

Calculations of Ground Band in even-even  $^{170-180}\text{W}$  nuclei by  
Interacting boson model (IBM-1)

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### Abstract

In this work, the properties of  $^{170-180}\text{W}$  isotopes are studied by the E-GOS curves and the relation between energy levels and  $E_2$  showed that  $^{170-180}\text{W}$  lie in SU(3)-O(6) transition region were investigated to calculate the energy levels of ground state band according to the Interacting Boson Model (IBM-1). The entire calculations and drawing of the figures implemented by one program written by matlab language. The results are compared with experimental data and showed good agreement.

**Keywords:** energy levels, IBM(1), SU(3)-O(6), E-GOS, W isotopes.

### حساب مستويات الطاقة للحزمة الأرضية لنظائر $^{170-180}\text{W}$ الزوجية – الزوجية باستخدام أنموذج البوزونات المتفاعلة (IBM-1)

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### الخلاصة

في هذا البحث تم دراسة خصائص نظائر التنكستن  $^{170-180}\text{W}$  بواسطة منحنيات كما مقسوماً على الزخم E-GOS وتبين من العلاقة بين مستويات الطاقة ومستوي  $E_2$  ان هذه النظائر تقع بين التحديدين SU(3)-O(6) أي في المنطقة الانتقالية بين النوى الدورانية والنوى ذات خصائص كما الناعمة. حسب مستويات الطاقة للنظائر المدروسة باستخدام انموذج البوزونات المتفاعلة IBM-1 من خلال برنامج حاسوبي كتب بلغة matlab يفي بهذا الغرض من اجراء الحسابات ورسم الأشكال المطلوبة. قورنت الحسابات الحالية مع النتائج العملية وتبين انها تتفق بشكل جيد.

**الكلمات المفتاحية:** نظائر التنكستن، منحنيات E-GOS، SU(3)-O(6)، أنموذج البوزونات المتفاعلة IBM-1، مستويات الطاقة.

## Calculations of Ground Band in even-even $^{170-180}\text{W}$ nuclei by Interacting boson model (IBM-1)

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### Introduction

Several phenomenological and geometrical models [1] have been proposed to investigate the nuclear structure by the prediction of the ground states and the description of electromagnetic transition rates [2]. The quadrupole correlations and electromagnetic transition in nuclei depends mainly on the neutron-proton interaction. However, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell [3,4]. The tungsten isotopes received considerable attention both theoretically and experimentally in recent years. Abdul Ameer and Al-Shimmary [5] calculated the energy levels,  $B(E2)$  transition probabilities and electric quadrupole moment of the even-even  $^{180-190}\text{W}$  isotopes in the transition region  $SU(3)-O(6)$ . The energy levels, electric quadrupole moments,  $B(E2)$  values of  $^{182-186}\text{W}$  isotopes have been calculated by, Salem *et al* [6] within the framework of the interacting boson model IBM-2. Due to the increased interest in this subject recent years, the energy levels and E-Gos of even-even  $^{170-180}\text{W}$  isotopes in  $SU(3)-O(6)$  transition region within framework are studied by the Interacting Boson Model (IBM-1).

### Theory

#### Energy Gamma Over Spin (E-GOS)

Many nuclei exhibit a decay sequence consistent with quasi-vibrational excitations at lower spins, the perfect harmonic vibrator of gamma-ray decay energies are given by :

$$E_{\gamma}(J \rightarrow J-2) = \hbar w \quad (1)$$

where  $J$  is spin state,  $\hbar$  is plank's constant and  $w$  is angular frequency. While, for an axially symmetric rotor,

$$E_{\gamma}(J \rightarrow J-2) = \frac{\hbar^2}{2\mathcal{I}}(4J-2) \quad (2)$$

where  $\mathcal{I}$  is moment of inertia of the nucleus.

The gamma-soft nucleus can be written as

$$E_{\gamma}(J \rightarrow J-2) = \frac{E2_1^+}{4}(J+2) \quad (3)$$

The ratio  $R = \frac{E_{\gamma}(J \rightarrow J-2)}{J}$  provides an effective way of distinguishing

axially symmetric rotational,  $\gamma$ -unstable and harmonic vibrational mode [7].

at  $J \rightarrow 0$

$$\text{for vibrator } R = \frac{\hbar w}{J} \rightarrow 0 \quad (4)$$

$$\text{Rotor } R = \frac{\hbar^2}{2\mathcal{I}}(4 - \frac{2}{J}) \rightarrow 4 \frac{\hbar^2}{2\mathcal{I}} \quad (5)$$

$$\gamma\text{-unstable } R = \frac{E2_1^+}{4}(1 + \frac{2}{J}) \rightarrow \frac{E2_1^+}{4} \quad (6)$$

Figure (1) shows these theoretical limits plotted for three schematic nuclei: (i) a vibrator in which the first  $2^+$  excited state lies at an energy of 500 keV, (ii) a rotor where this energy is

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100 keV and (iii) a  $\gamma$ -unstable of energy 300 keV (These values were taken to represent typical nuclear vibrator, rotor and  $\gamma$ -unstable energies, respectively.)

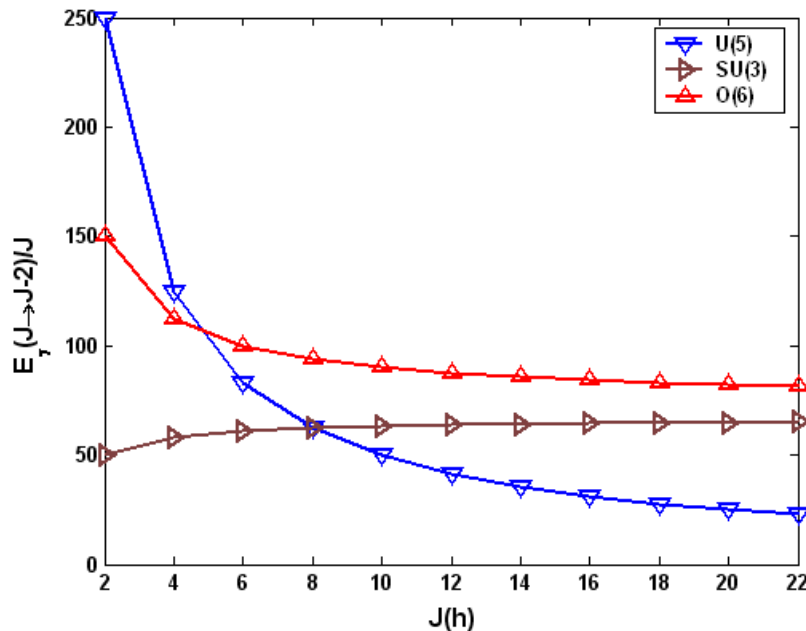


Figure (1): Standard curve of E-GOS plot for vibrational U(5), gamma unstable O(6) and rotational bands SU(3)

Interacting Boson Model (IBM-1)

In the present work, the IBM-1 states of the low lying collective state of even-even nuclei can be described by the interaction of s and d-bosons, carrying angular momentum  $L = 0$  and  $L = 2$ , respectively. The IBM-1 Hamiltonian is written

$$H = \varepsilon \hat{n}_d + a_0 \hat{P}^+ \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad (7)$$

Where  $a_0$ ,  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$  are strength of pairing, angular momentum and multipole terms. The Hamiltonian tends to reduce into three limits, the vibration U(5),  $\gamma$ -soft O(6) and the rotational SU(3) nuclei [8]. In U(5) limit, the effective parameter is  $\varepsilon$ , in the  $\gamma$ -soft limit O(6) the effective parameter is the pairing  $a_0$  and in the SU(3) limit the effective parameter is the quadrupole  $a_2$ .

The eigenvalues for the SU(3)-O(6) limit is given by [9]

$$E(\lambda, \mu, J) = K_3[N(N+4) - \sigma(\sigma+4)] + K_2(\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda\mu) + K_5J(J+1) \quad (8)$$

for low-lying  $N=\sigma$ ,  $\mu=0$  and  $\lambda=2N$  therefore

$$E(\lambda, J) = K_2(\lambda^2 + 3\lambda) + K_5J(J+1) \quad (9)$$

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**Discussion and Conclusions**

The parameters of Eq.(9) were calculated for  $^{170-180}\text{W}$  nucleus and listed in table (1), it was found that  $K_2$  decreases and  $K_5$  increases with mass number.

**Table(1): The Parameters  $K_2$ ,  $K_5$  and the ratio  $R=E_4/E_2$  for  $^{170-180}\text{W}$  isotopes**

| Nucleus          | $K_2$ (keV) | $K_5$ (keV) | $R=E_4/E_2$ |
|------------------|-------------|-------------|-------------|
| $^{170}\text{W}$ | 25.7672     | 4.1226      | 2.95        |
| $^{172}\text{W}$ | 18.4796     | 5.2798      | 3.0609      |
| $^{174}\text{W}$ | 13.9465     | 7.7379      | 3.154       |
| $^{176}\text{W}$ | 12.6059     | 8.4789      | 3.2151      |
| $^{178}\text{W}$ | 10.7882     | 9.5505      | 3.2364      |
| $^{180}\text{W}$ | 10.4675     | 9.6598      | 3.26        |

Figure (2) shows that the ratio  $E_J/E_2$  of  $^{170-180}\text{W}$  versus spin ( $J$ ) in which the limit lies between  $SU(3)$ - $O(6)$ . Eq.(9) used to calculate the energy levels of ground state band.

Figure (3) shows the E-GOS curves of the ground states band of isotopes. Comparing these curves with the ideal limits of vibration, rotational and  $\gamma$ -soft show the evolution in the property of these isotopes, were the slow reduce of all the curves from the first to the last excited states confirms that the  $^{170-180}\text{W}$  isotopes lies between  $SU(3)$ - $O(6)$  limit.

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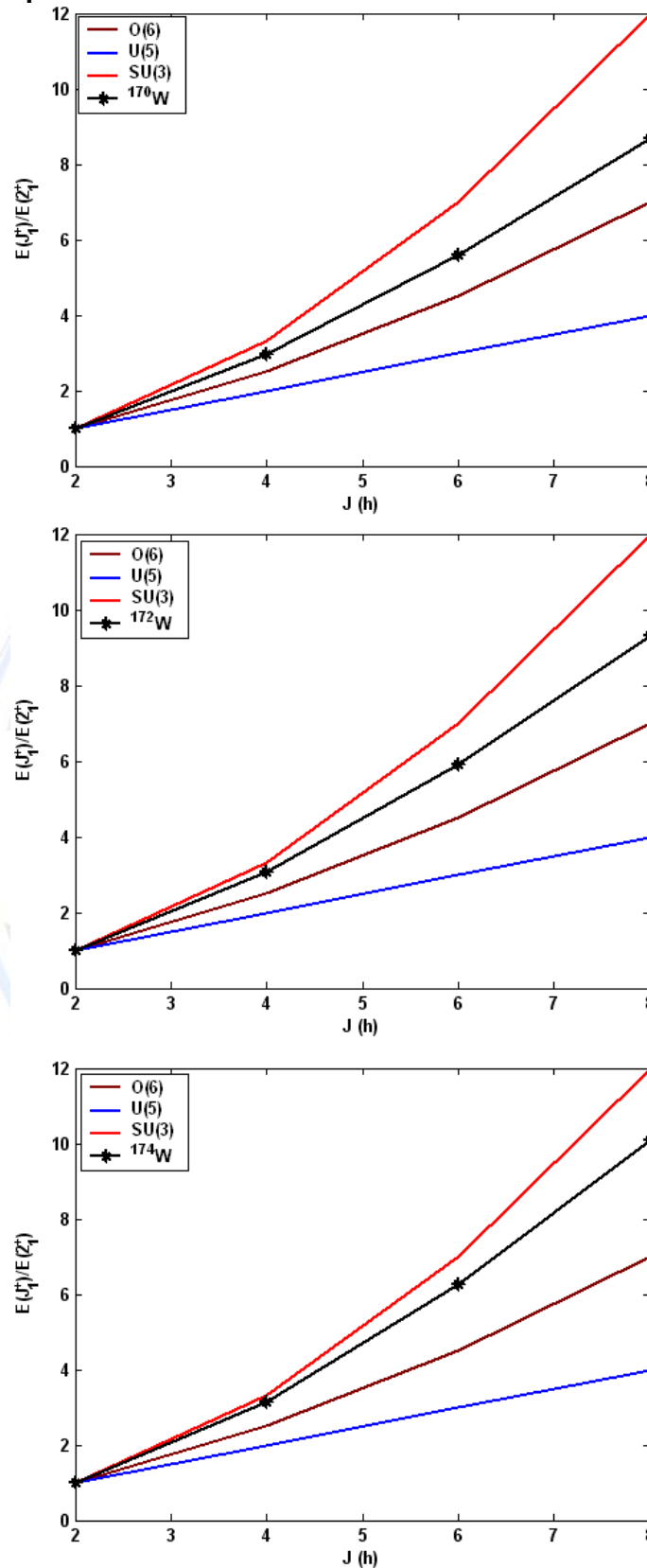


Figure (2):  $E(J_1)/E(2_1)$  versus spin  $J$

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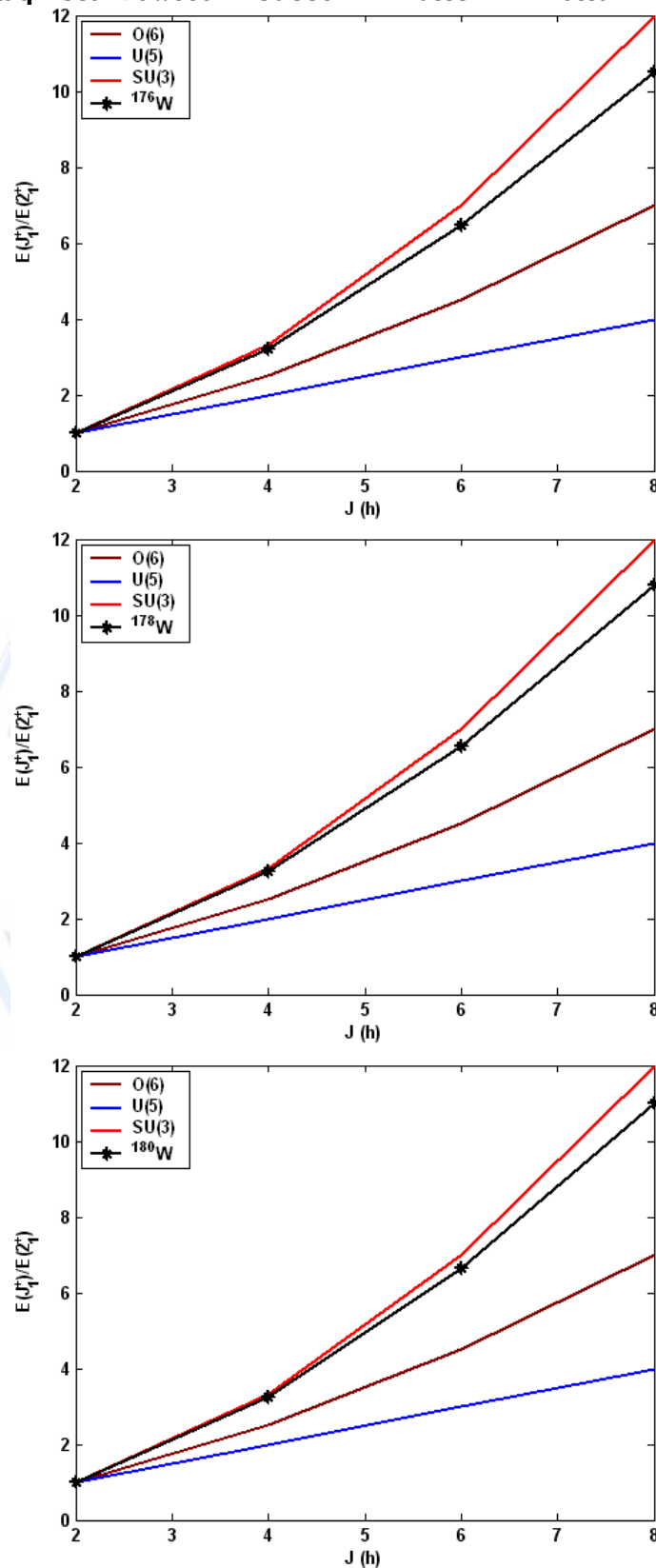


Fig. 2. (continued)

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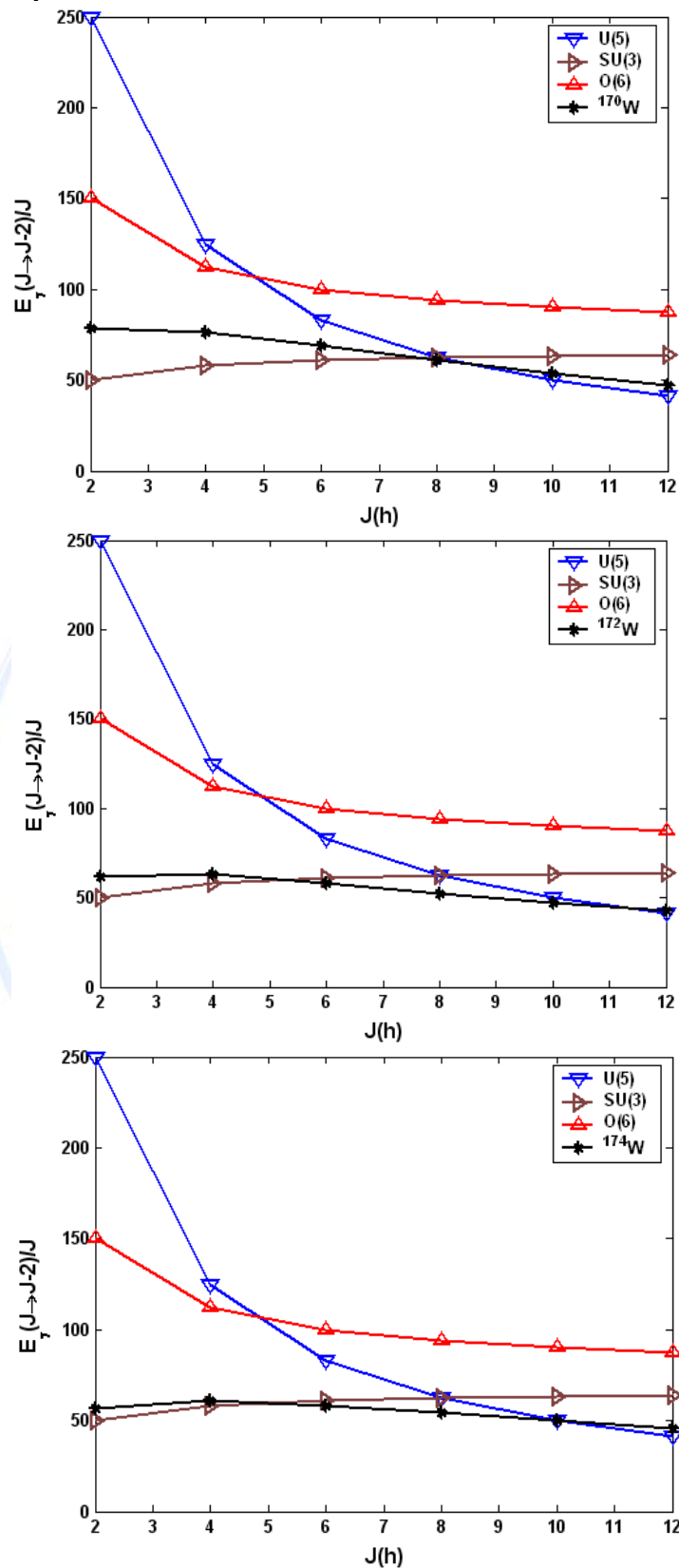


Figure (3): The E-GOS of  $^{170-180}\text{W}$  isotopes.

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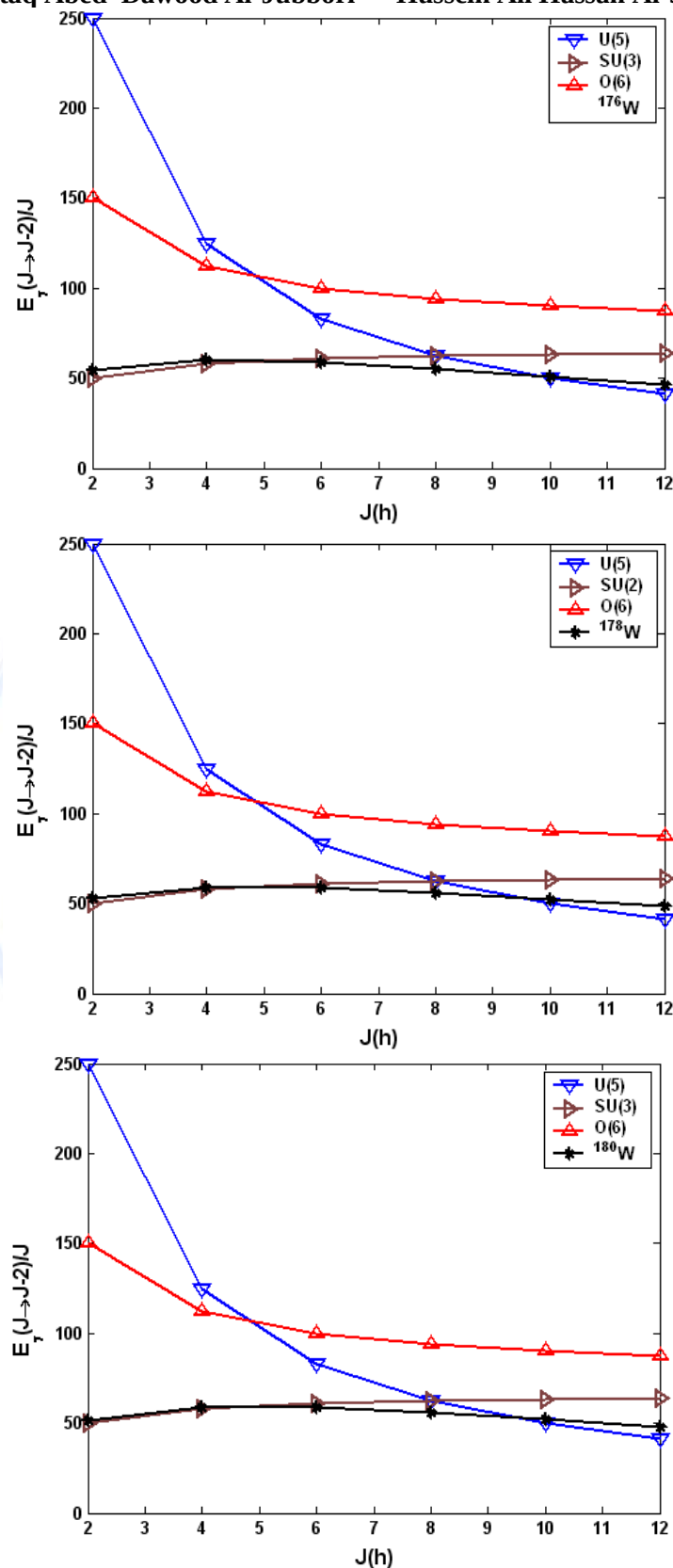


Fig. 3. (continued)

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Figure (4) shows the present calculations of the energy levels were found to be in a reasonable agreement with measured values.

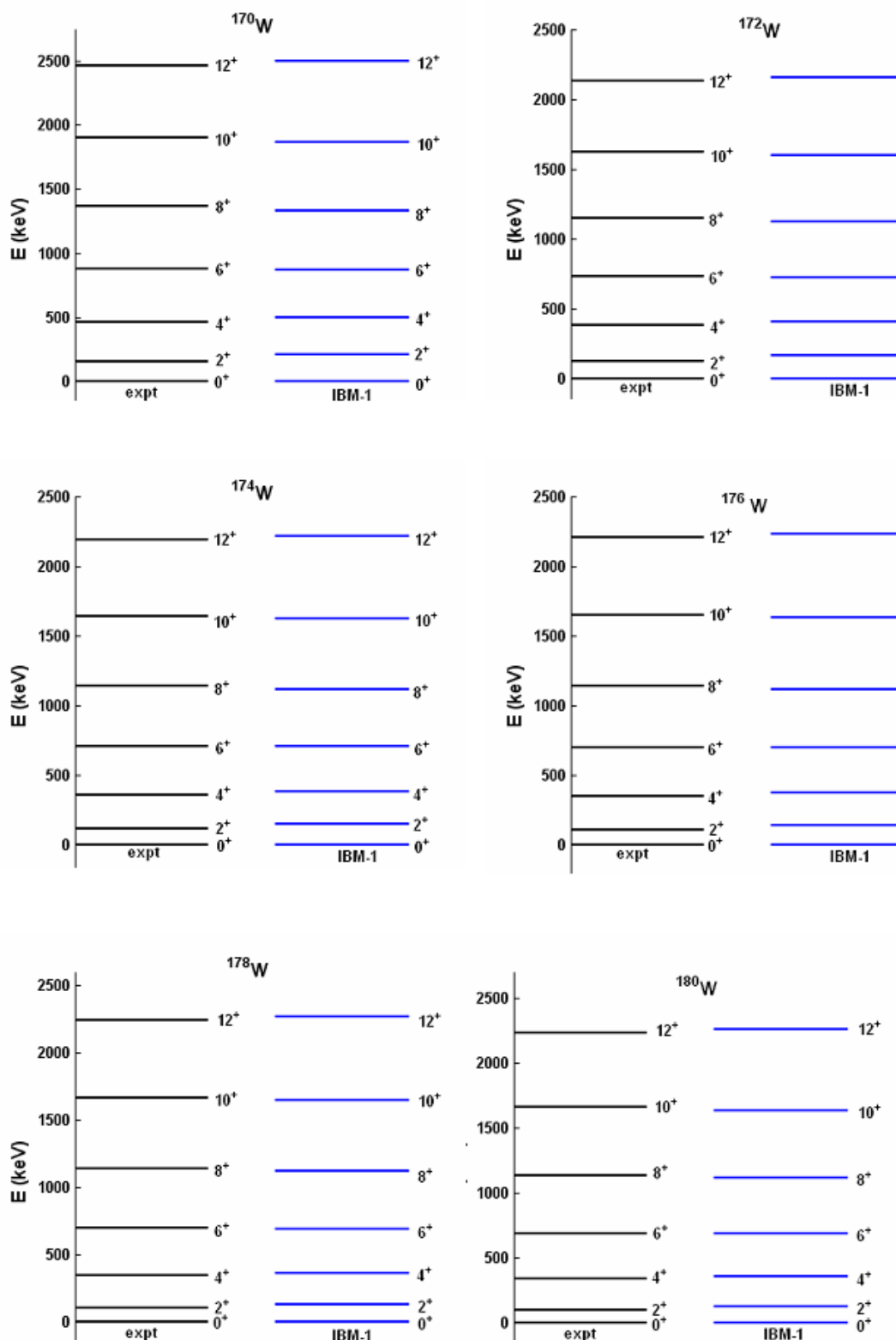


Figure (4): Comparison between experimental energy levels and calculated data

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**References**

- [1] H. M. Mittal and Vidya Devi, Armen. J. Phys. vol. 2, issue 3, pp. 146-156, (2009).
- [2] Hossain, Hewa Y. Abdullah, I. M. Ahmed, M. A. Seed, S. T. Ahmad. Armen. J. Phys. vol. 5, issue 3, pp. 101-104, (2012).
- [3] R. F. Casten, Nucl. Phys. A. vol. 347, pp. 173-204, (1980).
- [4] M. A. D. Al-Jubbori, Dyalia Journal Pure. Science. vol. 9 No: 2, pp.45-56, (2013).
- [5] M. Abdul Ameer and M. A. H. Al-Shimmery. Armenian Journal of Physics, vol. 4, issue 3, pp. 146-153, (2011).
- [6] A. M. Salem, S. Olaf and S. M. Daw. the Fourth International Conference of Science & Development, Gaza (2011) (Abstract).
- [7] P. H. Regan, C.W. Beausang, N.V. Zamfir, R. F. Casten, Jing-yeZhang, A. D. Yamamoto, M. A. Caprio, G. Gurdal, A. A. Hecht, C. Hutter, R. Krucken, S. D. Langdown, D. A. Meyer, and J. J. Ressler. Phys. Rev. Lett. 90, pp. 152502, (2003).
- [8] A. Arima and F. Iachello, Phys. Rev. Lett., 40, pp.385-391, (1978).
- [9] NSDD Workshop, Triste, (2005).

