Pairwise locally compact and compactification in double topological spaces

Rewayda R. Mohsin,
Math-Dep. Coll. of Math. and Com. Sci. .
University of Kufa, Najaf , Iraq
Email: ruwaida.mohsin@uokufa.edu.iq

Received Dec. 2, 2018. Accepted for publication Dec. 3, 2018

Abstract *The concept of intuitionistic topological space was introduced by Çoker. The aim of this paper is to generalize notions between bitopological spaces and double topological spaces and also give a notion of pairwise locally compact and compactification for double-topological spaces.*

Keywords Pairwise; Locally compact; Compactification; Topology space.

1-Introduction

The concept of a fuzzy topology introduced by Çhange[8] , after the introduction of fuzzy sets by Zadeh . Later this concept was extended to intuitionistic fuzzy topological spaces by Çoker in [10]. In [13] Coker studied continuity, connectedness, compactness and separation axioms in intuitionistic fuzzy topological spaces. In this paper we follow the suggestion of J.G. Garcia and S.E. Rodabaugh [15] that (double fuzzy set)is a more appropriate name than (intuitionistic fuzzy set) ,and therefore adopt the term (double-set) for the intuitionistic set , and (double-topology) for the intuitionistic topology of Dogan Çoker , (this issue) we denote by **Dbl-Top** the construct (concrete texture over Set) whose objects are pairs (X,τ) where τ is a double-topology on X .In Section three we discuss making use of this relation between bitopological spaces and double-topological spaces , we generalize a nation of locally compact and compactification for double- topological space in section four .

2-Preliminaries

Throughout the paper by X we denote a non-empty set. In this section we shall present various fundamental definitions and propositions. The following definition is obviously inspired by Atanassov [3].

2.1. Definition. [9] A double-set (Ds in brief) A is an object having the form A= <x, A_1, A_2 >, Where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of non- members of A.
Throughout the remainder of this paper we use the simpler A= (A_1, A_2) for a double-set.

2.2. Remark. Every subset A of X is may obviously be regarded as a double-set having the form $A' = (A, A^c)$, where $A^c = X\backslash A$ is the complement of A in X.
We recall several relations and operations between DS'^s as follows:

2.3. Definition. [9] Let the DS's A and B on X be the form A = (A_1, A_2), B= (B_1, B_2), respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of DS's in X, where $A_j = (A_j^{(1)}, A_j^{(2)})$. Then

(a) $A \subseteq B$ *if and only if* $A_1 \subseteq B_1$ *and* $A_2 \supseteq B_2$

(b) $A \supseteq B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$;

(c) $\overline{A} = (A_2, A_1)$ denotes the complement of A ;

(d) $\bigcap A_j = (\bigcap A_j^{(1)}, \bigcup A_j^{(2)})$;

(e) $\bigcup A_j = (\bigcup A_j^{(1)}, \bigcap A_j^{(2)})$;

(f) $[]A = (A_1, A_1^c)$;

(g) $\langle\rangle A = (A_2^c, A_2)$;

(h) $\underset{\sim}{\phi} = (\phi, X)$ and $\underset{\sim}{X} = (X, \phi)$.

In this paper we require the following:

(i) $()A = (A_1, \phi)$ and $)(A = (\phi, A_2)$.

Now we recall the image and preimage of DS's under a function.

2.4. Definition. [9, 12] Let $x \in X$ be a fixed element in X. Then:

(a)The DS given by $\underset{\sim}{x} = (\{x\}, \{x\}^c)$ is called a double–point (DP in brief X).

(b)The DS $\underset{\approx}{x} = (\phi, \{x\}^c)$ is called a vanishing double-point (VDP in brief X).

2.5. Definition. [9, 12]

(a) Let $\underset{\sim}{x}$ be a DP in X and A= (A_1, A_2) be a DS in X. Then $\underset{\sim}{x} \in A$ iff $x \in A_1$.

(b) Let $\underset{\approx}{x}$ be a VDP in X and A = (A_1, A_2) a DS in X. Then $\underset{\approx}{x} \in A$ iff $x \notin A_2$.

It is clear that $\underset{\sim}{x} \in A \Leftrightarrow \underset{\sim}{x} \subseteq A$ and that $\underset{\approx}{x} \in A \Leftrightarrow \underset{\approx}{x} \subseteq A$.

2.6. Definition. [13] A double-topology (DT in brief) on a set X is a family τ of DS's in X satisfying the following axioms:

T1: $\underset{\sim}{\phi}, \underset{\sim}{X} \in \tau$,

T2: $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

T3: $\bigcup G_j \in \tau$ for any arbitrary family $\{G_j : j \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a double-topological space (DTS in brief), and any DS in τ is known as a double open set (DOS in brief). The complement $\overline{A}$ of a DOS A in a DTS is called a double closed set (DCS in brief) in X.

2.7. ***Definition.*** [13] Let (X, τ) be a DTS and A = (A_1, A_2) be a DS in X.

Then the interior and closure of A are defined by:

$$\text{int}(A) = \bigcup\{G : G \text{ is a DOS in } X \text{ and } G \subseteq A\},$$

$$cl(A) = \bigcap(H : H \text{ is a DCS in } X \text{ and } A \subseteq H\},$$

Respectively.

It is clear that cl(A) is a DCS in and int(A) a DOS in X . Moreover, A is a DCS in X iff cl(A) = A , and A is a DOS in X iff int(A) = A.

2.8. ***Example***. [13] Any topological space (X, τ_0) gives rise to a DT of the form $\tau = \{A' : A \in \tau_0\}$ by identifying a subset A in X with its counterpart A$'$= (A, A^c), as in Remark 2.2.

3- The Constructs Dbl-Top and Bitop: [7]

We begin recalling the following result which associates a bitopology with a double topology.

3.1. ***Proposition.*** [13] Let (X, τ) be a DTS.

(a) $\tau_1 = \{A_1 : \exists A_2 \subseteq X \text{ with } A = (A_1, A_2) \in \tau\}$ is a topology on X.

(b) $\tau_2^* = \{A_2 : \exists A_1 \subseteq X \text{ with } A = (A_1, A_2) \in \tau\}$ is the family of closed sets of the topology $\tau_2 = \{A_2^c : \exists A_1 \subseteq X \text{ with } A = (A_1, A_2) \in \tau\}$ on X.

(c) Using (a) and (b) we may conclude that (X, τ_1, τ_2) is a bitopological space.

3.2. Proposition. **[7]** Let *(X,u,v)* be a bitopological space. Then the family

$$\{(U, V^c) : U \in u,\ V \in v, U \subseteq V\}$$

Is a double topology on X.

3.3. Definition. **[7]** Let *(X,u,v)* be a bitopological space. Then we set

$$\tau_{uv} = \{(U, V^c) : U \in u,\ V \in v, U \subseteq V\}$$

And call this the double topology on X associated with *(X,u,v)* .

3.4. Proposition. **[7]** If *(X,u,v)* is a bitopological space and τ_{uv} the corresponding DT on X, then $(\tau_{uv})_1 = u$ and $(\tau_{uv})_2 = v$.

4- Locally compact and compactification in double-topological space

In this section we introduced the definition of pairwise locally compact and pairwise compactification in double-topological space and prove some theorems through it.

4.1. Definition. [19] By a double open cover of a subset A of a double topological space (X,τ), we mean a collection $C=\{G_j : j\in J\}$ of double open subsets of X such that $A\subset\bigcup\{G_j : j\in J\}$ then we say that C covers A. In particular, A collection C is said to be an open cover of the space X iff $X=\bigcup\{(G_j^1,G_j^2) : j\in J\}$ of double open subsets of X.

4.2. Definition. [19] A double-set A of DTS in (X,τ) is said to be double compact set iff for every double open cover there is double sub cover , that is iff for every collection $\{G_j : j\in J\}$ of DOS's for which $A\subset\{G_j : j\in J\}\ \ for\ A=(A_1,A_2)$ such that $(A_1,A_2)\subset(G_{j1}^1,G_{j1}^2)\bigcup\ldots\bigcup(G_{jn}^1,G_{jn}^2)$.

4.3. Definition. Let (X,τ) be double topological space and (X^*,τ^*) be double compact topological space such that X is homeomorfic to a double dense subspace of X*. Then (X^*,τ^*) is called a double compactification of (X,τ). Thus a compact space (X^*,τ^*) is a double compactification of (X,τ) iff there exists a mapping *f* of X into X* such that *f* is a homeomorphism of X onto the subspace f[X] of X* and f [X] is dense in X*.

4.4. Proposition. Let (X,τ) be a non – double compact topological space and let $X^*=X\bigcup\{\infty\}$ where ∞ is an object not belonging to X. Let τ^* consist of all those double subsets G* of X* such that either $G^*\in\tau\ or\ X^*-G^*\ a\ \tau$- double closed and double compact subset of X. Then τ^* is a double topology for X* such that the space (X^*,τ^*) is double compact and (X,τ) is double dense subspace of (X^*,τ^*).

Proof: We first show that τ^* is a double topological for X*.

[T$_1$]: Since $\underset{\sim}{\phi}=(\phi,X)\in\tau$, by definition of $\tau^*,\underset{\sim}{\phi}\in\tau$. Also since $\underset{\sim}{X}^*-\underset{\sim}{X}^*=\underset{\sim}{\phi}$ and $\underset{\sim}{\phi}$ is double closed and double compact subset of X, we have $\underset{\sim}{X}^*\in\tau^*$.

[T$_2$]: Let $G_1^*,G_2^*\in\tau^*$ where $G_1^*=(G_1^{1*},G_2^{1*}),\ G_1^*=(G_1^{2*},G_2^{2*}),$

Three cases arise

(i) If $G_1^*,G_2^*\in\tau^*$ then

$(G_1^{1*},G_2^{1*})\bigcap(G_1^{2*},G_2^{2*})=$

$(G_1^{1*}\bigcap G_1^{2*},\ G_2^{1*}\bigcup G_2^{2*})$

$\because G_1^{1*}\bigcap G_1^{2*}\in\tau\ and\ \ G_2^{1*}\bigcup G_2^{2*}\in\tau\Rightarrow$

$G_1^*\bigcap G_2^*\in\tau$

And hence by definition of

$\tau^*,G_1^*\bigcap G_2^*\in\tau^*$

(ii) If $K_1=X^*-G_1^*$ and $K_2=X^*-G_2^*$ are double closed compact subsets of X, such that

$K_1=(K_1^1,K_2^1),\ K_2=(K_1^2,K_2^2),$

$(K_1^1,K_2^1)=\ X^*-(G_1^{1*},G_2^{1*})\Rightarrow K_1^1=$

$X^*-G_1^{1*},K_2^1=\ X^*-G_2^{1*}$

$(K_1^2,K_2^2)=\ X^*-(G_1^{2*},G_2^{2*})\Rightarrow K_1^2=$

$X^*-G_1^{2*},K_2^2=\ X^*-G_2^{2*}$

$[(G_1^{1*},G_2^{1*})\bigcap(G_1^{2*},G_2^{2*})]$

$=X^*-(G_1^{1*}\bigcap G_1^{2*},\ G_2^{1*}\bigcup G_2^{2*})$

$=X^*-(G_1^{1*}\bigcap G_1^{2*},\ X^*-(G_2^{1*}\bigcup G_2^{2*})$

$X^*-(G_1^{1*}\bigcap G_1^{2*})=$

$(X^*-G_1^{1*})\bigcup(X^*-G_1^{2*})=K_1^1-K_1^2$

$X^*-(G_2^{1*}\bigcup G_2^{2*})=$

$(X^*-G_2^{1*})\bigcup(X^*-G_2^{2*})=K_2^1-K_2^2$

$=(K_1^1\bigcup K_1^2,K_2^1\bigcap K_2^2)=K_1\bigcup K_2$

Which is double closed and double compact, being a union of tow double closed and double compact sets, hence $G_1^*\bigcap G_2^*\in\tau^*$.

(iii) If one of the double sets, say $G_1^*\in\tau$ and $K=X^*-G_2^*$ where K= (K$_1$, K$_2$) is a double closed compact subset of X, then

$G_1^* \bigcap G_2^* = (G_1^* \bigcap X) \bigcap G_2^* = G_1^* \bigcap (X \bigcap G_2^*)$
$= (G_1^{1*}, G_2^{1*}) \bigcap [X - (X^* - (G_1^{2*}, G_2^{2*}))]$
$= (G_1^{1*}, G_2^{1*}) \bigcap [X - (X^* - G_1^{2*}, X^* - G_2^{2*})]$
$= (G_1^{1*}, G_2^{1*}) \bigcap$
$[X - (X^* - G_1^{2*}), X - (X^* - G_2^{2*})]$
$= [G_1^{1*} \bigcap (X - K_1), G_2^{1*} \bigcup (X - K_2)]$
$= G_1^* \bigcap (X - K)$

Which belong to τ since $G_1^* \in \tau$ and X-K$\in \tau$, K being τ - double closed. Hence by definition of τ^*, $G_1^* \bigcap G_2^* \in \tau^*$. Thus in either case $G_1^* \bigcap G_2^* \in \tau^*$

[T^3]: Let $\{G_\lambda^* : \lambda \in \Lambda\}$ be the collection of τ^*- double open sets and let
$G^* = (G_1^*, G_2^*) = \bigcup\{G_{1\lambda}^*, G_{2\lambda}^*) : \lambda \in \Lambda\}$
If
$\infty \notin G_\lambda^* \textit{ for all } \lambda \in \Lambda, \textit{ then each } G_\lambda^* \in \tau$. Therefore $G^* \in \tau$ and if
$\infty \in G_\lambda^* \textit{ for some } \lambda_0 \in \Lambda,$ then
$X^* - G_{\lambda_0}^* = (X^* - G_{1\lambda_0}^*, X^* - G_{2\lambda_0}^*)$
$= (X - G_{1\lambda_0}^*, X - G_{2\lambda_0}^*) = X - G_{\lambda_0}^*$
Hence $X^* - G^* = X - G^* = X - \bigcup$
$\{(G_{1\lambda}^*, G_{2\lambda}^*) : \lambda \in \Lambda\}$

$= \bigcap\{(X - G_{1\lambda}^*, X - G_{2\lambda}^*) : \lambda \in \Lambda\}$

Thus $X^* - G^*$ is the intersection of double closed subsets of X, at least one of which is double compact and consequently $X^* - G^*$ is also double compact subset of X. Hence $G^* \in \tau^*$.

We now show that (X^*, τ^*) is double compact. Let C be any τ^*- double open cover of X*. Then ∞ is in some member of C and that member must be of the form X*-K where K is some double closed compact subset of X. Since K is covered by C and K is compact, there exists a finite subfamily C` of C such that C` covers K. Then $C^{`} \bigcup (X^* - K)$ is double compact. Finally we show that (X, τ) is a double dense subspace of (X^*, τ^*).

In order that X may be a subspace of X*, we must show that $\tau = \tau_X^*$ where τ_X^* denotes the relativization of τ^* to X. If $G \in \tau$, G = (G_1, G_2), then $G \in \tau_X$ since X is a τ^*- double open set such that $G = G \bigcap X$. conversely, let $G \in \tau_X^*$. Then $G = G^* \bigcap X$. for some $G^* \in \tau^*$, If $\infty \notin G_1^*$,then
$G_1^* \subset X \textit{ so that } G = G^*$ and therefore G is τ - open.

If $\infty \in G^*$ then $X^* - G^*$ is a τ - double closed subset of X. Hence
$G = (G_1, G_2) = X \bigcap (G_1^*, G_2^*)$
$= X - (X^* - (G_1^*, G_2^*))$
is a τ - double open subset of X. Thus we have shown that $G \in \tau \Leftrightarrow G \in \tau_{X^*}$. It follows that (X, τ) is a double subspace of (X^*, τ^*). We now show that X is double dense in X*. Since X is double compact subset of a double space X*, X is a not double closed in X*. Since X is a non-double compact subset of a double compact space X*, X is not double closed in X. Hence its complement X* - X = {∞} is not double open in X. It follows that every neighborhood of ∞ must contain a point of X. Thus $\infty \in \overline{X} \textit{ and so } \overline{X} = X$ which is shows that X is dense in X*.

4.5. Definition. In a bitopological space (X, u, v), *u* is said to be locally compact with respect to v if for each point $x \in X$ there is u-open neighborhood of x whose v- closure is compact. (X, u, v), locally compact if u is locally compact w.r.t. v, and v is locally compact w.r.t. u.

4.6. Definition. Let (X, τ) be DTS and let $N \in X$. A double set N of X is said to be τ -nhd of $\underset{\sim}{x}$ iff there exists τ -DOS, G such that $\underset{\sim}{x} \in G \subset N$, similarly N is called a τ -double nhd of $A \subset X$ iff there exists an DOS, G such that $A \subset G \subset N$.

4. 7. Proposition. If a bitopological space (X, u, v) is pairwise locally compact then (X, τ_{uv}) is pairwise locally compact.

Proof: Let G be open neighborhood of x, such that $\underset{\sim}{x} \in G \in \tau_{uv}$, G = (U, V^c) with in particular $x \in U \in u,$ U is open neighborhood of x. (X,u,v) is locally compact then cl(U) is pairwise compact then $\{N_j : j \in J\}$ and there exists a finite subcover such that

$cl(U) \subset N_{j1} \cup ... \cup N_{jn}$

Take $(cl(U), \phi) \in \tau_{uv}$ then

$(cl(U), \phi) \subset (N_{j1}, \phi) \cup ... \cup (N_{jn}, \phi)$

Then (X, τ_{uv}) is locally compact. □

4. 8. Definition. The DTS (X, τ) is said to be pairwise locally compact if for each point $x \in X$ there is double open neighborhood of x whose closure is pairwise compact.

4.9. Proposition. If a bitopological space (X, τ) is pairwise locally compact then (X, τ_1, τ_2) is pairwise locally compact.

Proof: Let $G = (A, B) \in \tau$ be pairwise locally compact such that $\underset{\sim}{x} \in G$ *then* $x \in A \in \tau_1$

$\because cl(G)$ is pairwise compact then there exist $\{N_j : j \in J\}$ open cover of cl(G) such that $cl(G) \subset \{N_j : j \in J\}$ and there exist subcover $cl(G) \subset N_{j1} \cup ... \cup N_{jn}$

Then

$(cl(A), cl(B)) \subset (N^1_{j1}, N^2_{j1}) \cup .. \cup (N^1_{jn}, N^2_{jn})$

$\therefore cl(A) \subset N^1_{j1} \cup ... \cup N^1_{jn}$

$N^2_{j1} \cap ... \cap N^2_{jn} \subset cl(B)$ *then*

$(cl(B))^c \subset (N^2_{j1})^c \cup ... \cup (N^2_{jn})^c$

So τ_1 is locally compact w.r.t τ_2, in the same way we prove τ_2 is locally compact w.r.t τ_1. And so (X, τ_1, τ_2) is pairwise locally compact. □

4.10. Proposition. Every double closed subspace of a pairwise locally compact space (X, τ) is pairwise locally compact. (X, τ).

Proof: Let Y be DCS of a locally compact space X, and let $\underset{\sim}{y} \in Y$ be arbitrary. Then $\underset{\sim}{y} \in X$. Since X is pairwise locally compact, there exists a double open neighborhood N of y such that cl(N) is double compact. But $N \cap Y$ is a double neighborhood of $\underset{\sim}{y}$ in Y such that $cl(N \cap Y) \subset cl(N)$.

Thus $cl(N \cap Y)$ is a DCS of the double compact set cl(N) and is therefore itself double compact. Also since Y is double closed in X, it is easy to see that the closure of $N \cap Y$ in X is the same as its closure in Y. Thus we have shown that every point in Y has a neighborhood in Y whose closure in Y is double compact. Hence the subspace Y is pairwise locally compact. □

4.11. Definition. [7] The DTS (X, τ) is called pairwise T_2 if given $x \neq y$ *in* X there exists $G, H \in \tau$ *satisfying* $\underset{\sim}{x} \in G, \underset{\approx}{y} \in H$ *and* $G \subseteq ()\overline{H}$.

4.12.Definition.[18] The DTS (X, τ) is called pairwise T_3 if $\forall DCS\ A \in \tau, a \in \text{int}\ A\ in\ X$ there exists $G, H \in \tau$ satisfying $\underset{\sim}{a} \in G, \underset{\approx}{a} \notin H$, $A \subseteq H$ *and* $G \subseteq ()\overline{H}$.

4.13. Proposition. Let (X, τ) be double topological space, then every pairwise locally compact, pairwise T_2 space is pairwise regular.

Proof: Let X be a pairwise locally compact, pairwise T_2 space and X^* its one-double compactification. Let $\underset{\sim}{x}$ be any double point in X^* and A any double closed set in X^* not containing $\underset{\sim}{x}$. Since X^* is pairwise compact. A is a double compact subset of X^*. Since X^* is pairwise T_2 there exists disjoint double open subsets G, H of X^* satisfying $\underset{\sim}{x} \in G, \underset{\approx}{y} \in H$ *for* $x \neq y$ *and* $G \subseteq ()\overline{H}$.

From above A is DCS in X, $x \in \text{int}\ A,$ *take* $G = (A, B), H = (C, D) \in \tau$,

$\underset{\sim}{x} \in G \to x \in A$ *and*

$\because ()\overline{H} = (D, \phi)$ *then* $x \in A \subseteq D$

$\rightarrow x \in D$ *then* $\underset{\sim}{x} \in G, \underset{\approx}{x} \notin H$ and $A \subseteq H$,So X is pairwise regular. □

4.14. Definition. [18] The DTS (X,τ) is called pairwise T_4 if

$\forall$ *DCS* A, B *in* X *there exists*

$N, M \in \tau$ *satisfying* $A \subseteq N, B \subseteq M$

and $N \subseteq ()\overline{M}$.

4.15. Proposition. Let (X,τ) be DTS, then every pairwise locally compact, pairwiseT_2 space is pairwise completely regular.

Proof: Let X^* be the one-double point compactification of pairwise locallycompact, pairwise T_2 space X. We first show that $X^*$is pairwise normal.

Let A, B be any disjoint DCS's in X^* is pairwise compact [17], A, B are double compact subsets of X^*, there exist disjoint DOS's G & H such that $A \subset G, B \subset H.$

$\because X^*$ is pairwise T_2 then for each $x \neq y, \underset{\sim}{x} \in G \,\&\, \underset{\approx}{y} \in H$ *and* $G \subseteq ()\overline{H}$

$\because X^*$ is pairwise T_1 then X^* is pairwise normal.

We now show that X is pairwise completely regular. Let $\underset{\sim}{x}$ be any double point in X, U be double open neighborhood of $\underset{\sim}{x}, \underset{\sim}{x} \notin U$ since X is double locally compact, there is a DOS N in containing $\underset{\sim}{x}$ such that cl(N) is double compact, $M = X^* - cl(N)$ is a DOS in X^*.Since X^* is pairwise T_2 , { $\underset{\sim}{x}$ }is DCS in X^*.

Now cl(M) and{ $\underset{\sim}{x}$ } are disjoint DCS's in the pairwise normal X^*. Hence there is a lower semi -continuous (l.s.c) and upper semi -continuous (u.s.c) function $f : X \rightarrow [0,1]$ *s.t* $f(\underset{\sim}{x}) = 1$ *and* $f(cl(M)) = \{0\}$, since X is subspace of X^* and $\overline{U} \subset cl(M)$ there is restriction $f_X : X \rightarrow [0,1]$ off to X is continuous such that $f_X(\underset{\sim}{x}) = 1$, $f_X[U] = 0,$ hence X is pairwise completely regular.

References

[1] Adamek, J. Herrlich, H. and Strecker, G.E. Abstract and Concrete Categories (John Wiley &Sons, Inc., 1990).

[2] Atanassov, K. Intuitionistic fuzzy sets , VII ITKR s Session (Sofia, June 1983 Central Sci. and Tech. Library) (V. Sgurev , ed.) , (Blug. Academy of Sciences , Sofia, 1984).

[3] Atanassov, K. Intuitionistic fuzzy sets , Fuzzy Sets and Systems 20(1),84-96,1986.

[4] Bayhan, S. and Coker, D. On Fuzzy Separation Axioms In Intuitionistic Fuzzy Topological Spaces, Busefal 67, 77-87,1996.

[5] Bayhan, S. and Coker, D. On Separation Axioms in Intuitionistic Topological Spaces, Int. J. of Math. And Math. Sci 27(10), 621-630,2001.

[6] Bayhan, S. and Coker, D. On T_1 and T_2 Separation Axioms In Intuitionistic Fuzzy Topological Spaces, J. Fuzzy Mathematics 11(3), 581-592,2003.

[7] Bayhan, S. and Coker, D. Pairwise Separation Axioms in Intuitionistic Topological Spaces, Int. J. of Math. And Math. Sci 34S, 101-114, (2005).

[8] Chang, C. L. Fuzzy Topological Spaces, J. Math. Appl. 24, 182-190, 1968.

[9] Coker, D. A note On Intuitionistic Sets And Intuitionistic Points, Turkish J. Math. 20(3), 343-351, 1996.

[10] Coker, D. and Es, A. H. On Fuzzy Compactness In Intuitionistic Fuzzy Topological Spaces, j. Fuzzy Mathematics 3(4), 899-909, 1995.

[11] Coker, D. An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems 88(1), 81-89,1997.

[12] Coker, D. and Demirci, M. On Intuitionistic Fuzzy Points, Notes IFS 1(2), 79-84, 1995.

[13] Coker, D. An Introduction to Intuitionistic Topological Spaces, BUSEFAL, 81, 51-56, 2000.

[14] P. Fletcher , H. B. Holye and C. W. Patty, The Comparison of Topologies , Duke Math. J.36,325-331, 1965.

[15] Garia, J.G. and Rodabaugh, S.E. Order-theoretic, topological, categorical redundancies of interval-valued sets, grey sets, vague sets, interval-valued

(intuitionistic) sets, (intuitionistic) fuzzy sets and topologies, Fuzzy Sets ans Systems 156(3),445-484, 2005.
[16] Kelly, J. G. Bitopological Space , Proc. London Math. Soc.13,71-89, 1963.
[17] Ozcag, S. and Coker, D. On Connectedness In Intuitionistic Fuzzy Special Topological Spaces, Int. J. Math. Math. Sci. 21(1),33-40, 1998.
[18] Rewayda, R. Pairwise Separation Axioms in Intuitionistic Topological Spaces, Kerbala journal 2011.
[19] Rewayda, R. Pairwise Compact in Intuitionistic Topological Spaces, Kufa journal 2011.
[20] Zadeh, L. A. Fuzzy Sets, Inform. And Control 8,338-353, 1965.