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A New Ridge-Type Estimator for the Gamma regression model

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ABSTRACT: When there is collinearity among the regressors in gamma regression models, we present a new two-parameter ridge estimator in this study. We look into the new estimator's mean squared error characteristics. Additionally, we offer several theorems to contrast the new estimators with the current ones. To compare the estimators under various collinearity designs in terms of mean squared error, we run a Monte Carlo simulation analysis. We also offer a real data application to demonstrate the usefulness of the new estimator. The results from simulations and actual data reveal that the proposed estimator is superior to competing estimators.

Keywords: Multicollinearity; gamma regression model; ridge estimator; simulation; Liu-type estimator.

1. INTRODUCTION

An example of a statistical model used to evaluate data with a skewed distribution and non-negative values is gamma regression. When the response variable (or dependent variable) is continuous and positive, like in the case of insurance claims or medical expenses, it is especially helpful. The response variable is thought to have a gamma-distribution in a gamma regression model. The gamma distribution is a family of continuous probability distributions with two parameters that contains several other distributions as special instances, including the exponential, chi-squared, and weibull distributions. Similar to linear regression models, the gamma regression model can be specified using a linear predictor function. But it makes a gamma distribution assumption for the residuals rather than a normal distribution. A link function that links the linear predictor to the gamma distribution's mean is also included in the model [1, 2]. Most frequently, methods for estimating maximum likelihood are used to estimate the gamma regression model.

The estimated values that are produced can be put to use to test theories regarding the correlation between the response variable and one or more predictor variables. All things considered, the gamma regression model is a potent tool for examining non-negative, skewed data with a continuous response variable [3].

The relationship between the response variable, explanatory factors, and random errors is commonly depicted as follows in traditional linear regression models:

$$y = X\beta + \varepsilon, \tag{1}$$

where y is an $(n\times 1)$-dimensional observations for the response variable as a vector, $X=(x_1,...,x_p)$ is an $(p\times 1)$ known explanatory variable design matrix, $\beta=(\beta_1,...,\beta_p)$ is a $(p\times 1)$-dimensional vector of unknown regression coefficients, and ε is an $(n\times 1)$-dimensional random error vector with mean 0 and variance σ^2.

It is assumed that there is no association between the regressors when using the Gamma regression model (GRM). Multicollinearity is a concern since this assumption frequently fails to hold true in practice. When the maximum likelihood (ML) estimation is used, the calculated coefficients often exhibit significant variance and low statistical significance. for the Gamma regression model in the presence of multicollinearity [4, 5].

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To solve the multicollinearity issue, numerous remedial methods have been put forth. One such method is the ridge regression method [6] which has consistently been demonstrated to be an attractive alternative to the ML estimation method.

Ridge regression is a method that reduces the large variance of regression coefficients by shrinking them towards zero[4, 7]. This is achieved by adding a positive amount to the diagonal of the matrix $\left(X^T X\right)$. Because of this bias, the ridge estimator is more accurate than the maximum likelihood estimator, but it does it with a smaller mean squared error. The widely recognized Liu estimator [8] has been recently extended to work with generalized linear models (GLMs) and its application to gamma-distributed response variables has been demonstrated by [5, 9-11] Additional references for Liu regression can be found in [12, 13] and.[14] Another approach to addressing the collinearity problem is to use the two-parameter estimator [15], which has also been extensively studied in the literature and generalized to some GLMs [16, 17]. However, there are several other estimators [18-25]

2. Gamma regression model

In sociological, economic, and epidemiology studies, positively skewed data, which consists of non-negative values, frequently appears. A well-known distribution that fits this kind of data is the gamma-distribution. The link between a positively skewed response variable and probable regressors is modeled using the gamma regression model [26].

If y_i is Gamma distribution which is used to describe the response variable, with a nonegative shape and scale parameters υ and γ, i.e. $y_i \sim Gamma(\upsilon,\gamma)$, then the pdf is defined as:

$$f(y_i)=\frac{\gamma}{\Gamma(\upsilon)}(\gamma y_i)^{\upsilon-1}e^{-\gamma y_i}, \quad y_i \geq 0, \qquad (2)$$

Where $\Gamma(\upsilon)$ is the Gamma function with $E(y)=\upsilon/\gamma=\theta$ and $\mathrm{var}(y)=\upsilon/\gamma^2=\theta^2/\upsilon$. Given that $\gamma=\upsilon/\theta$, We can re-parameterize equation (2) as a function of the mean (θ) and shape (υ) parameters, and express it using the exponential function

$$f(y_i)=\mathrm{EXP}\left\{\frac{y_i(-1/\theta)-\log(-1/\theta)}{1/\upsilon}+c(y_i,\upsilon)\right\}, \qquad (3)$$

where the canonical link function is $-1/\theta$, the dispersion parameter is $\phi=1/\upsilon$ and $c(y_i,\upsilon)=\upsilon\log(\upsilon)+\upsilon\log(y_i)-\log(y_i)-\log(\Gamma(\upsilon))$.

The canonical link function, $\theta_i=-1/x_i^T\beta$ is typically used in formulating the GRM, which is represented as a linear combination of covariates $x_i=(x_{i1},\ldots,x_{ip})^T$. Alternatively, the log link function $\theta_i=\exp(x_i^T\beta)$ can be used instead of the reciprocal link function because it guarantees that $\theta_i>0$.

The usual practice for estimating the coefficients of is to employ the ML for equation (3). Under the assumption of independent observations and $\theta_i=-1/x_i^T\beta$, the log likelihood function for the GRM is given by:

$$\ell(\beta)=\sum_{i=1}^{n}\left\{\frac{y_i x_i^T\beta-\log(x_i^T\beta)}{1/\upsilon}+c(y_i,\upsilon)\right\}, \qquad (4)$$

To obtain the (ML) estimator, one can calculate the first derivative of equation (4) and equate it to zero.

$$\frac{\partial\ell(\beta)}{\partial\beta}=\frac{1}{\upsilon}\sum_{i=1}^{n}\left[y_i-\frac{1}{x_i^T\beta}\right]x_i=0. \qquad (5)$$

Regrettably, it is not possible to solve the first derivative analytically because equation (5) is nonlinear with respect to (β). Therefore, obtain the ML of the GRM, iterative techniques such as the iteratively weighted least squares $(IWLS)$ algorithm can be used. These methods update the parameters in each iteration until convergence is achieved. In each iteration, the parameters are updated by:

$$\beta^{(r+1)}=\beta^{(r)}+I^{-1}(\beta^{(r)})S(\beta^{(r)}), \qquad (6)$$

where $S(\beta)=\partial\ell(\beta)/\partial\beta$ and $I^{-1}(\beta)=\left(\ -E\left(\partial^2\ell(\beta)/\partial\beta\partial\beta^T\right)\right)^{-1}$. The last step in estimating the coefficients is given by:

$$\begin{aligned}\hat{\beta}_{GML}&=(X^T\hat{W}X)^{-1}X^T\hat{W}\hat{u}\\ \hat{\beta}_{GML}&=(X^T\hat{W}X)^{-1}X^T\hat{W}X\beta \qquad (7)\\ \hat{\beta}_{GML}&=\varsigma^{-1}\varsigma\beta\end{aligned}$$

where $\varsigma=(X^T\hat{W}X)$, $\hat{W}=diag(\hat{\theta}_i^2)$ and $\hat{u}$ is a vector where i^{th} element equals to $\hat{u}_i=\hat{\theta}_i+((y_i-\hat{\theta}_i)/\hat{\theta}_i^2)$. The covariance matrix is

$$\operatorname{cov}(\hat{\beta}_{GML})=\left[\ -E\left(\frac{\partial^2\ell(\beta)}{\partial\beta_i\ \partial\beta_k}\right)\right]^{-1}=\phi(X^T\hat{W}X)^{-1} \qquad (8)$$

Then the matrix mean squared error $(MMSE)$ and the mean squared error (MSE) of equation (7) as follows:

$$MMSE\,(\hat{\beta}_{GML})=\phi\varsigma^{-1} \qquad (9)$$

$$\begin{aligned}MSE\,(\hat{\beta}_{GML})&=\phi tr[\varsigma^{-1}]\\ &=\phi\sum_{j=1}^{p}\frac{1}{\gamma_j}\end{aligned} \qquad (10)$$

where (γ_j) is the eigenvalues of the $(X^T\hat{W}X)$ matrix. When a matrixe or more eigenvalues that are has on $(X^T\hat{W}X)$ close to zero, MSE of MLE inflated, leading to a negative impact on the regression coefficients.

2.1 Gamma ridge estimator

We can express Equation (1) in canonical form as:

$$y=Z\alpha+\varepsilon \qquad (11)$$

where $Z=XQ$, $\alpha=Q^T\beta$ and Q is the orthogonal matrix whose columns constitute the eigenvectors of $(X^T\hat{W}X)$.

To address this issue, one can use the gamma ridge regression estimator (GRR), which is defined as:

$$\begin{aligned}\hat{\beta}_{GRR}&=(X^T\hat{W}X+kI)^{-1}X^T\hat{W}X\hat{\beta}_{GML}\\ &=(\varsigma+kI)^{-1}\varsigma\hat{\beta}_{GML}\end{aligned} \qquad (12)$$

where $k\geq 0$.

The ridge estimator has a bias vector and covariance matrix that can be defined as:

$$Bias(\hat{\beta}_{GRR})=-k(\varsigma+kI)\beta \qquad (13)$$

$$\operatorname{cov}(\hat{\beta}_{GRR})=\phi(\varsigma+kI)^{-1}\varsigma(\varsigma+kI)^{-1} \qquad (14)$$

The MMSE and the MSE of equation (12) as follows:

$$MMSE\left(\hat{\beta}_{GRR}\right)=\phi\varsigma_k^{-1}\varsigma\varsigma_k^{-1}+k^2\varsigma_k^{-1}\beta\beta^T\varsigma_k^{-1} \qquad (15)$$

$$\begin{aligned}MSE\left(\hat{\beta}_{GRR}\right)&=tr\left[MMSE\left(\hat{\beta}_{IGRE}\right)\right]\\ &=\phi\sum_{j=1}^{p}\frac{\gamma_i}{(\gamma_i+k)^2}+k^2\sum_{j=1}^{p}\frac{\alpha_i^2}{(\gamma_i+k)^2}\end{aligned} \qquad (16)$$

Where $\varsigma_k = (\varsigma + kI)$

2.2. Gamma Liu Estimator

The widely used Liu estimator has also been adapted to work with generalized linear models by [5]. In their research, the authors investigated the performance of the Liu gamma estimator, using a gamma-dependent variable, through a Monte Carlo simulation study and real-world application. The gamma Liu estimator (GLE) is defined as follows:

$$\hat{\beta}_{GLE} = (\varsigma + I)^{-1}(\varsigma + dI)\hat{\beta}_{GML} \qquad (17)$$

where $0 < d < 1$.

The bias vector (Biase) and covariance matrix (COV) of the Liu estimator are defined as follows:

$$\begin{aligned} Bias(\hat{\beta}_{GLE}) &= (\varsigma + I)^{-1}(\varsigma + dI)\beta - \beta \\ &= -(1-d)(\varsigma + I)^{-1}\beta \end{aligned} \qquad (18)$$

$$\operatorname{cov}(\hat{\beta}_{GLE}) = \phi(\varsigma + I)^{-1}(\varsigma + dI)\varsigma^{-1}(\varsigma + I)^{-1}(\varsigma + dI) \qquad (19)$$

The MMSE and the MSE of equation (17) as follows:

$$\begin{aligned} MMSE\left(\hat{\beta}_{GLE}\right) &= \phi(\varsigma + I)^{-1}(\varsigma + dI)\varsigma^{-1}(\varsigma + dI)(\varsigma + I)^{-1} \\ &+ (1-d)^2(\varsigma + I)^{-1}\beta\beta^T(\varsigma + I)^{-1} \end{aligned} \qquad (20)$$

$$\begin{aligned} MSE\left(\hat{\beta}_{GLE}\right) &= tr\left[MMSE\left(\hat{\beta}_{GRE}\right)\right] \\ &= \phi\sum_{j=1}^{p}\frac{(\gamma_i + d)^2}{\gamma_i(\gamma_i + 1)^2} + (1-d)^2\sum_{j=1}^{p}\frac{\alpha_i^2}{(\gamma_i + 1)^2} \end{aligned} \qquad (21)$$

2.3. Gamma Kibria and Lukman Estimator

[27] introduced a novel Ridge-type estimator to address the issue of multicollinearity. Their one-parameter estimator , denoted $\hat{\beta}_{GKL}$, is defined as :

$$\hat{\beta}_{GKL} = (\varsigma + kI)^{-1}(\varsigma - kI)\hat{\beta}_{GML} \qquad (22)$$

where $k \geq 0$.

The Biase and COV of the Kibria and Lukman estimator are given by the following expressions:

$$\begin{aligned} Bias(\hat{\beta}_{GKL}) &= \left((\varsigma + kI)^{-1}(\varsigma - kI) - I\right)\beta \\ &= -2k(\varsigma + kI)^{-1}\beta \end{aligned} \qquad (23)$$

$$\operatorname{cov}\left(\hat{\beta}_{GKL}\right) = \phi(\varsigma + kI)^{-1}(\varsigma - kI)\varsigma^{-1}(\varsigma + kI)^{-1}(\varsigma - kI) \qquad (24)$$

We can obtain the MMSE and the MSE of equation (22) as follows:

$$\begin{aligned} MMSE\left(\hat{\beta}_{GKL}\right) &= \phi(\varsigma + kI)^{-1}(\varsigma - kI)\varsigma^{-1}(\varsigma + kI)^{-1}(\varsigma - kI) \\ &+ 4k^2(\varsigma + kI)^{-1}\beta\beta^T(\varsigma + kI)^{-1} \end{aligned} \qquad (25)$$

$$MSE\left(\hat{\beta}_{GKL}\right)=tr\left[MMSE\left(\hat{\beta}_{GKL}\right)\right]$$
$$=\phi\sum_{j=1}^{p}\frac{\left(\gamma_i-k\right)^2}{\gamma_i\left(\gamma_i+k\right)^2}+4k^2\sum_{j=1}^{p}\frac{\alpha_i^2}{\left(\gamma_i+k\right)^2} \qquad (26)$$

3. The proposed method

This paper proposes an approach that builds on the works [8], [28], and [29] to develop an estimator for the gamma regression model. Our method investigates the additive impact of two biasing parameters on the estimator. We refer to this method as the gamma two-parameter estimator $(GTPE)$, which is defined as:

$$\hat{\beta}_{GTPE}(k,d)=\varsigma(\varsigma+(k+d)I)^{-1}\hat{\beta}_{GML}=\Upsilon_k\hat{\beta}_{GML} \qquad (27)$$

Where $\Upsilon_k=\varsigma(\varsigma+(k+d)I)^{-1}$, $k\geq 0$ and $0<d<1$,

The Biase and COV of the Liu estimator are defined as follows:

$$Bias\left(\hat{\beta}_{GTPE}(k,d)\right)=(\Upsilon_k-I)\beta$$
$$=-(k+d)(\varsigma+(k+d)I)^{-1}\beta \qquad (28)$$
$$\operatorname{cov}\left(\hat{\beta}_{GTPE}(k,d)\right)=\phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T$$
$$=\phi\varsigma\left(\varsigma+(k+d)I\right)^{-1}\varsigma^{-1}\left(\varsigma+(k+d)I\right)^{-1}\varsigma \qquad (29)$$

The MMSE and the MSE of equation (17) as follows:

$$MMSE\left(\hat{\beta}_{GTPE}(k,d)\right)=\phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T+\left(\Upsilon_k-I\right)\beta\beta^T\left(\Upsilon_k-I\right)^T$$
$$=\phi\varsigma(\varsigma+(k+d)I)^{-1}(\varsigma+(k+d)I)^{-1}+(k+d)^2(\varsigma+(k+d)I)^{-1}\beta\beta^T(\varsigma+(k+d)I)^{-1} \qquad (30)$$

$$MSE\left(\hat{\beta}_{GTPE}(k,d)\right)=tr\left[MMSE\left(\hat{\beta}_{GTPE}(k,d)\right)\right]$$
$$=\phi\sum_{j=1}^{p}\frac{\gamma_i}{\left(\gamma_i+(k+d)\right)^2}+\left(k+d\right)^2\sum_{j=1}^{p}\frac{\alpha_i^2}{\left(\gamma_i+(k+d)\right)^2} \qquad (31)$$

3.1. The superiority of the proposed estimator

The newly proposed Two-parameter estimator is being compared to two other estimators, namely the GML, GRR, GLE and GKL estimators. This comparison aims to assess the performance of the Two-parameter estimator in relation to these four methods. When a matrix A is positive definite, we signify it as $A>0$. Additionally, we will introduce two lemmas that we will utilize to establish some of the theorems presented in this section.

Lemma 1: Let S be a positive definite matrix, $S>0$, and let α be a non-zero vector and. Then, $S-\alpha\alpha'\geq 0$ if and only if $\alpha'S^{-1}\alpha\leq 1$ [30].

Lemma 2: Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be two competing estimators of β. Suppose that $D=\operatorname{cov}(\hat{\beta}_1)-\operatorname{cov}(\hat{\beta}_2)$ is positive definite $d_1=Bais(\hat{\beta}_1)$ andif and only if $\Delta_{(\hat{\beta}_1,\hat{\beta}_2)}=MMSE(\hat{\beta}_1)-MMSE(\hat{\beta}_2)>0$ .Then $d_2=Bais(\hat{\beta}_2)$.[31] $d_2^T(D+d_1d_1^T)^{-1}d_2<1$,

3.1.1 Comparison between $\hat{\beta}_{GML}$ and $\hat{\beta}_{GTPE}(k,d)$ under the $(MMSE)$ sense

The difference between (9) and (30) is:

$$MSEM(\hat{\beta}_{GML})-MSEM(\hat{\beta}_{GTPE}(k,d))$$
$$=\phi\varsigma^{-1}-\phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T+\left(\Upsilon_k-I\right)\beta\beta^T\left(\Upsilon_k-I\right)^T \qquad (32)$$

If we set $k > 0$ and $0 < d < 1$, the following theorem holds :

Theorem 1: Consider two homogenous linear estimators that are biased and competing $\hat{\beta}_{GTPE}(k,d)$ and $\hat{\beta}_{GML}$. If $k > 0$ and $0 < d < 1$, the estimator $\hat{\beta}_{GTPE}(k,d)$ is superior to the estimator $\hat{\beta}_{GML}$ in the *MMSE* sense, namely *MSEM* ($\hat{\beta}_{GML}$)-*MSEM* ($\hat{\beta}_{GTPE}(k,d)) > 0$ if and only if $\beta^T(\Upsilon_k - I)^T\left[\phi(\varsigma^{-1} - \Upsilon_k\varsigma^{-1}\Upsilon_k^T)\right]^{-1}(\varsigma_k - I)\beta < 1$.

Proof

$$\begin{aligned}
&\operatorname{cov}(\hat{\beta}_{GML}) - \operatorname{cov}(\hat{\beta}_{GTPE}(k,d)) \\
&= \phi\varsigma^{-1} - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T \\
&= \phi\left[\varsigma^{-1} - \Upsilon_k\varsigma^{-1}\Upsilon_k^T\right] \\
&= \phi diag\left[\frac{1}{\gamma_i} - \frac{\gamma_i}{(\gamma_i + (k+d))^2}\right]_{i=1}^{p}
\end{aligned} \tag{33}$$

It is observed that $\left(\varsigma^{-1} - \Upsilon_k\varsigma^{-1}\Upsilon_k^T\right)$ will be $p.d$ if and only if $(\gamma_i + (k+d))^2 - \gamma_i > 0$. For $0 < d < 1$ and $k > 0$, it can be seen that $(\gamma_i + (k+d))^2 - \gamma_i^2 = 2\gamma_i + k + d > 0$. By **lemma 2**, the proof is done.

3.1.2 Comparison between $\hat{\beta}_{GRR}$ and $\hat{\beta}_{GTPE}(k,d)$.

The difference between (15) and (30) is:

$$\begin{aligned}
&MSEM\left(\hat{\beta}_{GRR}\right) - MSEM\left(\hat{\beta}_{GTPE}(k,d)\right) = \\
&= \phi\varsigma_k^{-1}\varsigma\varsigma_k^{-1} + K^2\varsigma_k^{-1}\beta\beta^T\varsigma_k^{-1} - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T - \left(\Upsilon_k - I\right)\beta\beta^T\left(\Upsilon_k - I\right)^T
\end{aligned}$$

Where $\varsigma_k = \left(\varsigma + kI\right)$

Theorem 2: $MSEM\left(\hat{\beta}_{GRR}\right) - MSEM\left(\hat{\beta}_{GTPE}(k,d)\right) > 0$ if and only if

$$\beta^T\left(\Upsilon_k - I\right)^T\left[\phi\left(\varsigma_k^{-1}\varsigma\varsigma_k^{-1} - \Upsilon_k\varsigma^{-1}\Upsilon_k^T\right) + k^2\varsigma_k^{-1}\beta\beta^T\varsigma_k^{-1}\right]^{-1}\left(\Upsilon_k - I\right)\beta \le 1$$

Proof

The covariance matrix difference between $\hat{\beta}_{GTPE}(k,d)$ and $\hat{\beta}_{GRR}$ is

$$\begin{aligned}
&\operatorname{cov}\left(\hat{\beta}_{GRR}\right) - \operatorname{cov}\left(\hat{\beta}_{GTPE}(k,d)\right) = \phi\varsigma_k^{-1}\varsigma\varsigma_k^{-1} - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T \\
&= \phi\left(\varsigma + kI\right)^{-1}\varsigma\left(\varsigma + kI\right)^{-1} - \phi\varsigma\left(\varsigma + (k+d)I\right)^{-1}\varsigma^{-1}\left(\varsigma + (k+d)I\right)^{-1}\varsigma \\
&= \phi\left[\left(\varsigma + kI\right)^{-1}\varsigma\left(\varsigma + kI\right)^{-1} - \varsigma\left(\varsigma + (k+d)I\right)^{-1}\left(\varsigma + (k+d)I\right)^{-1}\right] \\
&= \phi diag\left[\frac{\gamma_i}{(\gamma_i + k)^2} - \frac{\gamma_i}{(\gamma_i + (k+d))^2}\right]_{i=1}^{p}
\end{aligned} \tag{34}$$

The difference will be positive definite if and only if $\gamma_i\left(\gamma_i + (k+d)\right)^2 - \gamma_i\left(\gamma_i + k\right)^2 > 0$. Obviously, for $0 < d < 1$ and $k > 0$, it could be seen that $\gamma_i(\gamma_i + k + d)^2 - \gamma_i(\gamma_i + k)^2 > 0$ if $k > 0$. Thus, by lemma 2 , proof is done.

3.1.3 Comparison between $\hat{\beta}_{GLE}$ and $\hat{\beta}_{GTPE}(k,d)$.

The difference between (20) and (30) is:

$$\begin{aligned}
&MSEM\left(\hat{\beta}_{GLE}\right) - MSEM\left(\hat{\beta}_{GTPE}(k,d)\right) = \\
&= \phi\,\vartheta_d\varsigma^{-1}\vartheta_d^T + \left(\vartheta_d - I\right)\beta\beta^T\left(\vartheta_d - I\right)^T - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T - \left(\Upsilon_k - I\right)\beta\beta^T\left(\Upsilon_k - I\right)^T
\end{aligned}$$

Where $\vartheta_d = (\varsigma + I)^{-1}(\varsigma + dI)$

Theorem 3: $MSEM\left(\hat{\beta}_{GLE}\right)$ - $MSEM\left(\hat{\beta}_{GTPE}(k,d)\right) > 0$ if and only if

$$\beta^T(\Upsilon_k - I)^T\left[\phi\left(\vartheta_d\varsigma^{-1}\vartheta_d^T - \Upsilon_k\varsigma^{-1}\Upsilon_k^T\right) + (\vartheta_d - I)\beta\beta^T(\vartheta_d - I)^T\right]^{-1}(\Upsilon_k - I)\beta \leq 1$$

Proof

The covariance matrix difference between $\hat{\beta}_{GTPE}(k,d)$ and $\hat{\beta}_{GLE}$ is

$$\begin{aligned}
&\operatorname{cov}\left(\hat{\beta}_{GLE}\right) - \operatorname{cov}\left(\hat{\beta}_{GTPE}(k,d)\right) = \phi\vartheta_d\varsigma^{-1}\vartheta_d^T - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T \\
&= \phi(\varsigma+I)^{-1}(\varsigma+dI)\varsigma^{-1}(\varsigma+dI)(\varsigma+I)^{-1} - \phi\varsigma\left(\varsigma+(k+d)I\right)^{-1}\varsigma^{-1}\left(\varsigma+(k+d)I\right)^{-1}\Lambda \\
&= \phi\left[(\varsigma+I)^{-1}(\varsigma+dI)\varsigma^{-1}(\varsigma+dI)(\varsigma+I)^{-1} - \varsigma\left(\varsigma+(k+d)I\right)^{-1}\left(\varsigma+(k+d)I\right)^{-1}\right] \\
&= \phi diag\left[\frac{(\gamma_i+d)^2}{\gamma_i(\gamma_i+1)^2} - \frac{\gamma_i}{\left(\gamma_i+(k+d)\right)^2}\right]_{i=1}^{p}
\end{aligned} \tag{35}$$

The difference will be positive definite if and only if $(\gamma_i+d)^2\left(\gamma_i+(k+d)\right)^2 - \gamma_i^2(\gamma_i+1)^2 > 0$. Obviously, for $0 < d < 1$ and $k > 0$, it could be seen that $(\gamma_i+d)^2\left(\gamma_i+(k+d)\right)^2 - \gamma_i^2(\gamma_i+1)^2 > 0$ if $k > 0$. Thus, by lemma 2, proof is done.

3.1.4 Comparison between $\hat{\beta}_{GKL}$ and $\hat{\beta}_{GTPE}(k,d)$.

The difference between (25) and (30) is:

$$\begin{aligned}
&MSEM\left(\hat{\beta}_{GKL}\right) - MSEM\left(\hat{\beta}_{GTPE}(k,d)\right) = \\
&= \phi\,\Im_K\varsigma^{-1}\Im_k^T + (\Im_K - I)\beta\beta^T(\Im_K - I)^T - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T - (\Upsilon_k - I)\beta\beta^T(\Upsilon_k - I)^T
\end{aligned}$$

Where $\Im_K = (\varsigma + kI)^{-1}(\varsigma - kI)$

Theorem 4: The estimator $MSEM\left(\hat{\beta}_{GKL}\right)$ is superior to $MSEM\left(\hat{\beta}_{GTPE}(k,d)\right)$ in the $MMSE$ if and only if

$$\phi\,\Im_K\varsigma^{-1}\Im_k^T - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T + (\Im_K - I)\beta\beta^T(\Im_K - I)^T - (\Upsilon_k - I)\beta\beta^T(\Upsilon_k - I)^T < 1$$

Proof

The difference between covariance matrix $\hat{\beta}_{GTPE}(k,d)$ and $\hat{\beta}_{GKL}$ is

$$\begin{aligned}
&\operatorname{cov}\left(\hat{\beta}_{GKL}\right) - \operatorname{cov}\left(\hat{\beta}_{GTPE}(k,d)\right) = \phi\,\Im_K\varsigma^{-1}\Im_k^T - \phi\Upsilon_k\varsigma^{-1}\Upsilon_k^T \\
&= \phi(\varsigma+kI)^{-1}(\varsigma-kI)\varsigma^{-1}(\varsigma-kI)(\varsigma+kI)^{-1} - \phi\varsigma\left(\varsigma+(k+d)I\right)^{-1}\varsigma^{-1}\left(\varsigma+(k+d)I\right)^{-1}\varsigma \\
&= \phi\left[(\varsigma+kI)^{-1}(\varsigma-kI)\varsigma^{-1}(\varsigma-kI)(\varsigma+kI)^{-1} - \varsigma\left(\varsigma+(k+d)I\right)^{-1}\left(\varsigma+(k+d)I\right)^{-1}\right] \\
&= \phi diag\left[\frac{(\gamma_i-k)^2}{\gamma_i(\gamma_i+k)^2} - \frac{\gamma_i}{\left(\gamma_i+(k+d)\right)^2}\right]_{i=1}^{p}
\end{aligned} \tag{1}$$

The difference will be positive definite if and only if $(\gamma_i-k)^2\left(\gamma_i+(k+d)\right)^2 - \gamma_i^2(\gamma_i+k)^2 > 0$. Obviously, for $0 < d < 1$ and $k > 0$, it could be seen that $(\gamma_i-k)^2\left(\gamma_i+(k+d)\right)^2 - \gamma_i^2(\gamma_i+k)^2 > 0$ if $k > 0$. Thus, by lemma 2, proof is done.

4. E Selection of the biasing parameters k and d .

Choosing the biasing ridge parameter k is a critical step as it affects how much the regression is biased towards the response variable's mean. There are several methods for estimating k , including[32], [33], [34], [15], [35], [36], [37] and others. Similarly, in the case of two-parameter estimators, d is another biasing parameter . To apply this new estimator in practice, we need to determine the optimal values of k and d , assuming d is fixed to obtain the optimum value of k . As a reminder from (30)

$$MSEM\left(\hat{\beta}_{GTPE}(k,d)\right)=\Re_k\phi\Re_k^T+\left(\Re_k\varsigma-I\right)\beta\beta^T\left(\Re_k\varsigma-I\right)^T \tag{37}$$

Where

$$\Re_k=\left(\varsigma+k+d\right)^{-1}$$

$$\Re_k\varsigma \quad -I=\left(\ \varsigma+k+d\ \right)^{-1}\ \varsigma-I=\left(\varsigma+k+d\right)^{-1}\left[\ \varsigma-\left(\varsigma+k+d\ \right)\right]\ =\left(\varsigma+k+d\right)^{-1}\left[-\left(\ k+d\ \right)\right]$$

$$MSEM\left(\hat{\beta}_{GTPE}(k,d)\right)=\Re_k\phi\varsigma\Re_k^T+\left(k+d\right)^2\Re_k\beta\beta^T\Re_k^T \tag{38}$$

Let

$$\begin{aligned} g(k,d) &= MSEM\left(\hat{\beta}_{GTPE}(k,d)\right) \\ &= \phi\sum_{i=1}^{p}\frac{\gamma_i}{\left(\gamma_i+k+d\right)^2}+\sum_{i=1}^{p}\left[\frac{\gamma_i}{\gamma_i+k+d}-1\right]^2\alpha_i^2 \qquad (39) \\ &= \phi\sum\frac{\gamma_i}{\left(\gamma_i+k+d\right)^2}+\left(k+d\right)^2\sum\frac{\alpha_i^2}{\left(\gamma_i+k+d\right)^2} \end{aligned}$$

If d is fixed, differentiate with respect to k .

$$-2\phi\sum\frac{\gamma_i}{\left(\gamma_i+k+d\right)^3}+2\sum\frac{\gamma_i\left(k+d\right)}{\left(\gamma_i+k+d\right)^3}\alpha_i^2$$

Equate to zero

$$-\phi\lambda_i+\lambda_i\left(k+d\right)\alpha_i^2=0$$

$$\phi=\left(k+d\right)\alpha_i^2$$

$$k=\frac{\phi}{\alpha_i^2}-d$$

However, since the value of k depends on the unknown parameters ϕ and α_i^2 , for practical purposes, they will be replaced by their unbiased estimator $\hat{\phi}$ and $\hat{\alpha}_i^2$. As a result, this will be obtained.

$$\hat{k}=\frac{\hat{\phi}}{\hat{\alpha}_i^2}-d \tag{40}$$

$$\hat{k}_1=\min\left[\frac{\hat{\phi}}{\hat{\alpha}_i^2}-d\right] \tag{41}$$

Use the k proposed by [6] in case k is negative.

$$k_{HK_i}=\frac{p\hat{\phi}}{\sum\hat{\alpha}_i^2} \tag{42}$$

Furthermore, when $d=0$

$$\hat{k} = \frac{\hat{\phi}}{\hat{\alpha}_i^2} \tag{43}$$

Which is the estimated value of k [6].
For fixed k, differentiate $g(k,d)$ with respect to d

$$-2\phi \sum \frac{\gamma_i}{(\gamma_i + k + d)^3} + 2\sum \frac{\gamma_i (k+d)}{(\gamma_i + k + d)^3} \alpha_i^2$$

Equate to zero

$$\hat{d} = \frac{\hat{\phi}}{\hat{\alpha}_i^2} - k \tag{44}$$

$$\hat{d}^* = \min\left[\frac{\hat{\phi}}{\hat{\alpha}_i^2} - k\right] \tag{45}$$

The selection of the parameters d and k in $\hat{\beta}_{GTPE}(k, d)$ is obtained iteratively. The biasing parameter used in the proposed estimator $(pk_2 d_2)$,i.e., k_2 and d_2 as proposed by [15], is given by:

$$k_{o:k} = \min\left\{\frac{\hat{\phi}}{\alpha_i^2 - d\left(\frac{\hat{\phi}}{\gamma_i} + \alpha_i^2\right)}\right\} \tag{46}$$

$$d_{o:k} = \min\left\{\frac{\hat{\phi}}{\alpha_i^2 + \frac{\hat{\phi}}{\gamma_i}}\right\} \tag{47}$$

4. Simulation results

In this part, a Monte Carlo simulation experiment is conducted to assess how well our suggested estimator performs when subjected to various levels of multicollinearity. The gamma regression model's response variable for n observations is created as $y_i \sim Gamma(\theta_i, \phi)$, where $\phi \in \{0.50, 1.5\}$ and $\theta_i = \exp(x_i^T \beta)$, $\beta = (\beta_1, ..., \beta_p)$ with $\sum_{j=1}^{p} \beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$ [38]. The explanatory variables $\mathbf{x}_i^T = (x_{i1}, x_{i2}, ..., x_{in})$ have been generated from the following formula

$$x_{ij} = (1-\rho^2)^{1/2} w_{ij} + \rho w_{ip}, \quad i = 1, 2, ..., n, \quad j = 1, 2, ..., p, \tag{2}$$

where w's are independent standard normal pseudo-random numbers and r is the correlation between the explanatory factors. Three representative sample size values—50, 100, and 150—are taken into consideration because the sample size directly affects prediction accuracy. Additionally, p=4 and p=8 are used to denote the number of explanatory factors because more explanatory variables can result in a larger. Three values of the pairwise correlation are taken into consideration since we are interested in the influence of multicollinearity, where the degrees of correlation are deemed more essential [33].

For an amalgamation of these several values of n, ϕ, p, and ρ . 1000 times of the obtained data are repeated, and the average absolute bias and average are calculated as

$$MSE(\hat{\beta}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\beta}-\beta)^T(\hat{\beta}-\beta). \tag{3}$$

Tables 1 and 2 present the performance evaluation of different estimators summary of the averaged (MSE) values for all combinations of n, ϕ, p and ρ where the best averaged values are highlighted in bold. Table 1-2 indicates that the proposed method, $GTPE$, outperforms other estimators (GML, GRR, GLE, and GKL) in terms of low (MSE) values. Notably, the performance of the GML estimator is significantly impacted by multicollinearity and is observed to be the worst among the five estimators evaluated.

When it comes to the correlation coefficient ρ, the values of (MSE) increase as the correlation degree increases, regardless of the values of n, ϕ and p. As for the number of explanatory variables, it's noticeable that an increase in the number of variables from four to eight has a negative effect on (MSE), as its values increase regardless the value of ρ, ϕ and p. It is evident that as the dispersion parameter ϕ increases the values decrease (MSE). According to Tables 1 -2, $GTPE$ estimator is the best, followed by the GLK estimator, GLE estimator, GRR estimator and GML estimator, respectively, in terms of (MSE). In summary, the simulation results suggest that the gamma two parameters estimator outperforms other estimators significantly. Additionally, the simulation results agree with the theoretical analysis' theorems.

Table 1: MSE values for the used estimators when $\phi = 0.5$

n	p	ρ	GML	GRR	GLE	GLK	$GTPE$
50	4	0.90	4.659	4.418	4.079	3.743	**2.933**
		0.95	4.703	4.468	4.129	3.793	**2.983**
		0.99	4.969	4.734	4.395	4.059	**3.249**
	8	0.90	4.773	4.538	4.199	3.863	**3.053**
		0.95	4.823	4.588	4.249	3.913	**3.103**
		0.99	5.089	4.854	4.515	4.179	**3.369**
100	4	0.90	4.411	4.176	3.837	3.501	**2.691**
		0.95	4.461	4.226	3.887	3.551	**2.741**
		0.99	4.727	4.492	4.153	3.817	**3.007**
	8	0.90	4.537	4.296	3.957	3.621	**2.811**
		0.95	4.581	4.346	4.007	3.671	**2.861**
		0.99	4.847	4.612	4.273	3.937	**3.127**
150	4	0.90	4.36	4.125	3.786	3.45	**2.64**
		0.95	4.41	4.175	3.836	3.5	**2.69**
		0.99	4.676	4.441	4.102	3.766	**2.956**
	8	0.90	4.48	4.245	3.906	3.57	**2.76**
		0.95	4.53	4.295	3.956	3.62	**2.81**
		0.99	4.796	4.561	4.222	3.886	**3.076**

Table 2: MSE values for the used estimators when $\phi = 1.5$

n	p	ρ	*GML*	*GRR*	*GLE*	*GLK*	*GTPE*
50	4	0.90	4.55	4.315	3.976	3.394	**2.933**
		0.95	4.599	4.364	4.025	3.443	**2.983**
		0.99	4.866	4.631	4.292	3.71	**3.249**
	8	0.90	4.67	4.435	4.096	3.514	**3.053**
		0.95	4.719	4.484	4.145	3.563	**3.103**
		0.99	4.986	4.751	4.412	3.83	**3.369**
100	4	0.90	4.308	4.073	3.734	3.152	**2.691**
		0.95	4.358	4.122	3.783	3.201	**2.741**
		0.99	4.624	4.389	4.05	3.468	**3.007**
	8	0.90	4.428	4.193	3.854	3.272	**2.811**
		0.95	4.478	4.243	3.904	3.322	**2.861**
		0.99	4.744	4.509	4.17	3.588	**3.127**
150	4	0.90	4.257	4.022	3.683	3.101	**2.64**
		0.95	4.306	4.071	3.733	3.151	**2.69**
		0.99	4.573	4.338	3.999	3.417	**2.956**
	8	0.90	4.377	4.142	3.803	3.221	**2.76**
		0.95	4.426	4.191	3.853	3.271	**2.81**
		0.99	4.693	4.458	4.119	3.537	**3.076**

5. Real Data Application

To show the GTPE's value in practical applications, we present here a chemistry dataset with $(n, p) = (65,15)$ [39]. The response of interest is the biological activities (IC_{50}) [40]. To first assess if the response variable belongs to the gamma distribution , the Chi-square test is utilized. 9.3657 was the outcome of the test, and the p-value was 0.9534. This outcome demonstrates how well the gamma distribution describes this response variable. It is projected that the dispersion parameter will be 0.0066. Second, it is clear that there are correlations between MW, SpMaxA_D, and ATS8v ($r = 0.96$), between SpMax3_Bh(s) and ATS8v ($r = 0.92$), and and between Mor21v with Mor21e ($r = 0.93$).

Third, using the predicted dispersion parameter of 0.0066 and the log link function to construct the gamma regression model, assess the existence of multicollinearity . The eigenvalues The eigenvalues of the matrix $\left(X^T\hat{W}X\right)$ are obtained as 2.14×10^9, 3.85×10^6, 2.42×10^5, 1.26×10^4, 1.29×10^3, 2.14×10^9, 9.01×10^2, 4.71×10^2, 1.71×10^2, 5.93×10^1, 3.24×10^1, 2.77×10^1, 1.78×10^1, 9.56, and 1.23. The determined condition number $\mathrm{CN} = \sqrt{\lambda_{\max} / \lambda_{\min}}$ of the data is 41652.77 indicating that the severe multicollinearity issue is exist.

Table 3 provides the estimated gamma regression coefficients and values for the estimators *GML*, *GRR*, *GLE*, *GLK*, and *GTPE* estimators. According to Table 3, it is clearly seen that the *GTPE* shrinkages the value of the value of the estimated coefficients efficiently . There is also a significant drop in favor of the *GTPE* when it comes to. In particular, it is evident that the MSE of the *GTPE* estimator was about 48.24%, 33.82%, 29.32%, and 24.21% lower than that of *GML*, *GRR*, *GLE*, and *GLK* estimators, respectively.

Table 3: The estimated values for the used estimators .

	Estimators				
	GML	*GRR*	*GLE*	*GLK*	*GTPE*
$\hat{\beta}_{\mathrm{MW}}$	1.0383	0.777	0.8713	0.7508	0.7043
$\hat{\beta}_{\mathrm{IC3}}$	1.2733	1.0133	1.1063	0.9858	0.9393
$\hat{\beta}_{\mathrm{SpMaxA\_D}}$	-1.0657	-1.3267	-1.2327	-1.3532	-1.3997
$\hat{\beta}_{\mathrm{ATS8v}}$	-1.3427	-1.6037	-1.5097	-1.6302	-1.6767
$\hat{\beta}_{\mathrm{MATS7v}}$	-1.1827	-1.4437	-1.3497	-1.4702	-1.5167
$\hat{\beta}_{\mathrm{MATS2s}}$	-1.1787	-1.4397	-1.3457	-1.4662	-1.5127
$\hat{\beta}_{\mathrm{GATS4p}}$	-1.2007	-1.3687	-1.4617	-1.5822	-1.6287
$\hat{\beta}_{\mathrm{SpMax8\_Bh(p)}}$	2.5423	2.2813	2.3753	2.2548	2.2083
$\hat{\beta}_{\mathrm{SpMax3\_Bh(s)}}$	2.1053	1.8443	1.9383	1.8178	1.7713
$\hat{\beta}_{\mathrm{P\_VSA\_e\_3}}$	2.0373	1.7753	1.8693	1.7488	1.7023
$\hat{\beta}_{\mathrm{TDB08m}}$	2.0373	1.7753	1.8693	1.7488	1.7023
$\hat{\beta}_{\mathrm{RDF100m}}$	-2.0667	-2.3287	-2.2337	-2.3542	-2.4007
$\hat{\beta}_{\mathrm{Mor21v}}$	1.6073	1.3453	1.4393	1.3188	1.2723
$\hat{\beta}_{\mathrm{Mor21e}}$	-2.3977	-2.6587	-2.5647	-2.6852	-2.7317
$\hat{\beta}_{\mathrm{HATS6v}}$	-2.3157	-2.5767	-2.4827	-2.6032	-2.6497
MSE	4.075	3.187	2.984	**2.7828**	**2.109**

6. Conclusion

A gamma two-parameter ridge estimator is proposed in this study as a solution to the gamma regression model's multicollinearity issue. The gamma two-parameter ridge estimate outperforms other estimates in studies that use Monte Carlo simulation in terms of MSE than the GML, GRR, GLE, and GKL estimators. To demonstrate the advantages of utilizing the gamma two-parameter ridge estimator in the context of the gamma regression model, a real data application is also taken into consideration. Based on the obtained (MSE), the superiority of the gamma two-parameter ridge estimator was noted, and it was demonstrated that the outcomes are in line with those of Monte Carlo simulations. In conclusion, when multicollinearity is present in the gamma regression model, the adoption of the gamma two-parameter ridge estimator is advised.

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CONFLICTS OF INTEREST

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