



17-Brauer trees of spin characters for S_n , $n=17, 18, 19, 20$

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Abstract: In this paper, we calculate the Brauer trees and the decomposition matrices of the symmetric groups S_n when $n= 17, 18, 19, 20$ for $p=17$ by using $(r, \bar{r})$ -inducing.

Keywords: Brauer tree, spin characters, decomposition matrices.

1- Introduction

The Brauer tree is a graph connection between the irreducible ordinary characters in the block of defect one [1]. Brauer expanded the theory of representation from the field with the characteristic zero to the field with the characteristic $p>0$ and proving that for

each ordinary representation, there is a modular representation on p -regular classes. We will found the Brauer trees by calculating the decomposition matrix of spin characters for the symmetric group S_n when $n=17,18,19,20$ and $p=17$ and the theoretical is depended on [2].

Notation

A_i	The principle indecomposable spin characters of S_{17}
B_j	The principle indecomposable spin characters of S_{18}
C_k	The principle indecomposable spin characters of S_{19}
D_l	The principle indecomposable spin characters of S_{20}
$\langle \alpha \rangle^*$	The double spin characters
$\langle \alpha \rangle, \langle \alpha \rangle'$	The associate spin characters

2- Brauer trees of spin characters for S_{17} :

In the symmetric group S_{17} there are 57 irreducible spin characters and the covering group has 56 of 17-regular classes then the decomposition matrix of S_{17} has 57 rows and 56 columns.

The symmetric group S_{17} has one block B_1 of defect one and the other spin blocks of defect zero.

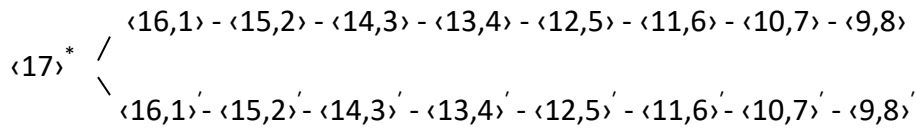
2.1 The block B_1 :

This block has the irreducible spin characters $\langle 17 \rangle^*, \langle 16,1 \rangle, \langle 16,1 \rangle', \langle 15,2 \rangle, \langle 15,2 \rangle', \langle 14,3 \rangle, \langle 14,3 \rangle', \langle 13,4 \rangle, \langle 13,4 \rangle', \langle 12,5 \rangle, \langle 12,5 \rangle', \langle 11,6 \rangle, \langle 11,6 \rangle', \langle 10,7 \rangle,$

$\langle 10,7 \rangle', \langle 9,8 \rangle, \langle 9,8 \rangle'$ which is an associate[3].

Lemma (2.1.1):

The Brauer tree of B_1 is:



Proof :

$\deg\{\langle 17 \rangle^*, \langle 15,2 \rangle, \langle 15,2 \rangle', \langle 13,4 \rangle, \langle 13,4 \rangle', \langle 11,6 \rangle, \langle 11,6 \rangle', \langle 9,8 \rangle, \langle 9,8 \rangle'\} \equiv 1 \pmod{17}$
 $\deg\{\langle 16,1 \rangle, \langle 16,1 \rangle', \langle 14,3 \rangle, \langle 14,3 \rangle', \langle 12,5 \rangle, \langle 12,5 \rangle', \langle 10,7 \rangle, \langle 10,7 \rangle'\} \equiv -1 \pmod{17}$ [4]
 by the $(r, \bar{r})$ -inducing of $\langle 16 \rangle, \langle 16 \rangle', \langle 15,1 \rangle^*, \langle 14,1 \rangle^*, \langle 13,3 \rangle^*, \langle 12,4 \rangle^*, \langle 11,5 \rangle^*, \langle 10,6 \rangle^*$

and $\langle 19,7 \rangle^*$ in S_{16} to S_{17} we have the columns $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}$ and A_{16} respectively[5].
 we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_1															
1	$\langle 17 \rangle^*$	1	1														
-1	$\langle 16,1 \rangle$	1		1													
-1	$\langle 16,1 \rangle'$		1		1												
1	$\langle 15,2 \rangle$			1		1											
1	$\langle 15,2 \rangle'$				1		1										
-1	$\langle 14,3 \rangle$					1		1									
-1	$\langle 14,3 \rangle'$						1		1								
1	$\langle 13,4 \rangle$							1		1							
1	$\langle 13,4 \rangle'$								1		1						
-1	$\langle 12,5 \rangle$									1		1					
-1	$\langle 12,5 \rangle'$										1		1				
1	$\langle 11,6 \rangle$											1		1			
1	$\langle 11,6 \rangle'$												1		1		
-1	$\langle 10,7 \rangle$													1		1	
-1	$\langle 10,7 \rangle'$														1		1
1	$\langle 9,8 \rangle$															1	
1	$\langle 9,8 \rangle'$																1
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}

(table 1)

3- Brauer trees of spin characters for S_{18} :

In the symmetric group S_{18} there are 69 irreducible spin characters and the covering group has 68 of 17-regular classes then the

decomposition matrix of S_{18} has 69 rows and 68 columns.

The symmetric group S_{18} has one block B_1 of defect one and the other spin blocks of

defect zero.

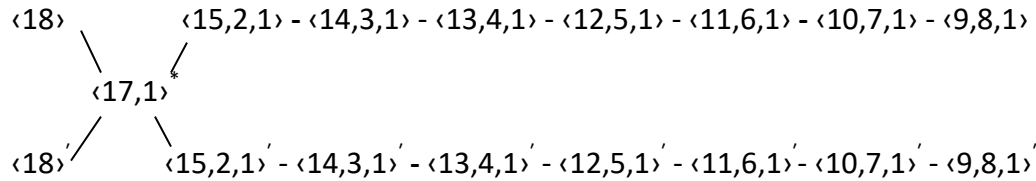
3.1- The block B_1 :

This block has the irreducible spin characters $\langle 18 \rangle$, $\langle 18 \rangle'$, $\langle 17,1 \rangle^*$, $\langle 15,2,1 \rangle$, $\langle 15,2,1 \rangle'$, $\langle 14,3,1 \rangle$, $\langle 14,3,1 \rangle'$, $\langle 13,4,1 \rangle$,

$\langle 13,4,1 \rangle'$, $\langle 12,5,1 \rangle$, $\langle 12,5,1 \rangle'$, $\langle 11,6,1 \rangle$, $\langle 11,6,1 \rangle'$, $\langle 10,7,1 \rangle$, $\langle 10,7,1 \rangle'$, $\langle 9,8,1 \rangle$, $\langle 9,8,1 \rangle'$ which is an associate [3].

Lemma (3.1.1):

The Brauer tree of B_1 is:



Proof:

$\deg\{\langle 18 \rangle, \langle 18 \rangle', \langle 15,2,1 \rangle, \langle 15,2,1 \rangle', \langle 13,4,1 \rangle, \langle 13,4,1 \rangle', \langle 11,6,1 \rangle, \langle 11,6,1 \rangle', \langle 9,8,1 \rangle, \langle 9,8,1 \rangle'\} \equiv 1 \pmod{17}$
 $\deg\{\langle 17,1 \rangle^*, \langle 14,3,1 \rangle, \langle 14,3,1 \rangle', \langle 12,5,1 \rangle, \langle 12,5,1 \rangle', \langle 10,7,1 \rangle, \langle 10,7,1 \rangle'\} \equiv -1 \pmod{17}$ [4]

by the $(r, \bar{r})$ -inducing of $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}$ and A_{16} in S_{17} to S_{18} we have the columns $B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_{15}$ and B_{16} respectively [5]. we get the following decomposition matrix

The degree mod 17	Spin characters	The decomposition matrix for B_1															
1	$\langle 18 \rangle$	1															
1	$\langle 18 \rangle'$		1														
-1	$\langle 17,1 \rangle^*$	1	1	1	1												
1	$\langle 15,2,1 \rangle$			1		1											
1	$\langle 15,2,1 \rangle'$				1		1										
-1	$\langle 14,3,1 \rangle$					1		1									
-1	$\langle 14,3,1 \rangle'$						1		1								
1	$\langle 13,4,1 \rangle$							1		1							
1	$\langle 13,4,1 \rangle'$								1		1						
-1	$\langle 12,5,1 \rangle$									1		1					
-1	$\langle 12,5,1 \rangle'$										1		1				
1	$\langle 11,6,1 \rangle$											1		1			
1	$\langle 11,6,1 \rangle'$												1		1		
-1	$\langle 10,7,1 \rangle$													1		1	
-1	$\langle 10,7,1 \rangle'$														1		1
1	$\langle 9,8,1 \rangle$															1	
1	$\langle 9,8,1 \rangle'$																1
		B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}

(table 2)

4- Brauer trees of spin characters for S_{19} :

In the symmetric group S_{19} there are 81 irreducible spin characters and the covering group has 79 of 17-regular classes then the decomposition matrix of S_{19} has 81 rows and 79 columns.
The symmetric group S_{19} has one block B_1

Lemma (4.1.1):

The Brauer tree of B_1 is:

$$\langle 19 \rangle^* - \langle 17, 1 \rangle = \langle 17, 1 \rangle' - \langle 16, 2, 1 \rangle^* - \langle 14, 3, 2 \rangle^* - \langle 13, 4, 2 \rangle^* - \langle 12, 5, 2 \rangle^* - \langle 11, 6, 2 \rangle^* - \langle 10, 7, 2 \rangle^* - \langle 9, 8, 2 \rangle^*$$

Proof :

$\deg\{\langle 19 \rangle^*, \langle 16, 2, 1 \rangle^*, \langle 13, 4, 2 \rangle^*, \langle 11, 6, 2 \rangle^*, \langle 9, 8, 2 \rangle^*\} \equiv 2 \pmod{17}$
 $\deg\{\langle 17, 2 \rangle + \langle 17, 2 \rangle', \langle 14, 3, 2 \rangle^*, \langle 12, 5, 2 \rangle^*, \langle 10, 7, 2 \rangle^*\} \equiv -2 \pmod{17}$ [4]
 by the $(r, \bar{r})$ -inducing of $B_1, B_3, B_5, B_7, B_9, B_{11}, B_{13}$ and B_{15} in S_{18} to S_{19} we have the

of defect one and the other spin blocks of defect zero.

4.1- The block B_1 :

This block has the irreducible spin characters $\langle 19 \rangle^*, \langle 17, 2 \rangle, \langle 17, 2 \rangle', \langle 16, 2, 1 \rangle^*, \langle 14, 3, 2 \rangle^*, \langle 13, 4, 2 \rangle^*, \langle 12, 5, 2 \rangle^*, \langle 11, 6, 2 \rangle^*, \langle 10, 7, 2 \rangle^*, \langle 9, 8, 2 \rangle^*$ which is a double [3].

columns $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ and C_8 respectively[5].

we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_1							
2	$\langle 19 \rangle^*$	1							
-1	$\langle 17, 2 \rangle$	1	1						
-1	$\langle 17, 2 \rangle'$	1	1						
2	$\langle 16, 2, 1 \rangle^*$		1	1					
-2	$\langle 14, 3, 2 \rangle^*$			1	1				
2	$\langle 13, 4, 2 \rangle^*$				1	1			
-2	$\langle 12, 5, 2 \rangle^*$					1	1		
2	$\langle 11, 6, 2 \rangle^*$						1	1	
-2	$\langle 10, 7, 2 \rangle^*$							1	1
2	$\langle 9, 8, 2 \rangle^*$								1
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8

(table 3)

5- Brauer trees of spin characters for S_{20} :

In the symmetric group S_{20} there are 96 irreducible spin characters and the covering group has 93 of 17-regular classes then the decomposition matrix of S_{20} has 96 rows and 93 columns.
The symmetric group S_{20} has two blocks B_1, B_2 of defect one and the other spin

blocks of defect zero.

5.1- The block B_1 :

This block has the irreducible spin characters $\langle 19, 1 \rangle^*, \langle 18, 2 \rangle^*, \langle 17, 2, 1 \rangle, \langle 17, 2, 1 \rangle', \langle 14, 3, 2, 1 \rangle^*, \langle 13, 4, 2, 1 \rangle^*, \langle 12, 5, 2, 1 \rangle^*, \langle 11, 6, 2, 1 \rangle^*, \langle 10, 7, 2, 1 \rangle^*, \langle 9, 8, 2, 1 \rangle^*$ which is a double [3].

Lemma (5.1.1):

The Brauer tree of B_1 is :

$$\langle 19,1 \rangle^* - \langle 18,2 \rangle^* - \langle 17,2,1 \rangle' = \langle 17,2,1 \rangle' - \langle 14,3,2,1 \rangle^* - \langle 13,4,2,1 \rangle^* - \langle 12,5,2,1 \rangle^* - \langle 11,6,2,1 \rangle^* - \langle 10,7,2,1 \rangle^* - \langle 9,8,2,1 \rangle^*$$

Proof :

$$\deg\{ \langle 19,1 \rangle^*, (\langle 17,2,1 \rangle' + \langle 17,2,1 \rangle'), \langle 13,4,2,1 \rangle^*, \langle 11,6,2,1 \rangle^*, \langle 9,8,2,1 \rangle^* \} \equiv 2 \pmod{17}$$

$$\deg\{ \langle 18,2 \rangle^*, \langle 14,3,2,1 \rangle^*, \langle 12,5,2,1 \rangle^*, \langle 10,7,2,1 \rangle^* \} \equiv -2 \pmod{17} [4]$$

by the $(r, \bar{r})$ -inducing of $\langle 18,1 \rangle$, C_2 , C_3 , C_4 , C_5 , C_6 , C_7 and C_8 in S_{19} to S_{20} we have the columns D_1 , D_2 , D_3 , D_4 , D_5 , D_6 , D_7 and D_8 respectively [5].

we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_1							
2	$\langle 19,1 \rangle^*$	1							
-2	$\langle 18,2 \rangle^*$	1	1						
1	$\langle 17,2,1 \rangle'$		1	1					
1	$\langle 17,2,1 \rangle'$		1	1					
-2	$\langle 14,3,2,1 \rangle^*$			1	1				
2	$\langle 13,4,2,1 \rangle^*$				1	1			
-2	$\langle 12,5,2,1 \rangle^*$					1	1		
2	$\langle 11,6,2,1 \rangle^*$						1	1	
-2	$\langle 10,7,2,1 \rangle^*$							1	1
2	$\langle 9,8,2,1 \rangle^*$								1
		D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8

(table 4)

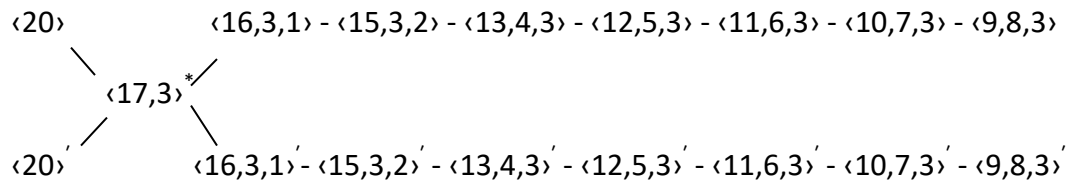
5.2- The block B_2 :

This block has the irreducible spin characters $\langle 20 \rangle$, $\langle 20 \rangle'$, $\langle 17,3 \rangle^*$, $\langle 16,3,1 \rangle$, $\langle 16,3,1 \rangle'$, $\langle 15,3,2 \rangle$, $\langle 15,3,2 \rangle'$, $\langle 13,4,3 \rangle$,

$\langle 13,4,3 \rangle'$, $\langle 12,5,3 \rangle$, $\langle 12,5,3 \rangle'$, $\langle 11,6,3 \rangle$, $\langle 11,6,3 \rangle'$, $\langle 10,7,3 \rangle$, $\langle 10,7,3 \rangle'$, $\langle 9,8,3 \rangle$, $\langle 9,8,3 \rangle'$ which is an associate [3].

Lemma (5.2.1):

The Brauer tree of B_2 is :



Proof :

$$\deg\{ \langle 20 \rangle, \langle 20 \rangle', \langle 16,3,1 \rangle, \langle 16,3,1 \rangle', \langle 13,4,3 \rangle, \langle 13,4,3 \rangle', \langle 11,6,3 \rangle, \langle 11,6,3 \rangle', \langle 9,8,3 \rangle, \langle 9,8,3 \rangle' \} \equiv 2 \pmod{17}$$

$$\deg\{ \langle 17,3 \rangle^*, \langle 15,3,2 \rangle, \langle 15,3,2 \rangle', \langle 12,5,3 \rangle, \langle 12,5,3 \rangle', \langle 10,7,3 \rangle, \langle 10,7,3 \rangle' \} \equiv -2 \pmod{17} [4]$$

by the $(r, \bar{r})$ -inducing of C_1 , $\langle 16,3 \rangle$, $\langle 16,3 \rangle'$, C_3 , C_4 , C_5 , C_6 , C_7 and C_8 in S_{19} to S_{20} we have the columns D_9 , D_{10} , D_{11} , D_{12} , D_{13} , D_{14} , D_{15} , D_{16} , D_{17} , D_{18} , D_{19} , D_{20} , D_{21} , D_{22} , D_{23} and D_{24} respectively [5].

we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_2															
2	$\langle 20 \rangle$	1															
2	$\langle 20 \rangle'$		1														
-2	$\langle 17, 3 \rangle^*$	1	1	1	1												
2	$\langle 16, 3, 1 \rangle$			1		1											
2	$\langle 16, 3, 1 \rangle'$				1		1										
-2	$\langle 15, 3, 2 \rangle$					1		1									
-2	$\langle 15, 3, 2 \rangle'$						1		1								
2	$\langle 13, 4, 3 \rangle$							1		1							
2	$\langle 13, 4, 3 \rangle'$								1		1						
-2	$\langle 12, 5, 3 \rangle$									1		1					
-2	$\langle 12, 5, 3 \rangle'$										1		1				
2	$\langle 11, 6, 3 \rangle$											1		1			
2	$\langle 11, 6, 3 \rangle'$												1		1		
-2	$\langle 10, 7, 3 \rangle$													1		1	
-2	$\langle 10, 7, 3 \rangle'$														1		1
2	$\langle 9, 8, 3 \rangle$															1	
2	$\langle 9, 8, 3 \rangle'$																1
		D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}

(table 5)

References

- [1] I. M. Isaacs: "Character Theory of Finite Group ", Academic press , (1976).
- [2] M. M. Jawad: Brauer trees and Decomposition Matrices of spin characters of S_n Modulo 13 ,11, M.Sc. Thesis, Basrah University, (2018).
- [3] J.F. Humphreys:"Projective Modular Representation of Finite Groups I ", J. London math. Society (2), 16(1977) 51- 66.
- [4] A.O. Morris: The Spin Representation of the Symmetric Group", proc. London Math. Soc. (3) 12 (1962) , 55-76
- [5] J. Muller: The 2-modular decomposition matrices of the symmetric groups S_{15} , S_{16} and S_{17} , com.Algebra 28(10) (2000)4997 – 5005.

17- أشجار براور للمشخصات الاسقاطية لـ S_n عندما $n=17, 18, 19, 20$

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المستخلص:

في هذا البحث تم حساب اشجار براور ومصفوفات التجزئة للزمر التناظرية عندما $n = 17, 18, 19, 20$ بحيث $p=17$ باستخدام $(r, \bar{r})$ - المولدة

الكلمات المفتاحية :

شجرة براور ، المشخصات الاسقاطية ، مصفوفات التجزئة