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Dynamical Symmetry and B(E2) Transitions of $^{94}_{42}Mo$ by Utilizing (IBM-1)

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التماثل الديناميكي وانتقالات B(E2) لـ $^{94}_{42}$Mo باستخدام (IBM-1)

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ABSTRACT

Background:
In this study, we utilize the IBM-1 model to theoretically investigate the structural characteristics of $^{94}_{42}Mo$, with a specific emphasis on a nucleus consisting of 5 bosons. Employing the IBM-1, an innovative interacting boson model, we analyze the nuclear structure.

Materials and Methods:
In our theoretical exploration, we delved into the nuclear architecture of Molybdenum. Employing the Interacting Boson Model (IBM-1), we identified the pertinent Hamiltonian to scrutinize the $^{94}_{42}Mo$ isotope in this research endeavor.

Results:
Our investigation delved into the $\frac{E4^+_1}{E2^+_1}$ symmetry limit of the proposed nucleus, unveiling its stable nature within the SU(5) dynamical symmetry domain. To optimize alignment with observed energy states in experiments, we meticulously determined the parameters in the IBM-1 Hamiltonian equation. Furthermore, our research extends to the computation of B(E2) transitions, quadrupole moments, and energy levels across all bands for the examined nucleus.

Conclusions:
Our investigation reveals the stability of the nucleus in the $^{94}_{42}Mo$ isotope, showcasing the presence of SU(5) dynamical symmetry. These findings offer fresh perspectives by suggesting additional electric quadrupole moments.

Keywords: IBM-1, dynamical symmetry, B(E2) transitions, Energy levels

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INTRODUCTION

The Interacting Boson Model-1 (IBM-1), pioneered by Arima and Iachello, serves as a comprehensive framework for investigating the structural intricacies and characteristics of even-even nuclei, shedding light on collective excitations within atomic nuclei [1]. By leveraging the algebraic constraints of boson operators, IBM-1 facilitates analytical interpretations of configurations and expectation values for the optimal bounds of nuclei in three distinct categories. The total number of bosons in IBM-1 is determined by the presence of proton and neutron pairs beyond their respective closed shells, with no differentiation between proton-type and neutron-type bosons [2]. IBM-1 is akin to a (s-d) boson system comprising six components, which can be conceptualized within a six-dimensional space. This model represents a sophisticated and advanced approach, offering valuable insights into the intricate nature of nuclear structure and dynamics [3]. Diverse forms of U(6) provide conception to a plethora of dynamic geometries in the field of group theory, each with relevance in the contexts of the vibrator SU(5), deformed rotor SU(3), and (γ-soft) axial symmetry O(6) [3, 4]. The significance of dynamical symmetry in nuclear structure is best understood via the view of the interacting boson model, in which these many U(6) configurations play a critical role in determining the system's complicated dynamics. This demonstrates the improved understanding of nuclear structure obtained from the interaction of group theory and the interacting boson model [4]. The energy of the second excited state relative to the first excited state was explored as the best technique for identifying the dynamical symmetry of the considered nucleus [4, 5]. To determine the overall count of bosons, it is essential to associate protons and neutrons with the nearest magic number. The ${}^{94}_{42}$Mo isotope possesses 5 bosons and is in close proximity to the magic number for neutrons, specifically 50. Moreover, the quantity of protons in the open shell is less than the magic number of 50. With 42 protons and 52 neutrons, the ${}^{94}_{42}$Mo nucleus illustrates that its proton bosons correspond to pairs of holes, while its neutron bosons correspond to pairs of particles [6, 7]. The study of ${}^{94}_{42}Mo$'s nuclear structure, particularly within the context of theoretical frameworks like the Interacting Boson Model-1, not only serves as a benchmark for advancing our understanding of exotic and neutron-rich nuclei but also holds implications for astrophysical processes, technological applications, and methodological advancements, making it a compelling subject for researchers seeking insights into nuclear physics. This study aims to evaluate the nuclear structure of ${}^{94}_{42}Mo$, encompassing aspects such as energy levels, B(E2) transitions, dynamical symmetry, nuclear shape, and quadrupole moment values. This assessment is conducted within the framework of the broader IBM-1 paradigm.

The origin of IBM-1

The Hamiltonian within the framework of the Interacting Boson Model-1 characterizes the interactions between pairs of s-bosons and d-bosons within the atomic nucleus. The IBM-1 model employs a straightforward Hamiltonian, consisting of six parameters derived from experimental data. This Hamiltonian operator ($\hat{H}$) incorporates both one-body and two-body operators [4, 5]

$$\hat{H} = \varepsilon_s s^\dagger \tilde{s} + \varepsilon_d \sum_m d^\dagger \tilde{d} + V \quad (1)$$

Where ε_s , and ε_d are s and d single – boson energies, V is boson-boson interaction potential, $s^\dagger$ ($\tilde{s}$) are creation and annihilation operators for the s-state, (s-boson), and $d^\dagger$($\tilde{d}$) are creation (annihilation) operators for the d-state, (d-boson). The IBM-1 Hamiltonian is indicated to be [6,7].

$$\hat{H} = \varepsilon (n_d) + a_0 (\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3 (\hat{T}_3.\hat{T}_3) + a_4 (\hat{T}_4.\hat{T}_4) \quad (2)$$

Where ε, a_o, a_1, a_2 , a_3 and a_4 are the model parameters, P, L, Q, T_3 and T_4 are the pairing, angular momentum, quadrupole, octopole and hexadecapole operators respectively. n_d is the d-boson number operator, and all operators in the Hamiltonian are the following

Pairing operator is

$$\hat{P} = \frac{1}{2}[(\tilde{d}.\tilde{d}) - (\tilde{s}.\tilde{s})] = \frac{1}{2}(\tilde{d}^2 - \tilde{s}^2) \quad (3)$$

$$T_l = [d^\dagger \otimes \tilde{d}]^l \quad l=0,1,2,3,4,\ldots \quad (4)$$

Angular momentum operator is

$$\hat{L} = \sqrt{10}\,[d^\dagger \otimes \tilde{d}]^1 = \sqrt{10}\hat{T}_1 \quad (5)$$

Quadrupole moment operator is

$$\hat{Q} = [d^\dagger \otimes \tilde{s} + s^\dagger \otimes \tilde{d}]^2 - \frac{\sqrt{7}}{2}[d^\dagger \otimes \tilde{d}]^2 \quad (6)$$

Octapole operator is

$$\hat{T}_3 = [d^\dagger \otimes \tilde{d}]^3 \quad (7)$$

Hexadecapole operator is

$$\hat{T}_4 = [d^\dagger \otimes \tilde{d}]^4 \quad (8)$$

Number of d-boson operator is

$$\hat{n}_d = \sqrt{5}\hat{T}_0 \quad (9)$$

In this form there appear terms that have, at least superficially, a more physical connotation, specifically an angular momentum operator, a quadrupole operator, octuplet and hexadecapole terms, as well as the so-called pairing operator P. Note, however, that these are operators acting on boson states, not in the fermion space. It is in this form, therefore, that we shall usually consider the application of the IBA-1 Hamiltonian to the set of basis states described earlier. The electromagnetic properties encapsulated by E2 transitions are of paramount importance, with

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B(E2) values computed using the E2 algorithm. Ensuring the conservation of the total boson count is imperative due to the nature of the E2 transition operator, characterized as a Hermitian tensor of rank two. This constraint holds significance, as it allows the expression of the universal E2 activator within these defined parameters [7],

$$T(E2) = e_B \; [(d^\dagger \tilde{s} + s^\dagger \tilde{d}) + \chi(d^\dagger \tilde{d})^{(2)}] = \alpha_B Q \qquad (10)$$

Where e_B plays the role of the effective boson charge. The parameter χ determines the relative importance of the two terms. The *E2* operator, which is identical in form to the *Q* operator in the Hamiltonian, consists of one piece that changes n_d by unity and another that leaves n_d unchanged, the ratio of the two terms being given by the parameter χ.

RESULTS AND DISCUSSION

An examination and elucidation of the energy levels throughout all bands of ${}^{94}_{42}Mo$ were carried out by meticulously adjusting the Hamiltonian parameters for the optimum match. This included incorporating the specified configurations from Table (1) into IBM-1's Hamiltonian equations. The resulting low-lying energy levels remained consistently below the SU(5) limit. The IBM-1 Hamiltonian was designed and calculated inside an SU(5) framework using the computer program PHINT [8]. The parameters, which were considered independent variables, were fine-tuned to exactly recreate the energy excitations of each positive parity level.

Table 1: Hamiltonian assumptions variable used in the current work for IBM-1 estimation pertain to the ${}^{94}_{42}Mo$

IBM-Parameter variable -MeV-	
$\boldsymbol{EPS\ (\varepsilon)}$	**0.74000**
$\boldsymbol{P.P}$	**0.00680**
$\boldsymbol{L.L}$	**0.00185**
$\boldsymbol{Q.Q}$	**0.01200**
$\boldsymbol{T3.T3}$	**-0.14100**
$\boldsymbol{T4.T4}$	**0.11900**

Table (1) showcases a remarkable alignment with the Hamiltonian components outlined in Eq.(2), utilized in this calculation. This precise alignment results in a high level of agreement between the calculated energy values and the experimental observations for ${}^{94}_{42}$Mo [9, 10], as visually demonstrated in Figure (1).

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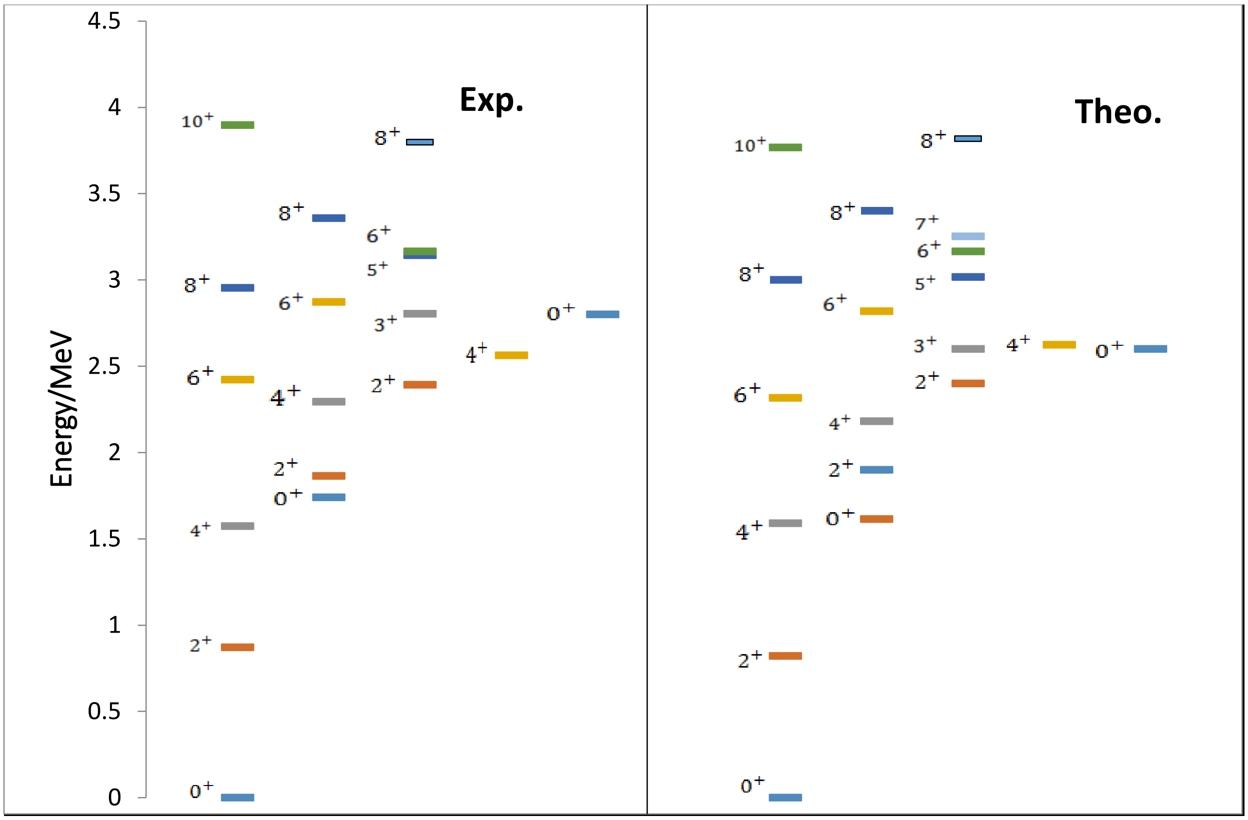

Figure (1): A juxtaposition of the measured as well as IBM-1 energy levels of ${}^{94}_{42}Mo$

In exploring the excited state of the examined ${}^{94}_{42}\mathrm{Mo}$, the E2 boson operator within IBM-1, along with its E2 transition values and quadrupole moments, were calculated using equation (10). The calculation of the IBM-1's effective charge e_B was based on the normalization to the experimental data obtained from trials for $B(E2{:}\,2_1^+ \rightarrow 0_1^+)$. Table 2 displays the B(E2) values employed in the present study, utilizing the IBMT-code.

Table 2: The juxtaposition of the estimated B(E2) transition (in unit e^2b^2) of ${}^{94}_{42}Mo$ with the measured amounts [7, 11], as well as quadrupole moments.

Parameter	$\alpha_B = 0.55300$ e. b.	
Transitions	Experimental	Theoretical
$2_1^+ \rightarrow 0_1^+$	1.40001	1.4146
$2_1^+ \rightarrow 0_2^+$		0.0957
$2_2^+ \rightarrow 0_1^+$	0.035	0.0242
$2_2^+ \rightarrow 0_2^+$		0.2032
$2_3^+ \rightarrow 0_1^+$	0.192	0.0011
$2_3^+ \rightarrow 0_2^+$		0.7665
$2_2^+ \rightarrow 2_1^+$		2.6212
$2_3^+ \rightarrow 2_1^+$	0.027	0.0002
$2_3^+ \rightarrow 2_2^+$		0.0192
$4_1^+ \rightarrow 2_1^+$	2.260	2.1441
$4_1^+ \rightarrow 2_2^+$		0.0924
$4_1^+ \rightarrow 2_3^+$		0.0507
$4_2^+ \rightarrow 2_1^+$		0.0275
$4_2^+ \rightarrow 2_2^+$		1.5132
$4_2^+ \rightarrow 2_3^+$		0.1321
$4_3^+ \rightarrow 2_2^+$		0.0476
$4_3^+ \rightarrow 2_3^+$		0.0198
$4_2^+ \rightarrow 4_1^+$		1.4989
$6_1^+ \rightarrow 4_1^+$		2.1753
$8_1^+ \rightarrow 6_1^+$		1.6064
$Q2_1^+$	-0.1300	-0.1580
$Q2_2^+$		-0.0572
$Q2_3^+$		-0.8135
$Q4_1^+$		-1.6713

The deductions regarding the estimates of electric quadrupole transition probability in the current study align well with the measured values. The identification of new B(E2) transition probabilities in ${}^{94}_{42}$Mo significantly enhances our grasp of its nuclear structure, characterizing its shape, collective vibrational motion, and excitation spectrum, while validating theoretical models and providing crucial insights into nuclear dynamics, stability, and implications for nuclear astrophysics, ultimately deepening our understanding of ${}^{94}_{42}$Mo's nuclear characteristics within the broader field of nuclear physics. Notably, the exceptional outcome arises from the ratio of

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excited states to the initial state, as determined by IBM-1 for ${}^{94}_{42}$Mo. This alignment with experimental results is clearly depicted in Figure (2). The findings from this study indicate that the molybdenum isotope behaves as a vibrational nucleus within the SU(5) chain group, given its proximity to stable nuclei and the ratio of $\frac{E4_1^+}{E2_1^+}$ being close to 1.93, a rough match to experimental data [10]. Overall, the IBM-1 calculations for the aforementioned ratio values closely correspond to the estimates derived from experimental energy ratios.

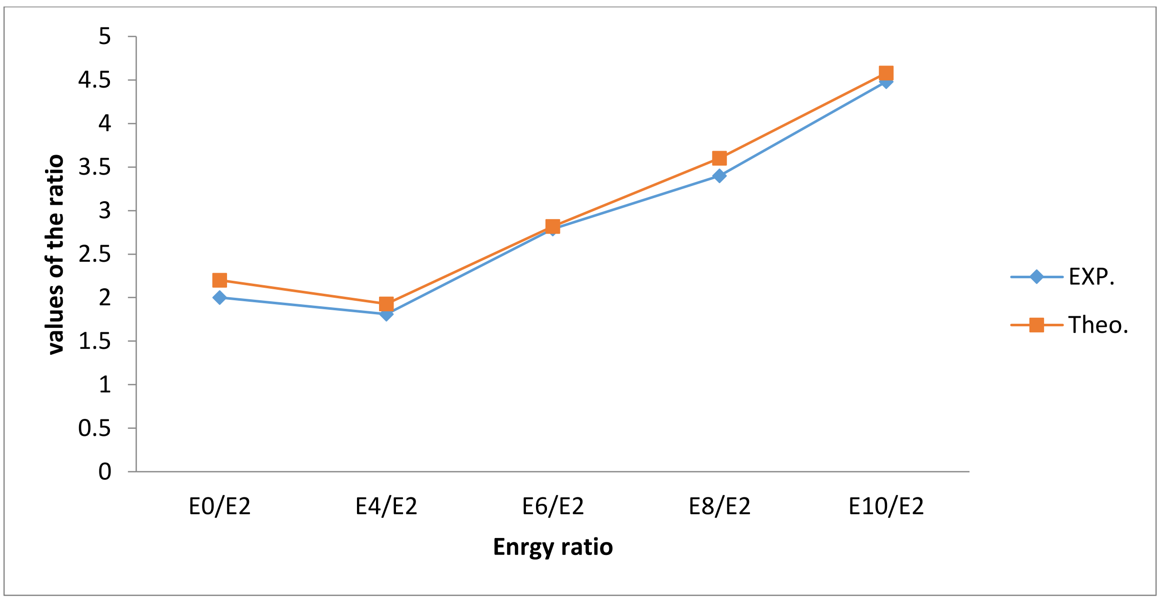

Figure (2): Energy ratios are compared among theoretical and experimental results [12, 13] for ${}^{94}_{42}Mo$.

As mentioned earlier, it is crucial to analyze the overall Hamiltonian within the framework of the U(6) group, encompassing all unitary transformations in six dimensions. Our current work indicates that the nucleus ${}^{94}_{42}Mo$ aligns closely with the vibrational chain group, exhibiting complete stability and it exhibits the highest level of conformity with the experimental data. The dynamical symmetry characteristics within the IBM-1 model can be calculated by knowing the ratio $\frac{E4_1^+}{E2_1^+}$. When this ratio closely approximates 2, the nuclei demonstrate behavior akin to SU(5) symmetry, indicating stability. The stability of molybdenum in the SU(5) group chain is visually illustrated in Figure (3).

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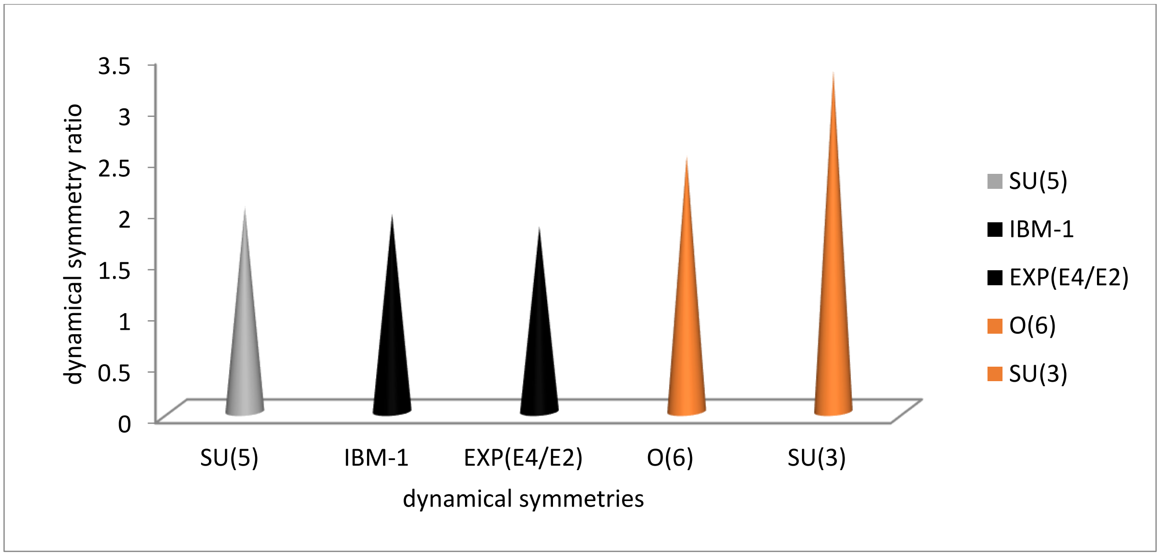

Figure (3): comparison of experimental and theoretical values of the energy ratios $(\frac{E4_1^+}{E2_1^+})$ according to SU(5) dynamical symmetry

The favorable alignment observed in the comparison of theoretical and experimental B(E2) values has motivated us to extend these findings to neutron-rich nuclei. B(E2) transition ratios serve as valuable tools for elucidating nuclear structural characteristics. These B(E2) ratios provide evidence that ${}^{94}_{42}$Mo exhibits dynamical symmetry within the SU(5) chain group, indicating proximity to stability. Figures (4) draw a comparison between the observed [11] and IBM-1 electric transition ratios for the molybdenum isotope.

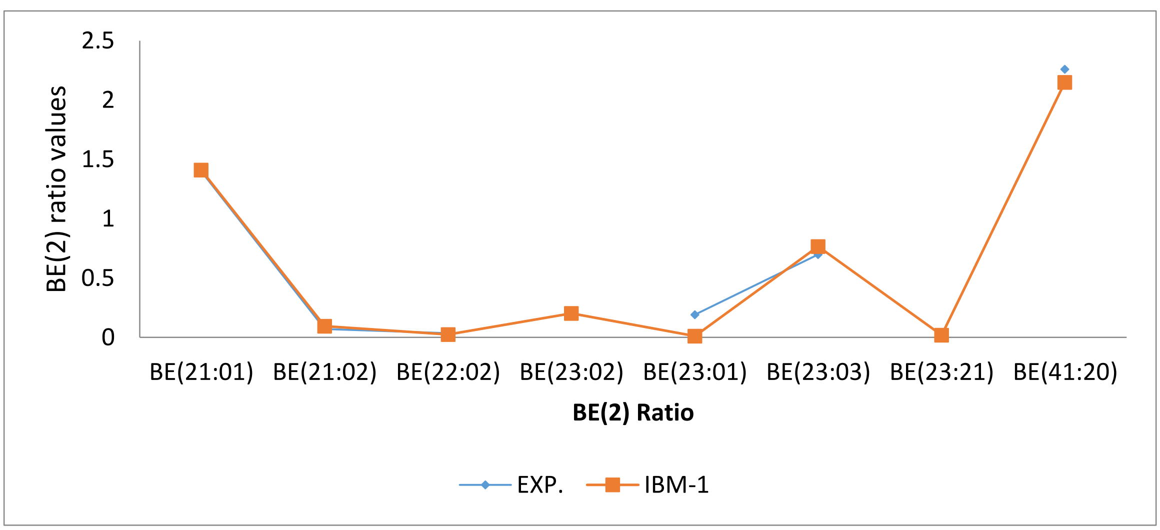

Figure 4: the contrast of experimental and IBM-1 ratios of $B(E2: j_i^+ \rightarrow j_f^+)$ for ${}^{94}_{42}Mo$ nuclide

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In Figure (4) of the investigation, a direct juxtaposition is undertaken, contrasting the experimentally observed B(E2) transition ratios with those derived from IBM-1 calculations for the molybdenum isotope. This graphical representation serves to emphasize the degree of concurrence between the theoretical forecasts and the empirical measurements, thereby reinforcing the credibility and applicability of the IBM-1 model in elucidating the nuclear structure of $^{94}_{42}Mo$.

CONCLUSION

Conclusively, our analysis reveals that the $^{94}_{42}Mo$ isotope exhibits a stable nucleus with five bosons, showcasing SU(5) dynamical symmetry. The closely aligned $\frac{E4_1^+}{E2_1^+}$ ratio of 1.93 and $\frac{E6_1^+}{E2_1^+}$ ratio of 2.81, in proximity to experimental data for a vibrator nucleus in the SU(5) group chain, underscores the robustness of our findings. The high level of consistency in the results underscores the efficacy of the IBM-1 model in accurately describing the energy levels of this isotope. Furthermore, our study indicates the need for additional investigation into the B(E2) values of the $^{94}_{42}Mo$ isotope to elucidate the strength of E2 transitions. The present findings introduce novel insights, proposing additional electric quadrupole moments and three new quadrupole moments, hitherto lacking experimental data. These discoveries offer promising avenues for further exploration of the proposed isotope, enhancing our understanding of its nuclear structure

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Conflict of interests.

There are non-conflicts of interest.

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الخلاصة

مقدمة:

في هذه الدراسة، نستخدم نموذج IBM−1 للتحقيق نظريًا في الخصائص الهيكلية لـ $^{94}_{42}Mo$، مع التركيز بشكل خاص على نواة تتكون من 5 بوزونات. باستخدام IBM−1، وهو نموذج بوزون تفاعلي مبتكر، نقوم بتحليل البنية النووية.

طرق العمل:

في استكشافنا النظري، بحثنا في البنية النووية للموليبدينوم. باستخدام نموذج البوزون المتفاعل (IBM−1)، حددنا هاملتونيان ذو الصلة لفحص نظير $^{94}_{42}Mo$ في هذا المسعى البحثي.

الاستنتاجات:

بحثنا في حد التناظر $\frac{E4_1^+}{E2_1^+}$ للنواة المقترحة، وكشف النقاب عن طبيعتها المستقرة داخل مجال التناظر الديناميكي SU(5). لتحسين التوافق مع حالات الطاقة المرصودة في التجارب، قمنا بتحديد المعلمات بدقة في معادلة IBM−1 هاملتونيان. علاوة على ذلك، يمتد بحثنا إلى حساب التحولات B(E2) ، والعزوم الرباعية، ومستويات الطاقة عبر جميع النطاقات للنواة التي تم فحصها.

الكلمات المفتاحية: IBM−1، التناظر الديناميكي، التحولات B(E2) ، مستويات الطاقة

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