



Generalized N^* ideal closed sets in Nano N^* ideal topological Spaces With Some Properties

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DOI: <http://dx.doi.org/10.25130/tjps.27.2022.045>

ARTICLE INFO.

Article history:

-Received: 3 / 3 / 2022

-Accepted: 27/ 4 / 2022

-Available online: / / 2022

Keywords: $N^*R\alpha I-o(x)$, $N^*R\beta I-o(x)$, $N^*g\alpha I-c(x)$, $N^*g\beta I-c(x)$, $N^*Rg\alpha I-c(x)$, $N^*Rg\beta I-c(x)$, $N^*gI-c(X)$, $N^*g^*I-c(X)$, $N^*g^*\beta I-c(X)$, $N^*g^*\alpha I-c(X)$.

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ABSTRACT

In this paper, we will study a new class of sets and said to be generalized N^* ideal -closed sets in nano N^* ideal topological spaces and its properties. Furthermore the relationships were introduced and notation.

1. Introduction

In 1913 [1] studied the notation of Nano topology. [2] study Nano generalized alpha closed sets in Nano topological Spaces, [3] introduced Nano $g^*\alpha$ Closed sets in nano topology. [4] introduced new class of open sets in nano topological spaces. [5] studied a new types of nano topology via nano ideals. [6] studied alpha generalized closed sets in ideal topology. (2018-2019) [7-8] studied simple forms of nano open sets in ideal nano topological spaces, unified approach of several sets in ideal nano topological spaces. [9] studied the nano ideal generalized closed set in nano ideal topological space. [10] presented continuous maps and hausdorff spaces in nano ideal spaces. In (2019) [11] studied the generalized classes of ideal nano topological space, [12] presented new sets in ideal topological space. [13] studied generalized $I\beta$ -closed sets in ideal topological spaces. [14] studied On Some New Notions in Nano Ideal Topological Spaces. In (2020) [15] presented nano $I\alpha g$ -closed sets and normality via $nI\alpha g$ -closed sets in nano ideal topology In (2019) presented New generalized classes of ideal nano topological space [17] studied ideal with micro topological space [18] introduced nano ideal α

regular- closed sets in nano ideal topological spaces. [19] presented topological approach for rough sets by using j-nearly concepts via ideals. In this paper, we introduce and investigate a new class of closed sets called generalized n^* ideal -closed sets in nano n^* ideal topological spaces and also discuss the relationship with some new existing closed sets in nano n^* ideal topological Spaces.

2 Preliminaries

Definition 2.1 [1] : Let w be a non-empty set and R be Relation Equivalence on w , then w is called the indistinguishable relationship. and (W, R) area of approximation, $X \subseteq W$.

A) $L_{R(X)} = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$.

B) $U_{R(X)}$. That is $U_{R(X)} = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

C) $B_{R(X)} = \{U_{R(X)} - L_{R(X)}\}$.

Definition 2.2 [1]: W is universe, R be an equivalence relation on W , $X \subseteq W$. $\tau_{R(X)} = \{W, \emptyset, L_{R(X)}, U_{R(X)}, B_{R(X)}\}$, $(W, \tau_{R(X)})$ is said to be Nano top. space. The sets of $\tau_{R(X)}$ are named as nano open sets.

Definition 2.3 [1] : $(\varphi, \tau_{R(X)})$ nano topological Space, $X \subseteq \varphi$, $P \subseteq \varphi$:

- $n \text{ int}(P) = \bigcup \{G : G \subseteq P, G \text{ is nano open sets}\}$
- $n \text{ cl}(P) = \bigcap \{G : P \subseteq G, G \text{ is nano closed sets}\}$.

Definition 2.4[20]: $(\omega, \tau_{R(X)}^*)$ is nano N^* topological Space, $A \subseteq \omega$ then A is said to be $N^*\beta$ regular -open set, if $\exists N$ regular open set $D \ni D \subseteq A \subseteq N^*\beta cl(D)$, symbolized by $N^*\beta o(x)$.

Definition 2.5[20]: The nano N^* topological Space $(\delta, \tau_{R(X)}^*)$, $A \subseteq \delta$, A named as $N^*\alpha$ - regular open set, if $\exists N^*$ Regular open set F , $F \subseteq A \subseteq N^*\alpha cl(F)$.

Definition 2.6[21]: $(W, \tau_{R(X)})$ is nano topological space, $\beta \subseteq W$ is called nano N^* open sets if β satisfy

1- For each $Q \in \beta$, $\exists N\alpha$ -Open Set $G \ni Q \in G \subseteq Ncl(\beta)$.

2- If $\beta \cap [U_{R(X)}]^c \neq \emptyset$ then $U_{R(X)} \subseteq \beta$, symbolized by $\tau_{R(X)}^*$.

Definition 2.7[21]: $(\gamma, \tau_{R(X)}^*)$ N^* Space, $Q \subseteq \gamma$ called

a) N^* -alpha-Open set if $Q \subseteq N^*int(N^*cl(N^*int(Q)))$, symbolized by $N^*\alpha$ -open set.

b) N^* -beta-Open set if $Q \subseteq N^*cl(N^*int(N^*cl(Q)))$, symbolized by $N^*\beta$ -open set.

Definition 2.8[22]: let I be an Ideal on a topological space (X, τ) which satisfies the following conditions.

(1) $H \in I$, $F \subset H \rightarrow F \in I$ (2) $H \in I$ and $F \in I \rightarrow H \cup F \in I$.

Definition 2.9[5]: (W, N) is nano topological space with ideal I on W called Ideal nano top. space symbolized by (W, N, I) . $M_{n(x)} = \{M_n | x \in M_n, M_n \in N\}$, is the collection of nano open set $(x) \supseteq x$.

Definition 2.10 [19]: $S \subseteq (U, N, I)$ is called N^* -closed if $S_n^* \subseteq S$. supplement of N^* -c(x) is called N^* -O(x)

Definition 2.11[5]: Let (μ, N) is nano top. S. And ideal I on μ . The Ncl^* called nano*-closure, $ncl^*(H) = H \cup H_n^*$, $H \subseteq X$.

Theorem 2.12 [5-9]: The Nano top. s. (μ, N) with an Ideal (j, k) on μ , $H, J \subseteq \mu$. Then

1. $H \subseteq J \Rightarrow H_n^* \subseteq J_n^*$.
2. $j \subseteq k \Rightarrow H_n^*(k) \subseteq H_n^*(j)$.
3. $H_n^* = N-cl(H_n^*) \subseteq N-cl(H)$ (H_n^* be nano closed $\subseteq ncl(H)$).
4. $(H_n^*)_n^* \subseteq H_n^*$

Lemma 2.13[5-9]: (μ, N, I) be nano topological space with ideal I , $M \subseteq M_n^*$, then $M_n^* = ncl(M_n^*) = ncl(M)$

Theorem 2.14 [5-9] ncl^* satisfies the conditions:

1. $M \subseteq ncl^*(M)$.
2. $ncl^*(\emptyset) = \emptyset$, $ncl^*(\mu) = \mu$.
3. If $H \subseteq F$, thus $ncl^*(H) \subseteq ncl^*(F)$.
4. $ncl^*(H) \cup ncl^*(F) = ncl^*(H \cup F)$
5. $ncl^*(ncl^*(H)) = ncl^*(H)$

Definition 2.15[7,11]: $H \subseteq (\gamma, N, I)$ is called

1. Nano α -I-Open set (briefly α -NI-open) if $H \subset nint(ncl^*(H))$
2. Nano β -I-Open set (briefly β -NI-open) if $H \subset ncl^*(nint(ncl^*(H)))$.
3. NI Regular Open set if $H = nint(ncl^*(H))$ and NI regular closed if $ncl^*(nint(H)) = H$.

Definition 2.16[18]: $(\gamma, \tau_{R(X)}^*)$ nano N^* space, $H \subseteq \gamma$, H is said to be N^* generalized α -closed set in nano N^* topological space $\leftrightarrow L \subseteq N^*\alpha Cl(H)$, $H \subseteq L$, L is $N^*\alpha$ -open set.

3 Generalized N^* ideal closed set in Nano N^* ideal topological Spaces With Some Properties.

Definition 3.1: $(\mu, \tau_{R(X),j}^*)$ be nano N^* ideal topological space.

Let $()_{n^{**}}$ a set operator from $F(\mu)$ to $F(\mu)$, $[F(\mu) \subseteq \mu]$.

For $A \subseteq \mu$, $A_{n^{**}}(j, N^*) = \{x \in \mu : G_{n^*} \cap A \notin j, \forall G_{n^*} \in G_{n^*}(x)\}$ said Nano n^* locally Function of A , j and n^* (briefly $A_{n^{**}}$).

Theorem 3.2. $(\mu, \tau_{R(X),j}^*)$ is nano N^* ideal topological space and ideals j, k on μ and $A, B \subseteq \mu$. Then

- (i) $A \subseteq B \Rightarrow A_{n^{**}} \subseteq B_{n^{**}}$
- (ii) $j \subseteq k \Rightarrow A_{n^{**}}(k) \subseteq A_{n^{**}}(j)$.
- (iii) $A_{n^{**}} = N^*-Cl(A_{n^{**}}) \subseteq N^*-Cl(A)$ [$A_{n^{**}}$ is Nano N^* closed subset of $N^*-Cl(A)$].
- (iv) $(A_{n^{**}})_{n^{**}} \subseteq A_{n^{**}}$.

Proof. (i) Let $A \subset B$ and $x \in A_{n^{**}}$. let $x \notin B_{n^{**}}$. $\rightarrow G_{n^*} \cap B \in j, G_{n^*} \in G_{n^*}(x)$. Since $G_{n^*} \cap A \subseteq G_{n^*} \cap B$ and $G_{n^*} \cap B \in j$, we obtain $G_{n^*} \cap A \in j$ from def. of ideal. Hence, $x \notin A_{n^{**}}$. This is a contradiction. Its Clear that, $A_{n^{**}} \subseteq B_{n^{**}}$.

(ii) $j \subseteq k$ and $x \in A_{n^{**}}(k)$, $G_{n^*} \cap A \notin k \forall G_{n^*} \in G_{n^*}(x)$. since $G_{n^*} \cap A \notin j$. then $x \in A_{n^{**}}(j)$.

(iii) $x \in A_{n^{**}}$. $\forall G_{n^*} \in G_{n^*}(x)$, $G_{n^*} \cap A \notin I \rightarrow G_{n^*} \cap A \neq \emptyset$. Hence $x \in N^*-cl(A)$.

(iv) From (iii), $(A_{n^{**}})_{n^{**}} \subseteq N^*-Cl(A_{n^{**}}) = A_{n^{**}}$, since $A_{n^{**}}$ Is Nano N^* closed set. Hence $(A_{n^{**}})_{n^{**}} \subseteq A_{n^{**}}$.

The opposite of [i, ii and iii] in Theorem 3.3. not satisfy.

Example 3.3: (i) Let $\mu = \{a, b, c, d\}$, $X = \{a, b\} \subset \mu$, $\mu/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $N = \{\mu, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$, $N^*O-(x) = \{\mu, \emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ and the ideal $I = \{\emptyset, \{a\}\}$. $A = \{a, c\}$, $B = \{a, d\}$, $A_{n^{**}} = \{c\} \subset B_{n^{**}} = \{b, c, d\}$ but $A \not\subset B$.

(ii) Let $\mu = \{a, b, c, d\}$ be the universe, $X = \{a, b\} \subset \mu$, $\mu/R = \{\{a\}, \{c\}, \{b, d\}\}$

$N = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$, $N^*O-(x) = \{U, \emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ and the ideals be $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$, $I \not\subset J$, $A = \{a, c, d\}$. $A_{n^{**}}(I) = \{c, d\} \subset A_{n^{**}}(J) = \{a, c, d\}$.

(iii) Let $\mu = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset \mu$, $\mu/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $N = \{\mu, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$, $N^*O-(x) = \{\mu, \emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ with ideal $I = \{\emptyset, \{a\}\}$. $A = \{b, c, d\}$, we have $N^*-cl(A) = \{b, c, d\}$, $A_{n^{**}} = \{c, d\}$, $N^*-cl A_{n^{**}} = \{c, d\}$, $N^*-cl(A) \not\subset A_{n^{**}} = N^*-cl(A_{n^{**}})$.

Theorem 3.4: If $(\mu, \tau_{R(X),j}^*)$ nano N^* ideal topological space, and $B \subseteq B_{n^{**}}$, then $B_{n^{**}} = N^*-cl(B_{n^{**}}) = N^*-cl(B)$.

Proof: $B \subseteq \mu$, $B_{n^{**}} = N^*-cl(B_{n^{**}}) \subseteq N^*-cl(B)$, by using Theorem 3.6 [iii] $B \subseteq B_{n^{**}} \rightarrow N^*-cl(B) \subseteq N^*-cl(B_{n^{**}})$ and So $B_{n^{**}} = N^*-cl(B_{n^{**}}) = N^*-cl(B)$.

Theorem 3.5: $(\mu, \tau_{R(X),j}^*)$ be nano N^* ideal topological space and ideal j on μ , $A \subseteq X$. If $A \subseteq$

A_n^{**} , thus $A_n^{**} = N^*-cl(A_n^{**}) = N^*-cl(A) = N^*-cl^{**}(A)$.

Definition 3.6: $(\mu, \tau_{R(X)}^*, I)$ is nano N^* ideal topological space. The set operator N^*cl^{**} is said to be n^* -closure ** [$n^*cl^{**}(A) = A \cup A_n^{**}$], $A \subseteq X$.

Definition 3.7: $(\mu, \tau_{R(X)}^*, I)$ be nano N^* ideal topological Space. The set operator N^*int^{**} is said to be n^* -interior ** [$n^*int^{**}(A) = A \cap A_n^{**}$] for $A \subseteq X$.

Theorem 3. 8: $(\mu, \tau_{R(X)}^*, j)$ is nano N^* ideal topological space and ideal j on μ , $A \subseteq \mu$. If $A \subseteq A_n^{**}$, then

(i) $n^*cl(A) = n^*cl^{**}(A)$

(ii) $n^*int(\mu - A) = n^*int^{**}(\mu - A)$.

Proof. (i) by using Th. 2.5.

Proof. (ii) let $A \subseteq A_n^{**}$, then $N^*cl(A) = N^*cl^{**}(A)$ by (i) and so $X - N^*cl(A) = X - N^*cl^{**}(A)$. Therefore, $N^*int(X - A) = N^*int^{**}(X - A)$.

Definition 3.9: Let $(\mu, \tau_{R(X)}^*, j)$ be nano N^* ideal topological space. The subset A of μ is said to be nano N^* I- open set (briefly $N^*I-o(x)$) if $A \subseteq N^*int(A_n^{**})$.

Example 3.10: $\mu = \{1, 2, 3, 4, 5\}$, $\mu/R = \{\{1\}, \{2, 3\}, \{4, 5\}\}$, $X = \{1, 2\}$

$I = \{\emptyset, \{2\}\}$, $\tau_{R(X)} = \{\emptyset, \mu, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$, $\tau_{R(X)}^* = \{\emptyset, U,$

$\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}\}$.

$N^*IO(x) = \{\emptyset, \mu, \{1\}, \{3\}, \{1, 3\}\}$

Remark 3.11: It is clear that N^*I -open and N^* -open are independent

Example 3.12: $\mu = \{a, b, c, d\}$ be the universe, $X = \{a, b\} \subset \mu$

$\mu/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $N = \{\emptyset, \mu, \{a\}, \{b, d\}, \{a, b, d\}\}$

$N^*O(x) = \{\emptyset, \mu, \{a\}, \{b\}, \{d\}, \{b, d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$

and the ideal $J = \{\emptyset, \{a\}\}$. (i) For $A = \{a, b, d\}$, we have $A_n^{**} = \{b, c, d\}$, $N^*int(A_n^{**}) = \{b, d\} \Rightarrow A \not\subseteq N^*int(A_n^{**})$, A is not an element an $N^*IO(x)$.

(ii) For $A = \{b\}$, $A_n^{**} = \{b, c, d\}$, $N^*int(A_n^{**}) = \{b, d\} \Rightarrow A \subseteq N^*int(A_n^{**})$. $A \in N^*I$ -open set, $A \notin N^*o(x)$.

Definition 3.13: $(\mu, \tau_{R(X)}^*, j)$ be nano N^* ideal topological space, $A \subseteq \mu$, A is said to be

a) $n^*RI-o(x)$ [$A = n^*int(ncl^{**}(A))$].

b) $n^*\alpha I-o(x)$ [$A \subseteq n^*int(n^*cl^{**}(n^*int(A)))$].

c) $n^*\beta I-o(x)$ [$A \subseteq n^*cl^{**}[n^*int(n^*cl^{**}(A))]$]. denoted by $N^*RIO(\mu, x)$ (Respectively, $N^*\alpha I(\mu, x)$, $N^*\beta I(\mu, x)$).

Example 3.14: Let $\mu = \{1, 2, 3, 4, 5\}$, $\mu/R = \{\{1\}, \{2, 3\}, \{4, 5\}\}$, $X = \{1, 2\}$, $I = \{\emptyset, \{2\}\}$

$\tau_{R(X)} = \{\emptyset, \mu, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$, $\tau_{R(X)}^* = \{\emptyset, \mu, \{1\}, \{2\}, \{3\}$

$\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}\}$.

$N^*IO(x) = \{\emptyset, \mu, \{1\}, \{3\}, \{1, 3\}\}$

$N^*RI-o(x) = \{\emptyset, \mu, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$.

$N^* \alpha I -o(x) = \{\emptyset, \mu, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}\}$.

$N^*\beta I o(x) = \{\emptyset, \mu, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 4\}, \{1, 5\}, \{3, 4\}, \{3, 5\}, \{1, 2, 4\}, \{1, 2, 5\}, \{2, 3, 4\}, \{2, 3, 5\}, \{3, 4, 5\}, \{1, 3, 4\}, \{1$

$, 3, 5\}, \{1, 2, 4, 5\}, \{2, 3, 4\}, \{1, 3, 4, 5\}, \{1, 4, 5\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}\}$.

Definition 3.15 : $(\sigma, T_{R(X)}^*, j)$ be nano N^* ideal topological space, $A \subseteq \sigma$, A is said to be N^* regular βI -open set. If there is a N^* regular $-I$ open set D . $\exists D \subseteq A \subseteq N^*\beta cl(D)$.

Recall example 3.13: $N^*R\beta I -o(x) = \{\emptyset, \mu, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$.

Definition 3.16: $(\mu, \tau_{R(X)}^*, j)$ is nano N^* ideal topological space, $A \subseteq \mu$, A called N^* regular $\alpha I -o(x)$ if there is N^* regular I open set D , $\exists D \subseteq A \subseteq N^*\alpha cl(D)$.

Recall example 3.13:

$N^*\alpha I o(x) = \{\emptyset, \mu, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 4\}, \{1, 5\}, \{1, 4, 5\}, \{2, 4, 5\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{3, 4, 5\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 3, 4, 5\}, \{2, 3, 4\}, \{2, 3, 5\}, \{2, 3, 4, 5\}\}$.

Proposition 3.17: $N^*I -o(x)$ is $N^*\alpha I -o(x)$, the opposite is not satisfies

Proof: Assume B is n^*I -open set $\Rightarrow B \subseteq n^*int(A_n^{**}) \subseteq n^*int(n^*cl^{**}(B))$.

$\subseteq n^*int((n^*cl^{**}(n^*int(B)))$. B is $n^*\alpha I$ -open set.

Recall example 3.13: $A = \{1, 2, 3, 4\}$ is $N^*\alpha I -o(x)$, not $N^*I -o(x)$.

Proposition 3.18: $N^*\alpha I$ -open set is $N^*\beta I$ -open set, the opposite is not satisfy.

Proof: Since $n^*int(A) \subseteq A$, $n^*cl^{**}(n^*int(A)) \subseteq n^*cl^{**}(A)$.

$n^*int(N^*cl^{**}(n^*int(A))) \subseteq n^*$

$int(n^*cl^{**}(A)) \subseteq n^*cl^{**}(n^*int(n^*cl^{**}(A)))$. A is $N^*\beta I -o(x)$.

Recall Example 3.13: $A = \{1, 2, 4\}$ is $N^*\beta I -o(x)$, not $N^*\alpha I -o(x)$.

Proposition 3.19: N^* regular βI -open set is N^* regular αI -open set, the opposite doesn't satisfies

Proof: Assume A is $n^*\beta \alpha I -o(x) \Rightarrow \exists D \subseteq A \subseteq n^*\beta cl(D)$ And D is n^*RI -open set Since $n^*\beta cl(D) \subseteq n^*\alpha cl(D) \Rightarrow D \subseteq A \subseteq n^*\alpha cl(D)$. A is $N^*\alpha I -o(x)$.

Recall example 3.13: $\mu = \{1, 2, 3, 4, 5\}$, $\mu/R = \{\{1\}, \{2, 3\}, \{4, 5\}\}$, $X = \{1, 2\}$, $I = \{\emptyset, \{2\}\}$

$A = \{3, 4, 5\}$ is $N^*\alpha I -o(x)$, not $n^*\beta I -o(x)$

Proposition 3.20: $N^*R\alpha I$ -open set is $N^*\alpha I$ -open set, the opposite doesn't satisfies

Proof: Assume A is $N^*R\alpha I -o(x) \Rightarrow \exists D \subseteq A \subseteq N^*\alpha cl(D)$

And D is N^*RI -open set $\Rightarrow D = N^*int(N^*cl^{**}(D)) \subseteq N^*int(D) \subseteq (N^*cl^{**}(N^*int(D))) \subseteq N^*int(N^*cl^{**}(N^*int(D))) \Rightarrow D$ is $N^*\alpha I -o(x)$.

Recall example 3.13: $A = \{1, 2, 3, 5\}$ $N^*\alpha I -o(x)$, not $N^*R\alpha I -o(x)$.

Proposition 3.21: $N^*R\beta I$ -open set is $N^*\beta I$ -open set, the opposite doesn't satisfied

Proof: Assume A is $n^*R\beta I - o(x) \Rightarrow \exists T \subseteq A \subseteq n^*\beta cl(T)$

And T is $n^*RI - o(x)$ take $A=T \Rightarrow A = n^*int(ncl^{**}(A)) \subseteq ncl^{**}(n^*int(ncl^{**}((A))) \Rightarrow A$ is $n^*\beta I - o(x)$. Recall Example 3.13: $A = \{1,2,4\}$ is $N^*\beta I - o(x)$, not $N^*R\beta I - o(x)$.

Definition 3.22 : $(\mu, \tau_{R(x), I}^*)$ is nano N^* ideal topological space, $E \subseteq \mu$, E is called n^* generalized αI -closed set in nano N^* ideal topological space $\Leftrightarrow Y \subseteq n^*\alpha cl(E)$, $E \subseteq Y$, Y is a $n^*\alpha I - o(x)$.

Recall Example 3.13: $E = \{1,2,3\}$, $G = \{1,2,3,4\} \Rightarrow G \subseteq n^*\alpha cl(E) = \mu \Rightarrow E$ is $N^*g\alpha I - c(x)$.

Definition 3.23: $(\mu, \tau_{R(x), I}^*)$ be nano N^* ideal topological space, $F \subseteq \mu$, F is said to be N^* regular generalized αI -closed set in nano N^* ideal topological space $\Leftrightarrow D \subseteq N^*\alpha cl(F)$, whenever $F \subseteq D$, D is a $n^*R\alpha I$ -open set.

Recall example 3.13: $U = \{2,3\}$, $D = \{2,3,4,5\}$, $D \subseteq N^*\alpha cl(U) \Rightarrow U$ is $n^*rg\alpha I - c(x)$.

Definition 3.24: $(\mu, \tau_{R(x), I}^*)$ be nano N^* ideal topological space, $Q \subseteq \mu$, Q is said n^* generalized $\beta I - c(x)$ in nano N^* ideal topological space $\Leftrightarrow D \subseteq N^*\beta cl(Q)$, $Q \subseteq D$, D is $N^*\beta I - o(x)$. Recall example 3.13: $Q = \{1,2,3\}$, $D = \{1, 2,3,4\} \Rightarrow N^*\beta cl(Q) = \mu \Rightarrow D \subseteq \mu \Rightarrow Q$ is $N^*g\beta I - c(x)$.

Definition 3.25: $(\mu, \tau_{R(x), I}^*)$ is nano N^* ideal topological space, $Q \subseteq \mu$, Q called n^* regular generalized $\beta I - c(x)$ in nano N^* ideal topological space $\Leftrightarrow Z \subseteq n^*\beta cl(Q)$, $Q \subseteq Z$, Z is $N^*R\beta I - o(x)$.

Recall example 3.13: $A = \{2,3\}$, $D = \{2,3\}$, $D \subseteq N^*\beta cl(A) \Rightarrow A$ is $N^*Rg\beta I - c(x)$.

Proposition 3.26: $n^*g\beta I - c(x)$ and $n^*g\alpha I - c(x)$ are independent.

Example 3.27: $\varphi = \{i, j, k, l\}$, $\varphi/R = \{\{i\}, \{j, k, l\}\}$, $X = \{j, k\}$, $I = \{\emptyset, \{j, l\}\}$

$\tau_{R(x)} = \{\emptyset, \varphi, \{j, k, l\}\}$, $N^*o(x) = \{\emptyset, \varphi, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{k, l\}, \{j, k, l\}\} = N^*\alpha o(x)$
 $N^*g\beta I o(x) = \{\emptyset, \varphi, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{k, l\}, \{j, k, l\}\}$

$N^*g\alpha I - o(x) = \{\emptyset, \varphi, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{i, j, k\}, \{k, j, l\}\}$.

$A = \{i, j, k\}$ is $N^*g\alpha I - c(x)$. but A is not $N^*g\beta I - c(x)$.

$A = \{k, i\}$ is $n^*g\beta I - c(x)$, not $n^*g\alpha I - c(x)$.

Proposition 3.28: $n^*rg\alpha I - c(x)$ With $n^*g\alpha I - c(x)$ are independent.

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Recall example 3.32: $N^*Rg\alpha I - o(x) = \{\emptyset, \mu, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{i, j, k\}, \{k, j, l\}, \{k, l\}, \{i, k, l\}\}$

$N^*g\alpha I - o(x) = \{\emptyset, \mu, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{i, k\}, \{k, j, l\}\}$.

$A = \{k, l\}$ is $N^*rg\alpha I - c(x)$ but A is not $N^*g\alpha I - c(x)$.

Proposition 3.29: $N^*Rg\beta I - c(x)$ is $N^*Rg\alpha I - c(x)$, the converse doesn't satisfy.

Proof: Assume K is $n^*rg\beta I - c(x) \Rightarrow D \subseteq N^*\beta cl(K)$, $k \subseteq D$, D is a $n^*R\beta I - o(x)$

$n^*\beta cl(K) \subseteq n^*\alpha cl(K)$, and every $N^*R\beta I - o(x)$ is $N^*R\alpha I - o(x) \Rightarrow D \subseteq N^*\alpha cl(K)$, D is $N^*R\alpha I - o(x) \Rightarrow K$ is $N^*Rg\alpha I - c(x)$ closed set ■

Recall example 3.32: $N^*Rg\alpha I - o(x) = \{\emptyset, \mu, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{i, j, k\}, \{k, j, l\}, \{k, l\}, \{i, k, l\}\}$

$N^*Rg\beta I - o(x) = \{\emptyset, \mu, \{j\}, \{k\}, \{l\}, \{j, k\}, \{j, l\}, \{k, l\}\}$.

$M = \{k, j\}$, $D = \{k, j, l\} \Rightarrow M$ is $N^*Rg\alpha I - c(x)$, not $N^*Rg\beta I - c(x)$. Since $\{k, j\}$, $N^*\beta - o(x)$, but $D \not\subseteq N^*\beta cl(M) = \{k, j\}$. M is not $N^*Rg\beta I - c(x)$.

Proposition 3.30: $n^*g\beta I - c(x)$ with $n^*rg\alpha I - c(x)$ are independent..

Recall example 3.32: $F = \{i, k\}$, $D = \{k, i\} \Rightarrow F$ is $N^*g\beta I - c(x)$, not $N^*R\alpha I - c(x)$, $D \subseteq n^*\alpha cl(F)$ but $\{k, i\}$ is not $N^*R\alpha I - o(x)$, not $N^*Rg\alpha I - c(x)$.

Recall example 3.14:

$N^*Rg\alpha I c(x) = \{\emptyset, U, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,4\}, \{1,5\}, \{1,4,5\}, \{2,4,5\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}, \{1,2,4\}, \{1,2,5\}, \{1,2,4,5\}, \{1,3,4\}, \{1,3,5\}, \{1,3,4,5\}, \{2,3,4\}, \{2,3,5\}, \{2,3,4,5\}\}$.

$N^*g\alpha I -$

$c(x) = \{\emptyset, U, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,5\}\}$.

$A = \{1,2,3,4\}$ is $N^*g\alpha I - c(x)$ but A is not $N^*Rg\alpha I - c(x)$.

Proposition 3.31: $n^*rg\beta I - c(x)$ is $n^*g\beta I - c(x)$. converse doesn't satisfy

Proof: Assume F is $n^*rg\beta I - c(x) \Rightarrow D \subseteq n^*\beta cl(F)$, $F \subseteq D$, D is $n^*R\beta I - o(x)$. $\therefore n^*R\beta I - o(x)$ is $n^*\beta I - o(x)$, $D \subseteq n^*\beta cl(F) \Rightarrow F$ is $n^*g\beta I - c(x)$. ■

Recall example 3.32: $M = \{i, j\}$, $D = \{i, j\} \Rightarrow M$ is $N^*g\beta I - c(x)$, not $N^*Rg\beta I - c(x)$, $D \subseteq N^*\beta cl(M)$, $\{i, j\}$ not $N^*R\beta I - o(x)$, not $N^*Rg\beta I c(x)$.

Conclusions

from our study in the light of the theoretical part and examples illustrated, we can conclude the generalized N^* ideal -closed set can be study in(micro, soft) topological spaces

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المجموعات المغلقة المعممة N^* المثالية في الفضاءات التبولوجية النانوية N^* المثالية

وخصائصها

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الملخص

سوف ندرس فئة جديدة من المجموعات اسميناها المجموعات المغلقة المعممة N^* المثالية في الفضاءات التبولوجية النانوية N^* المثالية وخصائصها. علاوة على ذلك ، تم تقديم العلاقات بينها.