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On Regular Semi Supra Open set and Regular Semi Supra Continuity

Marwa Fathi Ahmed, Taha H. Jasim

Department of Mathematics, Computer Science and Mathematics, Tikrit University, Tikrit, Iraq

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Corresponding Author:

Name: Marwa Fathi Ahmed

E-mail:

marwaahmed557@gmail.com

tahahameed@tu.edu.iq

Tel:

ABSTRACT

The aim of this paper is to present the concepts of bi^i -supra continuous function, bi^i -semi supra continuous function, bi^i -regular open set and bi^i -regular semi supra open set in bi^i -supra topological space. We studied relations between these functions and check up from some of its characteristics.

1. Introduction

In 1963 Kelly [1] presented the concept of bi-topological space which was called a set "equipped with two topologies a bi-topological space and is denoted by $(X, S\mathfrak{J}, P\mathfrak{J})$ where $\mathfrak{J}_1, \mathfrak{J}_2$ are two topologies defined on X . In 1983 Mashhoure, M. S. Allam, A. A. Mohamoud F.S. and Khedr, F.H.[3] presented the supra topological space. In 2012 Jamunarani, R. and Jeyanthi, P., [8] present the Regular sets in generalized topological spaces. New concepts after we introduced the new definitions called bi^i -supra topological spaces on the topological space $(X, \mathfrak{J})$ and comparable between bi^i -supra topological space which is induced from bi-topological spaces $(X, \mathfrak{J}_i, \mathfrak{J}_j)$ where $i, j \in \{1, 2\}$ such that $i \neq j$ and our new definitions bi^i -supra topological space.

2. Bi-Supra Topological Space

Definition 2.1:

Let X be a non-empty set. Let $S\mathfrak{J}$ is the set of all semi open subsets of X (for short $S(X)$ [6], and Let $P\mathfrak{J}$ be the set of all pre-open subsets of X (for short $Po(X)$ [3].

then we say about $(X, S\mathfrak{J}, P\mathfrak{J})$ is a bi-supra topological space. where each of $(X, S\mathfrak{J})$ and $(X, P\mathfrak{J})$ are supra topological spaces.

Definition 2.2 [3]:

A subfamily $\mathfrak{W}$ of a family of all subset of non empty set X is said to be a supra topology on X if :

1., $\emptyset \in \mathfrak{J}$.

2. If $\mathfrak{K}_i \in \mathfrak{W}$, $\forall i \in I \Rightarrow \bigcup \mathfrak{K}_i \in \mathfrak{W}$, where I is any index set. $(X, \mathfrak{W})$ its called a supra topological spaces. The elements of $\mathfrak{W}$ are called supra open sets in $(X, \mathfrak{W})$ and the complement of a supra open sets is called a supra closed sets.

Definition 2.3:

A subset $\mathfrak{K}$ of a supra topological space $(X, \mathfrak{W})$ be called:

1. semi supra open set if $\mathfrak{K} \subseteq cl_{\mathfrak{W}}(int_{\mathfrak{W}}(\mathfrak{K}))$, the complement of semi supra open set is said to be semi supra closed set [5].

2. pre supra open set if $\mathfrak{K} \subseteq int_{\mathfrak{W}}(cl_{\mathfrak{W}}(\mathfrak{K}))$, the complement of pre supra open set is said to be pre supra closed set [1].

3. $\aleph$ -supra open set if $\aleph \subseteq \text{int}_{\aleph}(\text{cl}_{\aleph}(\text{int}_{\aleph}(\aleph)))$, the complement of $\aleph$ -supra open set is said to be $\aleph$ -supra closed set. [2]

4. $\mathfrak{P}$ -supra open if $\aleph \subseteq (\text{cl}_{\aleph}(\text{int}_{\aleph}(\text{cl}_{\aleph}(\aleph))))$, the complement of $\mathfrak{P}$ -supra open set is said to be $\mathfrak{P}$ -supra closed set. [4]

5. regular supra open if $\aleph = \text{int}_{\aleph}(\text{cl}_{\aleph}(\aleph))$, the complement of regular supra open set is said to be regular supra closed set [7].

3. Regular Semi Supra Open in Bi- Supra Topological Space

Definition 3.1:

Let $(\aleph, \text{So}(\aleph), \text{Po}(\aleph))$ be a bi^i - supra topological space, a subset $\aleph$ of $\aleph$ is regular semi supra open set if $\aleph = \text{int}_{\aleph}(\text{cl}_{\aleph}(\aleph))$ and denoted by $\mathcal{RSO}(\aleph)$.

Now we introduce some remarks which are needed in our work

Remark 3.2:

1. Finite union of bi^i - regular supra open sets is not necessary to be a bi^i -regular supra open sets.

2. Finite intersection of bi^i -regular supra open sets is not necessary to be a bi^i -regular supra open sets.

This will be shown by the example:

Let $\aleph = \{1, 2, 3, 4\}$

$\aleph = \{\varphi, \aleph, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,3,4\}, \{1,2,4\}, \{1,2,3\}, \{1,4\}, \{2,4\}\}$

$\aleph^c = \{\varphi, \aleph, \{2,3,4\}, \{1,3,4\}, \{3,4\}, \{2\}, \{3\}, \{4\}, \{1,2,3\}, \{2,3\}, \{1,3\}\}$

bi^i -supra open =

$\{\varphi, \aleph, \{1\}, \{3\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}\}$

$(bi^i - \text{supra})^c = \{\varphi, \aleph, \{2,3,4\}, \{1,2,4\}, \{1,2\}, \{1,4\}, \{1,2\}, \{4\}, \{3\}, \{2\}, \{1\}\}$

bi^i -Regular supra open = $\{\varphi, \aleph, \{1\}, \{3\}, \{1,2,4\}, \{2,3,4\}\}$

Note that $\{1\}, \{3\}$ are bi^i -regular supra open but $\{1\} \cup \{3\} = \{1,3\}$ is not bi^i -regular supra open

And so $\{2,3,4\}, \{1,2,4\}$ are bi^i -regular supra but $\{2,3,4\} \cap \{1,2,4\} = \{2,4\}$ is not bi^i -regular supra open.

Remark 3.3:

1. A finite union of $\mathcal{RSO}(\aleph)$ is not necessary to be $\mathcal{RSO}(\aleph)$.

2. A finite intersection of $\mathcal{RSO}(\aleph)$ is not necessary to be $\mathcal{RSO}(\aleph)$ open sets

It will be shown in the following example:

Let $\aleph = \{1, 2, 3, 4\}$

$\aleph = \{\varphi, \aleph, \{1\}, \{2,3\}, \{1,2,3\}, \{1,3,4\}, \{3,4\}\}$

$\aleph^c = \{\varphi, \aleph, \{2,3,4\}, \{1,4\}, \{4\}, \{2\}, \{1,2\}\}$

$\text{So}(\aleph) =$

$\{\varphi, \aleph, \{1\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$

$\text{Po}(\aleph) =$

$\{\varphi, \aleph, \{1\}, \{3\}, \{1,3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}\}$

bi^i -supra =

$\{\varphi, \aleph, \{1\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}, \{3\}, \{1,3\}, \{1,2,4\}\}$

$(bi^i - \text{supra})^c =$

$\{\varphi, \aleph, \{2,3,4\}, \{1,4\}, \{1,2\}, \{4\}, \{2\}, \{1\}, \{1,2,4\}, \{2,4\}, \{3\}\}$

bi^i -So($\aleph$) =

$\{\varphi, \aleph, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$

bi^i -Po($\aleph$) =

$\{\varphi, \aleph, \{1\}, \{3\}, \{1,3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$

$(bi^i - \text{Po}(\aleph))^c =$

$\{\varphi, \aleph, \{2,3,4\}, \{1,2,4\}, \{2,4\}, \{1,4\}, \{1,2\}, \{4\}, \{3\}, \{2\}, \{1\}\}$

$bi^i - \mathcal{RSO}(\aleph) =$

$\{\varphi, \aleph, \{1\}, \{3\}, \{1,2\}, \{1,2,4\}, \{2,3,4\}\}$

we note $\{2,3,4\}$ and $\{1,2,4\}$ are $bi^i - \mathcal{RSO}(\aleph)$ but $\{2,4\}$ is not belong to $bi^i - \mathcal{RSO}(\aleph)$. As that respect to the union we find $\{1,2\}$ and $\{3\}$ belong to $bi^i - \mathcal{RSO}(\aleph)$, but $\{1,2,3\}$ is not belong to $bi^i - \mathcal{RSO}(\aleph)$.

Definition 3.4:

A subset $\aleph$ of a space $(\aleph, \text{So}(\aleph), \text{P}(\aleph))$ is said to be

1. $\aleph$ -So($\aleph$)-supra open if $\aleph \subseteq \text{int}_{\aleph}(\text{cl}_{\aleph}(\text{int}_{\aleph}(\aleph)))$.

2. $\mathfrak{P}$ -So($\aleph$)-supra open if $\aleph \subseteq \text{cl}_{\aleph}(\text{int}_{\aleph}(\text{cl}_{\aleph}(\aleph)))$.

Lemma 3.5

Every $\mathcal{RSO}(\aleph)$ and $\mathfrak{P}$ -So($\aleph$)-supra open and $\aleph$ -So($\aleph$) are independent.

Proof

Directly from definition.

Definition 3.6:

A subset $\mathcal{S}$ of a bi^i - supra topological space is called bi^i -supra clopen set if it is both bi^i -supra open and bi^i - supra closed set in $(\aleph, \text{So}(\aleph), \text{Po}(\aleph))$.

Corollary 3.7:

Let $\aleph$ be a bi^i - regular supra clopen set. Suppose that $\mathcal{F}$ is a bi^i -supra closed set. Then $\aleph \cap \mathcal{F}$ is a bi^i -regular semi supra closed sets.

Proof:

We have the $\aleph \cap \mathcal{F}$ be supra closed in. Hence $\text{cl}_{\aleph}(\aleph \cap \mathcal{F}) = \aleph \cap \mathcal{F}$ in $\aleph$. Let $\aleph \cap \mathcal{F} \subset \mathcal{U}$, where $\mathcal{U}$ is regular supra open in $\aleph$. Hence $\aleph \cap \mathcal{F}$ is a regular semi supra closed set in the regular supra closed. Apply Theorem $\mathfrak{B} \subset \aleph \subset \aleph$.

Theorem 3.8:

suppose that $\mathfrak{B} \subset \aleph \subset \aleph$, $\mathfrak{B}$ is a bi^i -regular semi supra closed set relative to $\aleph$ and that $\aleph$ is a bi^i -regular supra clopen subset of $\aleph$. Then $\mathfrak{B}$ is bi^i -regular semi supra closed relative to.

Proof

Let $\mathfrak{B} \subset \mathcal{U}$ and let $\mathcal{U}$ be bi^i -regular supra open. We have $\mathfrak{B} \subset (\aleph \cap \mathcal{U})$. But $\mathfrak{B}$ is a

bi^i -regular supra closed set relative to $\aleph$. Hence $bi^i - \text{cl}_{\aleph}(\mathfrak{B}) \subset (\aleph \cap \mathcal{U}) \dots 1$,

Note that $\aleph \cap \mathcal{U}$ is regular open in $\aleph$. But $bi^i - \text{cl}_{\aleph}(\mathfrak{B}) = bi^i - \text{cl}_{\aleph}(\mathfrak{B}) \cap \aleph \dots 2$,

From (1) and (2) it follows that

$(\aleph \cap bi^i - \text{cl}_{\aleph}(\mathfrak{B})) \subset (\aleph \cap \mathcal{U})$. Consequently $\aleph \cap bi^i - \text{cl}_{\aleph}(\mathfrak{B}) \subset \mathcal{U}$.

Hence $\aleph \cap (bi^i - \text{cl}_{\aleph}(\mathfrak{B})) \cup ((bi^i - \text{cl}_{\aleph}(\mathfrak{B}))^c \subset (\mathcal{U} \cup (bi^i - \text{cl}_{\aleph}(\mathfrak{B}))^c)$.

That is $(\mathfrak{K} \cap \mathfrak{X}) \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{B}}(\mathfrak{B})))^c$

so $\mathfrak{K} \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{B}}(\mathfrak{B})))^c = G \dots (3)$

But then G be a supra open set. since $\mathfrak{K}$ be a bi^i -regular-supra closed in $\mathfrak{X}$,

from (3) we have

$$(bi^i - cl_{\mathfrak{B}}(\mathfrak{K}))^c \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{B}}(\mathfrak{B})))^c = G \dots (4)$$

$$\text{But } bi^i - cl_{\mathfrak{B}}(\mathfrak{B}) \subset bi^i - cl_{\mathfrak{B}}(\mathfrak{K}) \dots (5)$$

From (4) and (5) we have $(cl \mathfrak{B}) \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{B}}(\mathfrak{B})))^c$.

Hence $bi^i - cl_{\mathfrak{B}}(\mathfrak{B}) \subset \mathcal{U}$ because $bi^i - cl_{\mathfrak{B}}(\mathfrak{B}) \cap (bi^i - cl_{\mathfrak{B}}(\mathfrak{B}))^c = \emptyset$.

This implies that $\mathfrak{B}$ is regular semi supra closed relative to $\mathfrak{X}$.

lemma 3.9:

If $\mathfrak{K}$ is bi^i -regular open and bi^i -regular semi closed then $\mathfrak{K}$ is bi^i -semi clopen.

4. Continuity in bi^i -Supra Topological Space

Definition 4.1

A function

$f: (\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X})) \rightarrow (\mathcal{Y}, So(\mathcal{Y}), Po(\mathcal{Y}))$ is called:

1. bi^i -Semi supra continuous function if the inverse image for any semi- supra open set in $\mathcal{Y}$ is semi-supra open set in $\mathfrak{X}$.
2. bi^i - Regular supra continuous function if the inverse image for any regular - supra open set in $\mathcal{Y}$ is regular- supra open set in $\mathfrak{X}$.
3. bi^i -regular semi supra-continuous function if the inverse image for any regular semi – supra open set in $\mathcal{Y}$ is regular semi- supra open set in $\mathfrak{X}$.

Theorem 4.2:

Let $f: (\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X})) \rightarrow (\mathcal{Y}, So(\mathcal{Y}), Po(\mathcal{Y}))$, be a bi^i -supra function then the following are equivalents:

1. f be a bi^i - $\mathcal{R}So(\mathfrak{X})$ supra continuous.
2. inverse image for every bi^i -regular supra open subset in $\mathcal{Y}$ is bi^i -regular semi supra open in $\mathfrak{X}$.
3. inverse image of every bi^i -regular supra closed subset in $\mathcal{Y}$ be a bi^i -regular semi supra closed in $\mathfrak{X}$.

proof

suppose 1) to prove 2)

since f is a bi^i - $\mathcal{R}So(\mathfrak{X})$ supra continuous

then inverse image for every bi^i -regular supra open subset in $\mathcal{Y}$ be bi^i -regular

semi supra open in $\mathfrak{X}$.

Thus, the condition is fulfilled.

suppose 2) to prove 3)

if we take the complement to the two condition we get

the inverse image for every bi^i -regular supra closed subset of $\mathcal{Y}$ be a bi^i -

regular semi supra closed of $\mathfrak{X}$.

It is the three condition.

suppose 3) to prove 1)

let G be a bi^i -regular supra closed subset of $\mathcal{Y}$

$\therefore G^c$ be a bi^i -regular supra open subset of $\mathcal{Y}$

$\therefore G$ be a bi^i -regular semi supra closed subset of $\mathfrak{X}$

$\therefore G^c$ be a bi^i -regular semi supra open subset of $\mathfrak{X}$.

Thus, the first condition is fulfilled.

Remark 4.3:

Every bi^i -regular supra continuous function is a bi^i -supra regular semi continuous function but the converse is not true.

Proof:

Let $f: (\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X})) \rightarrow (\mathcal{Y}, So(\mathcal{Y}), Po(\mathcal{Y}))$ be a bi^i -regular supra continuous function and $\mathfrak{K}$ is regular supra open set in $\mathcal{Y}$.

Then there exist $\mathfrak{B}$ in $\mathfrak{X}$ s.t $f(\mathfrak{B}) = \mathfrak{K}$.

$\therefore f$ is bi^i -regular semi supra continuous

$\therefore f(\mathfrak{B}) = \mathfrak{K}$ and $f^{-1}(\mathfrak{K}) = \mathfrak{B}$.

For that $f^{-1}(\mathfrak{K})$ be a regular supra open set in $\mathfrak{X}$.

Hence supra regular semi open set in $\mathfrak{X}$. Hence f is bi^i -regular semi supra continuous function.

The converse for the previous theory is an incorrect, this will be shown by example.

Example 4.4:

Let $\mathfrak{X} = \{1, 2, 3\}$

$\mathfrak{B} = \{\emptyset, \mathfrak{X}, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$

$\mathcal{Y} = \{a, b, c\}$

$\mathfrak{B} = \{\emptyset, \mathcal{Y}, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$f(1) = a, f(2) = b, f(3) = c$.

We find $\{a, b\}$ is bi^i -regular semi supra continuous function but not bi^i -regular supra continuous function.

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الملخص

الغرض من هذا البحث هو تقديم المفاهيم .الدالة المستمرة الفوقية و الدالة المستمرة الشبه الفوقية والمجموعة المفتوحة المنتظمة والمجموعة المفتوحة شبه الفوقية المنتظمة في فضاء ثنائي التبولوجي الفوقي من النمط τ العلاقات بين هذه الدوال قد درست وتحرينا بعض الخواص لها.