part in the rule $R^{(l)}$ for the input vector x^v, in overall FS.

$$w^l = \prod_{i=1}^{n} \mu_{f_1^l}(x_i) \tag{4}$$

Fig. 1, shows the basic configuration of TSK fuzzy system:

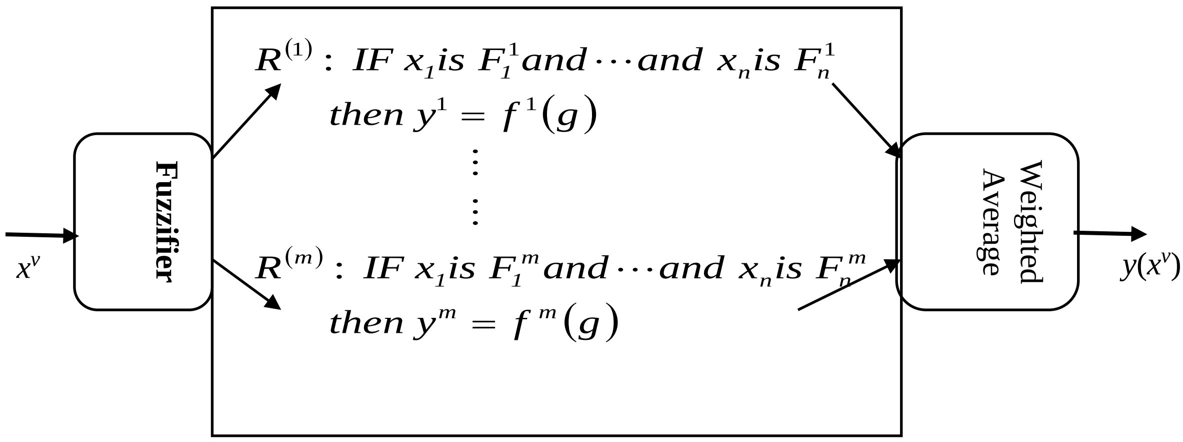


Fig. 1: Basic configuration of TSK fuzzy system

Disadvantages of TSK Model

TSK model solves the need for using real valued variables in the rule base instead of using fuzzy sets in THEN part of rule [6], by replacing it by a nonlinear function of inputs. Thus, the accuracy of overall system can be improved because the THEN part of the rule will contain many parameters can be tuned to reach faster convergence of solution .Therefore ; training time of parameters will increase when inputs of the system increase[7, 12].

In the next section, we will cover some of these disadvantages and suggest a new TSK rule base model.

The Proposed Rule Base Model

TSK model dose not use any fuzzy set in THEN part of the rules. Therefore, we need good training algorithm to treat and train random parameters in THEN part of every rule in the rule base to reach exact values of parameters that make error very small [8], and an acceptable accuracy; Therefore; we will introduce a new TSK rule base model that merges fuzzy sets properties and the nonlinear function in the THEN part of the rules in rule base to improve traditional TSK model:

The rules in the proposed TSK come in the form:

$$R^{(l)}: IF\ \ x_1\ is\ F_1^l\ and \cdots and\ \ x_n\ is\ F_n^l$$

$$then\ y^l = \left(p_0^l + \cdots + p_n^l x_n\right) \bullet \frac{c_{y^l}}{c_{F_1^l} + \cdots + c_{F_n^l}} \tag{5}$$

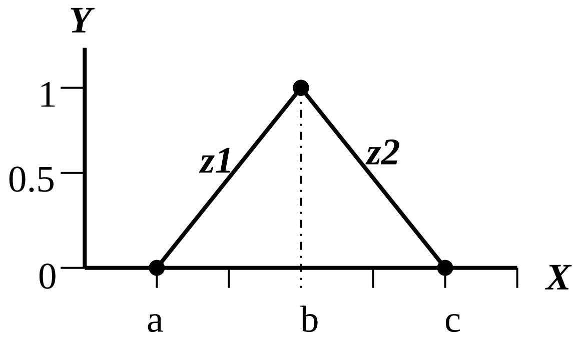
$$where\quad c_{y^l} : center \text{ of the fiered output}$$

$$c_{F_1^l} \cdots c_{F_n^l} : centers \text{ of the fiered inputs}$$

This means that we will use outputs centers in every rule in the rule base of the TSK model, so our proposed TSK model used rules depending not on just random parameters p, but also on the universe of discourse outputs centers of the plant.

Random parameters may put the actual point just far away from the desired one in the solution space; and make the convergence to the solution take a long time. Outputs center will correct the direction of the solution toward the aim to reach stable point of the plant in a short time.The proposed TSK model will cover the disadvantages of the original TSK model, because the output centers will decrease training time of parameters.

In the proposed rule base model ;we used triangle membership function because of it's simplicity to define it's shape by connected two lines , as shows in fig.2 [1,2] .



$$Triangle = \begin{cases} a \le x \langle b & \mathrm{y = z1} \\ x = b & \mathrm{y = 1} \\ b \langle x \le c & \mathrm{y = z2} \end{cases} \tag{6}$$

Fig. 2: Triangle membership function shape

Next figure fig.3 Shows a block diagram that describe the proposed rule base model.

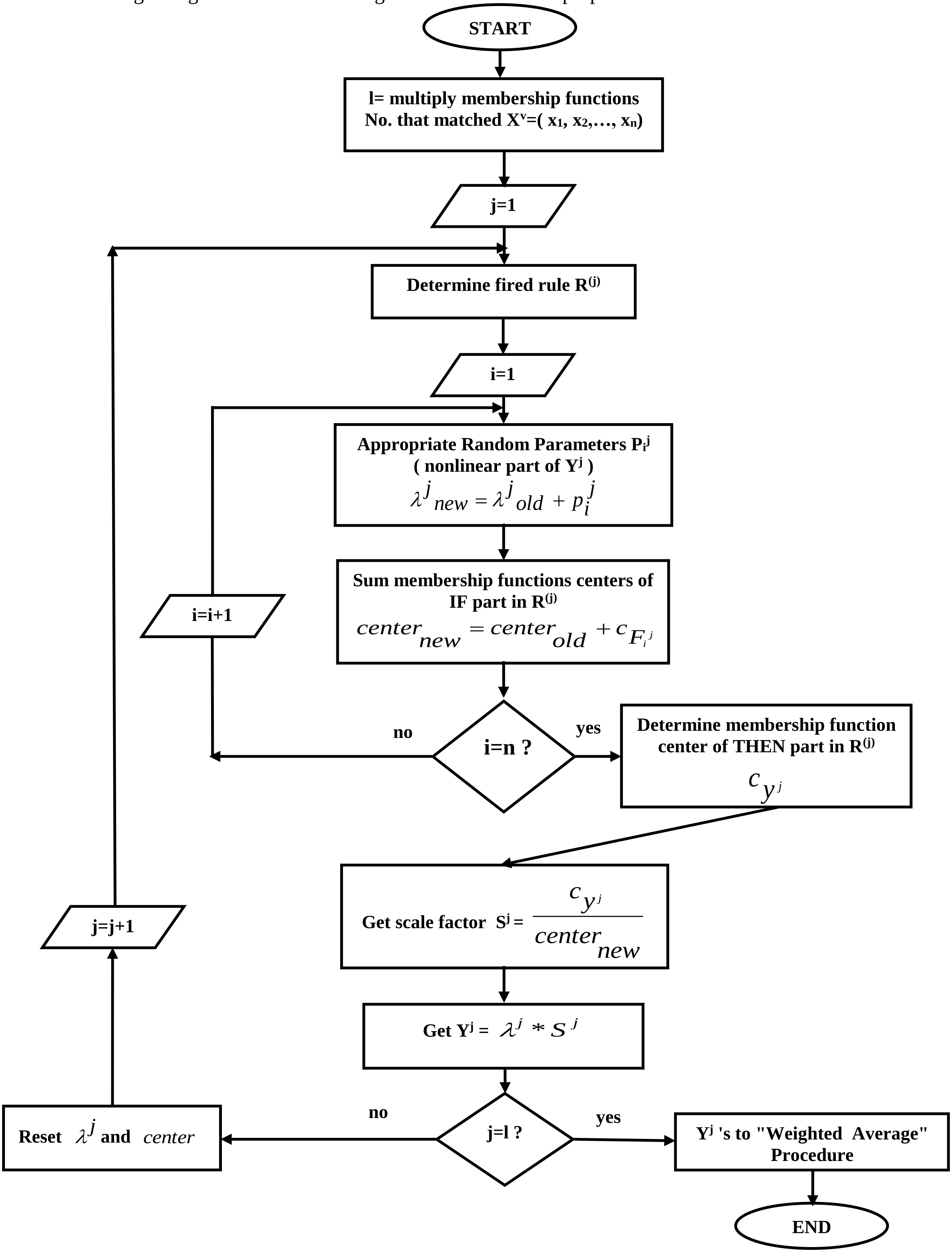


Fig. 3: Block diagram of the proposed rule base model

Experimental Results

Let us take a real problem, balancing an inverted pendulum on a cart [5]. The problem is how to control the pendulum, to prevent it's falling far away from the vertical ***v***. This means that the angle ***y*** between the position of the pendulum ***p*** and the vertical ***v*** must be close to zero as shown in fig. 4.

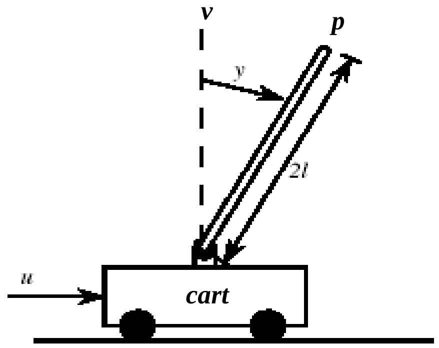


Fig. 4: An inverted pendulum on a cart

In the pendulum example, we can notice that the force ***u(t)*** which we want to use to control the condition above is affected by error ***e(t)*** between ***p*** and ***V***, and change-in-error Δ***e(t)*** according to time *t*. We will use the traditional TSK rule base model to simulate the problem, then try to improve this simulation.

The three variables, error, change-in-error, and force take the values shown in the fig. 5.

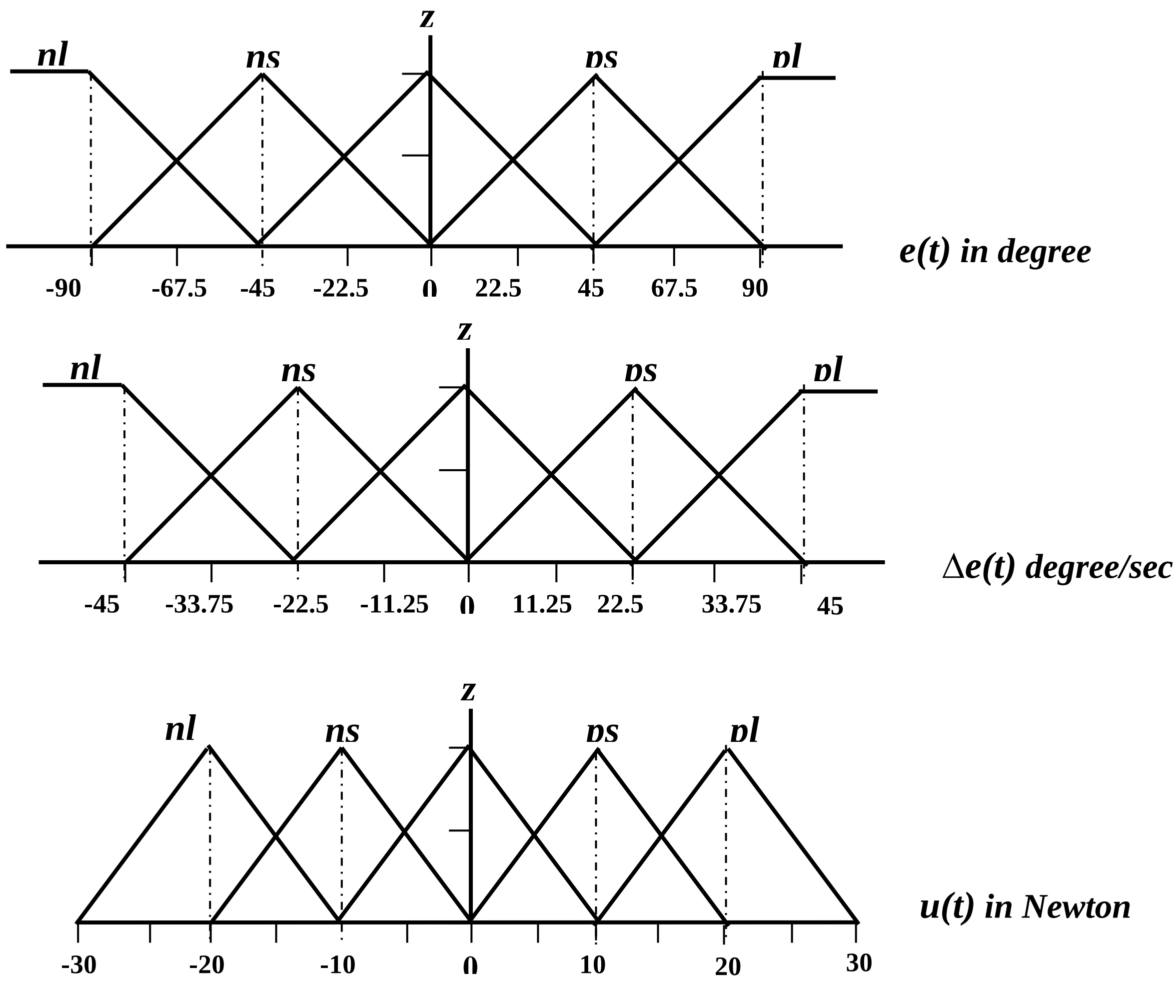


Fig. 5: Universe of discourse of the inverted pendulum variables

When the inputs to the inverted pendulum look likes:

e(t) = 22.5 degree/sec

Δe(t) = 22.5 degree/sec

The rules will be fired are:-

IF e(t) is Z AND Δe(t) is ps THEN y1= p0,0 + p0,1 e(t) + p0,2 Δe(t)

IF e(t) is ps AND Δe(t) is ps THEN y2= p1,0 + p1,1 e(t) + p1,2 Δe(t)

Or:-

IF e(t) is Z AND Δe(t) is ps THEN y1=0.42 +0.33 22.5 +0.14 22.5

IF e(t) is ps AND Δe(t) is ps THEN y2=0.12 +0.52 22.5 +0.92 22.5

While all parameters p i ,j i=0, 1 and j=0, 2 are random

Then y1= 0.42 +7.425 + 3.15 = 10.995

y2= 0.12 +11.7 + 20.7 = 32.52

$$w^l = \prod_{i=1}^{n} \mu_{f_1^l}(x_i)$$

w1= 0.5 * 1 = 0.5

w2= 0.5 * 1 = 0.5

$$y(x^v) = \frac{\sum_{l=1}^{m} w^l y^l}{\sum_{l=1}^{m} w^l}$$

$$where \qquad x^v = (x_1, x_2 \cdots x_n)$$

$$y(x^v) = \frac{\sum_{l=1}^{m} w^l y^l}{\sum_{l=1}^{m} w^l} = \frac{0.5*10.995 + 0.5*32.52}{0.5+0.5}$$

$$= \frac{5.4975 + 16.26}{1} = 21.7575$$

Now if we reduce random parameters in every rule in the same example by making

p0, 0= p1, 0 =0, we will obtain:

IF e(t) is Z AND Δe(t) is ps THEN y1= p0,0 + p0,1 e(t) + p0,2 Δe(t)

IF e(t) is ps AND Δe(t) is ps THEN y2= p1,0 + p1,1 e(t) + p1,2 Δe(t)

Or:-

IF e(t) is Z AND Δe(t) is ps THEN y1=0 +0.33 22.5 +0.14 22.5

IF e(t) is ps AND Δe(t) is ps THEN y2=0 +0.52 22.5 +0.92 22.5

Then y1= 7.425 + 3.15 = 10.575

y2=11.7 + 20.7 = 32.4

$$y(x^v) = \frac{\sum_{l=1}^{m} w^l y^l}{\sum_{l=1}^{m} w^l} = \frac{0.5*10.575 + 0.5*32.4}{0.5+0.5}$$

$$= \frac{5.2875 + 16.2}{1} = 21.4875$$

We can see that the two results above are very different because they both represent pushing the inverted pendulum to the same direction of it's position ***pl*** , while the condition of balancing need to pull the inverted pendulum to the position ***nl***.

The following table shows the relationship between force ***u(t)***, error ***e(t)***, and change-in-error Δ***e(t)***:

Tab.1: Rule base of the inverted pendulum problem

	nl	ns	z	ps	pl
nl	*pl*	*pl*	*pl*	*ps*	*z*
ns	*pl*	*pl*	*ps*	*z*	*ns*
z	*pl*	*ps*	*z*	*ns*	*nl*
ps	*ps*	*z*	*ns*	*nl*	*nl*
pl	*z*	*ns*	*nl*	*nl*	*nl*

Now when we use our proposed rule base, we obtain:

IF e(t) is Z AND Δe(t) is ps THEN y1= [e(t) + Δe(t)]* C1 u(t) / [C e(t) + CΔe(t)]

IF e(t) is ps AND Δe(t) is ps THEN y2= [e(t) + Δe(t)]* C2 u(t) / [C e(t) + CΔe(t)]

Or :-

IF e(t) is Z AND Δe(t) is ps THEN y1=[22.5 + 22.5] * [-10] / [0 + 45 + 22.5]

IF e(t) is ps AND Δe(t) is ps THEN y2=[22.5 + 22.5] * [-20] / [0 + 45 + 22.5]

While there are two centers corresponding to e (t): 0 and 45

Then y1= [45] *[– 0.1481] = -6.6645

y2= [45] *[– 0.2962] = -13.329

$$y(x^v)=\frac{\sum_{l=1}^{m} w^l y^l}{\sum_{l=1}^{m} w^l}=\frac{0.5*[-6.6645]+0.5*[-13.329]}{0.5+0.5}$$

$$=\frac{-3.33225-6.6645}{1}=-9.99675$$

We can see that the last result is very near to the condition of balancing because it is recommend to pull the inverted pendulum toward position ***nl***.

We will use our proposed TSK rule base model to simulate the dynamic inverted pendulum real problem which is given by the plant [8, 9, 11]:

$$G(s)=\frac{1}{1+0.0011\ s\ +\ 0.0264\ s^2} \qquad (7)$$

We used traditional TSK fuzzy model to simulate dynamic inverted pendulum with LMS error learning algorithm [14], which compute the difference between the plant output and the output of the TSK model and with the proposed TSK rule base model.

Fig. 6 show the behavior of the plant above when the initial input plant ***G(0)***=10°.

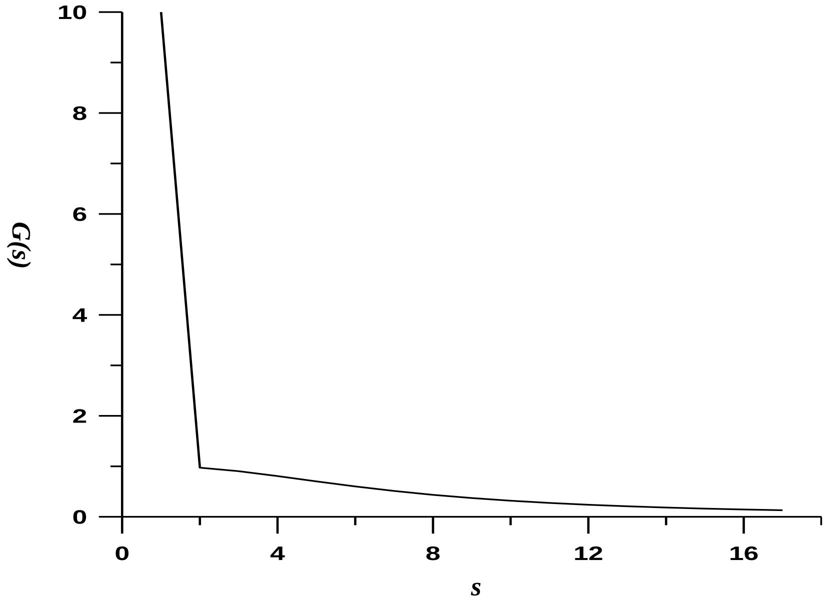


Fig. 6: Behavior of the inverted pendulum input plant at G(0)=10°

In this paper, we divide the suggested output interval into 5 singleton ***(outputs centers)*** {-10,-5, 0, 5, 10} respectively. Fig. 7, will show the desired output of the proposed TSK model.

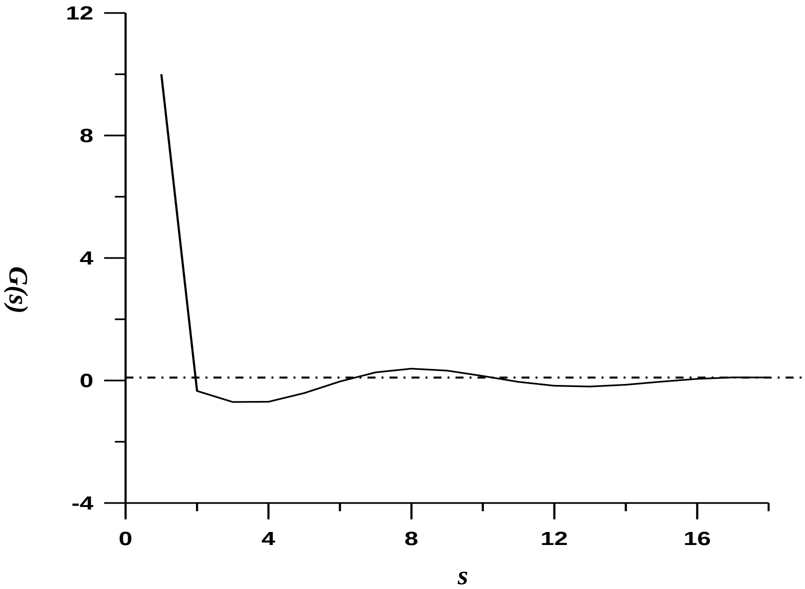


Fig. 7: Desired output of the inverted pendulum using our proposed TSK rule base model

Comparing fig. 7 with fig. 8, the dashed line will show the difference between the desired output and the stable point 0 . Fig. 8 also show a big difference between desired and stable point at s=16 (local end).

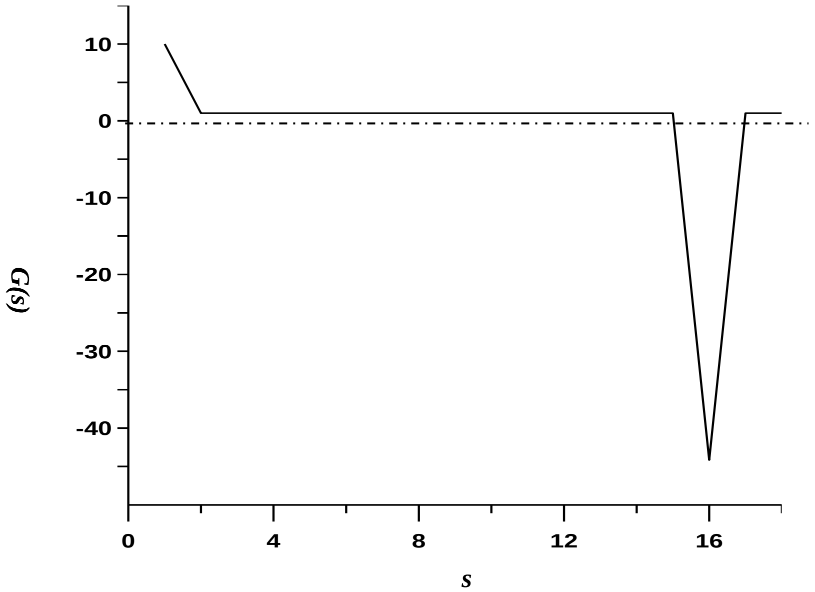


Fig. 8: Desired output of the inverted pendulum using traditional TSK rule base model

Tab. 2: Comparison between traditional/proposed TSK rule base models of the inverted pendulum example results

	Updated Parameters	*Epochs*	*Error*
TSK Rule base Model	*5 X 2 X 2 = 20*	*3*	*10.101*
Proposed Rule base Model	*5 X 2 X 2 = 20*	*3*	*06.595*

Conclusions

We can conclude several points from this paper:-

1. Many input-output fuzzy system variables with many random parameter make solution space very huge. Therefore, reducing these parameters or aiming them toward the solution may produce good results.
2. Merging fuzzy sets properties and nonlinear function properties in THEN part of the rule in TSK model will increase accuracy and decrease training time.
3. Suggesting a good universe of discourse output centers interval may improve the accuracy of overall fuzzy system.
4. We can lead the desired output plant to the aim without falling in the local end problem.
5. Our proposed rule base model make THEN part of every rule in rule base contain a relationship between input space and output space; instead of containing just nonlinear function of inputs, not referring to any relationship in it.

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الخلاصة

رُكّزَ الباحثون أهتمامهم حول نموذج TSK وذلك بسبب نَتائِجه الجيدةِ في التعامل مع المشاكلِ الحقيقيةِ وأيجاد حل لها . من أهم مساويء هذا النموذج أعتماده على البارامترات العشوائيِة–أبتداءاً–، وقد يسبب ذلك تشتيت تقارب النموذج نحو الحل .

ناقشنا في هذا البحث مساويء نموذج قاعدة القوانين TSK ، وحاولنا تَغْطية هذه المساوئ عن طريق أقتراح نموذج TSK جديد لقاعدة قوانين بدْمج خصائص المجموعاتِ المضببة مع الدالة اللاخطية في القانون؛ وذلك يعني: أن أخراج كل قانون في القاعدة لن يَعتمد على البارامتراتِ العشوائية في الدالة اللاخطية فقط ، لكن سيَعتمد أيضاً على مراكزِ الأخراج في كُلّ قانون .

إستعمل النموذجَ المُقتَرَحَ لمحاكاة مشكلةِ البندولِ الحقيقيةِ، حيث أنْتجت التجربـةَ نتائج جيدة وبدقةَ متزايدةَ ووقتَ تدريب متناقص مقارنة مع نموج TSK التقليدي.