Journal of Basrah Researches ((Sciences)) Vol. (46). No.2 (2020)

Available online at: https://www.iasj.net/iasj/journal/260/issues

ISSN –1817 –2695

Received date: 13-7-2020 Accepted date: 16-8-2020 Available online date: 14-3-2021

Theoretical Study of Dopant impurities Diffusion in Multi-quantum Well Heterostructure

Mohammad. H. Khalaf[a] , Fatima. H. Saeed[b]

Email[a]: mohammadhkh335@gmile.com , Email[b]: fatima.saeed@uobasrah.edu.iq

University of Basrah – college of Education for pure Sciences – Dept of Physics

Abstract. The thermal diffusion of dopant impurity in multi-quantum-well heterostructures can be modeled with the second Fick's law equation by using temperature dependent diffusion coefficient and equilibrium segregation coefficient at the interface between the layers of the heterostructures. The model adopted analytical methods by using the Laplace transform technique to solve the diffusion equation in different circumstances. The model has applied to practical structures such as $InGaAs/InP$ detectors and $InP/In_{0.53}Ga_{0.47}As/InP$ transistor. The results encourage to improvement the model to accommodate the complex types of heterostructure devices. The model may be a useful reference for a laboratory process controlling the dopant diffusion in multi-quantum-well heterostructures devices after future development.

Keywords: quantum well, heterostructure, thermal diffusion, semiconductors, segregation coefficient, Fick's equation.

1. Introduction

Most modern electronic devices comprise of multi-quantum well produced by various manufacturing processes such as the multi-crystallization, super-growth of semiconductor, thermal growth of oxide, and other methods that require high accuracy when manufacturing devices with nano-scales devices. The process of forming a quantum well is often followed by the process of doping with impurities using the methods of thermal diffusion or ion implantation as needed [1]. Diffusion and ion implantation are two commonly used doping techniques in the industry to add impurities in the semiconductor (shown in Figure (1)). Diffusion involves the movement of dopant particles from higher concentration to lower concentration regions in random motion. Whereas, ion implantation involves bombarding accelerated ions to the surface of the substrate [2].

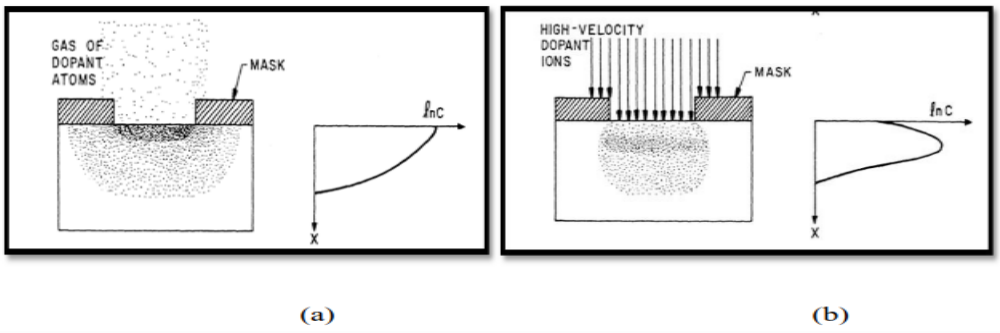


Figure (1): Doping techniques (a) Thermal diffusion and (b) Ion implantation [2].

The Diffusion process is widely used in the production of electronic and optical devices similar to $InGaAs/InP$ in order to build a $p \pm n$ link, the Diffusion process is the preferred technique because it can form a high p-doped area, of the most common materials used are zinc and cadmium. The challenges associated with using zinc as impurities are controlling the depth of diffusion and the concentration of restaurants for high-speed diode production [3].

In this study, we will present an analytical theoretical model to study and investigating the diffusion of dopant impurities into a multi- (QW) heterostructure device using the modified Fick's - diffusion equation.

2.Dopant Diffusion in Multi-Quantum Well

In the multi-quantum well heterostructures, dopant defects concentration changes with position in the layers. The simplest form heterostructure consists of a single heterojunction, which is an interface within a semiconductor crystal via a chemical composition changes [1]. Quantum well heterostructures are the building blocks of the most advanced semiconductor devices presently being developed and produced. They are imperative elements of the highest performance optical sources and detectors and are being employed increasingly in high-speed and high-frequency digital and analog devices [4]. The benefit of multi-(QW) heterostructures is that, they offer precise controlling the states and motions of charge carriers in semiconductors nano-devices [5]. The (QW) heterostructures attracted scientific attention due to their peculiar properties arising from the role of quantum confinement in nano-layer when one of the dimensions of these structures is smaller than twice the radius of Bohr's of the exciton, the quantum confinement occurs, and this limitation causes a difference in the electronic and optical behavior of nano-devices [6].

3. The Equation of Motion for Dopant Diffusion

For ions or atoms injected in a specific layer of thickness L during the time of interval t, by neglecting any surface sputtering, the equation of motion for ion diffusion process can be considered the form of one-dimension Fick's second low [7]:

$$\frac{\partial}{\partial Z}\left(D\frac{\partial C(Z,t)}{\partial Z}\right) = \frac{\partial C(z,t)}{\partial t} \quad (1)$$

$D\frac{\partial C(Z,t)}{\partial Z}$ is related to the gas flux. The diffusion coefficient empresses number of However, diffusion coefficients depend on temperature and may depend on the concentrations of impurities in the system, crystal defects, strain, stress, diffusion depth, and so on. In this research, the temperature dependence diffusion coefficients will be determined which is experimentally obey Arrhenius's relation [8],

$$D(T) = D_o \exp(\frac{E_a}{K_B T}) \quad (2)$$

where, D_o is a preexponential factor, E_a is the activation energy, K_B is Boltzmann constant and T is the absolute temperature.

3.1. Analytical Solution for the Diffusion Equation in One (QW)

For gaseous and liquid sources, there is a steady concentration of impurities at the surface. There is a vapor of impurities (it is also true for a solid source) that is keeping the steady concentration on the surface. Also, the diffusion length *Dt* is much smaller than the layer thickness *L,* so that the layer can be approximated as a semi-infinite medium. Then to calculate the concentration of the impurities as a function of depth from the layer surface, second Fick's law of diffusion can be used for the case of D only temperature dependent [9-10].

$$\frac{\partial C(z,t)}{\partial t} = D(T)\frac{\partial^2 C(z,t)}{\partial z^2} \quad (3)$$

Where $C(z,t)$ is the distribution of the ion gas concentration as a function of z and time t and D is the diffusion coefficient, The term

particles that diffuse across a unit area in unit time.

The initial and boundary condition for this problem is: at time $t = 0$, impurity concentration at depth z and time t, is given by, $C(z,0) = 0$. At the surface, we have, $C(0,t) = C_{init.}$ and at $z = L$, $C(L,t) = 0$. Where the surface concentration, $C_{ini.}$ remains a constant at a constant temperature T.

To solve equation (3) that satisfies the initial and boundary conditions we use Laplace transformation method and then equation (3) will be,

$$D(T)\int_0^\infty \frac{\partial^2 C(z,t)}{\partial z^2} e^{-st} dt = D(T)\frac{\partial^2}{\partial z^2}\int_0^\infty C(z,t)e^{-st}\,dt \quad (4)$$

and by using the Laplace transformation rules, equation (4) becomes,

$$D(T)\frac{\partial^2}{\partial z^2}\mathrm{C}(z,s) = [C(z,t)e^{-st}]_0^\infty + s\int_0^\infty C(z,t)e^{-st}\,dt \quad (5)$$

by using the boundary conditions, we get

$$D(T)\frac{\partial^2}{\partial z^2}\,C(z,s) = S\,\mathrm{C}(z,s) \quad (6)$$

Equation (6), is linear homogenous second-order ordinary differential equation and its solution is

$$C(z,s) = A\,exp\left(z\sqrt{\frac{S}{D(T)}}\right) + Bexp\left(-z\sqrt{\frac{S}{D(T)}}\right) \quad (7)$$

Where A and B are constants, so the solution of the equation (7) that satisfies the initial and boundary conditions is given by,

$$C(z,s) = \frac{C_{init.}}{S}\,exp\left(-z\sqrt{\frac{S}{D(T)}}\right) \quad (8)$$

By taking the inverse Laplace transform of equation (8), one can get,

$$C(z,t) = C_{init.}erfc\left(\frac{z}{2\sqrt{D(T)\,t}}\right) \quad (9)$$

Where $erfc$ is the complement of the error function, and $\sqrt{D(T)\,t}$ is the propagation length. This relationship represents the solution of the time-dependent propagation equation for one layer of matter at fixed surface concentration ions.

3.2. Analytical Solution for the Diffusion Equation in (QW) Heterostructure

To introducing the analytical solution for a special one-dimensional case, we consider two segments M_1 and M_2 of different quantum wells connected with the interface I. The impurity with concentration C diffuses under intrinsic dopant diffusion in both segments M_1 and M_2 with different temperature dependent diffusion coefficients $D_1(T)$ and $D_2(T)$, respectively.

Suppose in region M_1 as $z < 0$ is of the concentration C_1 and the region M_2 as $z > 0$ the concentration is C_2, as shown in figure (2). In this case, the dopant diffusion equations in each segment can be written as [9],

$$\frac{\partial C_i(z,t)}{\partial t} = D_i(T)\frac{\partial^2 C_i(z,t)}{\partial z^2}\,, \quad i = 1,2 \quad (10)$$

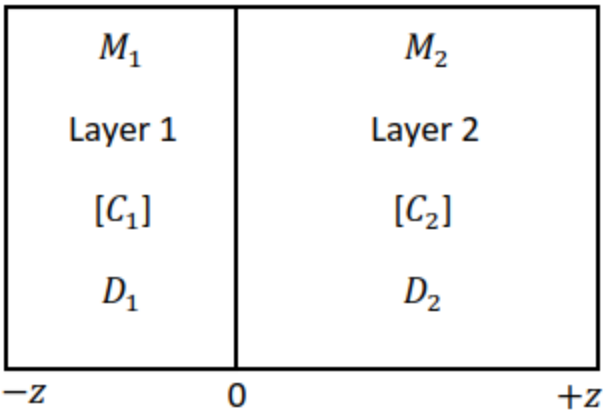


Figure (2): Schematic diagram for two-layer of different phases.

The initial condition in the region M_1 is at a uniform concentration $C_{init.}$ and in M_2 the concentration is zero initially:

$$\left.\begin{aligned} C_1(z\,,0) &= C_{init.} \quad , \quad z > 0 \\ C_2(z\,,0) &= 0 \qquad , \quad z < 0 \end{aligned}\right\} \tag{11}$$

The boundary conditions at the interface $z = 0$ can be written as:

$$\left.\begin{aligned} C_2(z,t) &= k\, C_1(\mathrm{z,t}) \\ D_1(T)\left.\frac{dC_1(z,t)}{dz}\right|_{z=0} &= D_2(T)\left.\frac{dC_2(z,t)}{dz}\right|_{z=0} \end{aligned}\right\} \tag{12}$$

$$C_1(z\,,t) = A_1 + B_1\,\mathrm{erfc}\frac{z}{2\sqrt{D_1(T)t}}$$

(13)

$$C_2(z\,,t) = A_2 + B_2\,\mathrm{erfc}\frac{|z|}{2\sqrt{D_2(T)t}} \tag{14}$$

where, A_1 , B_1, A_2, B_2 are constants, which are got by the substituting the initial and the boundary conditions in equations (13) and (14), and hence get a system of linear equation comprising four equations which solved by suitable algebraic methods,

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -k & 0 & 1 & 0 \\ 0 & \frac{D_1(T)}{\sqrt{\pi}\sqrt{D_1(T)t}} & \frac{D_2(T)}{\sqrt{\pi}\sqrt{D_2(T)t}} & 0 \end{bmatrix}\begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} C_o \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

where k is the segregation coefficient (The ratio between the balanced concentration of impurities in the region M_2 to their concentration in the region M_1) [11].

Since ions do not accumulate at the insulating surface between the two layers, the boundary condition in the relationship (12) clarifies the overflow of the boundary between the two layers.

Using the solution in the equation (10) and the boundary conditions (12) the results of the problems are given by [11],

Then by substituting the values of constants A_i and B_i in equations (13) and (14), we obtain relationships that describing the concentration profile of dopant impurities in the case of single heterojunction.

$$C_1(z\,,t) = \frac{C_o}{1+k\sqrt{\frac{D_2(T)}{D_1(T)}}}\left(1 + k\sqrt{\frac{D_2(T)}{D_1(T)}}\,erf\,\frac{z}{2\sqrt{D_1(T)t}}\right) \tag{16}$$

$$C_2(z,t) = \frac{k\,C_o}{1+k\sqrt{\frac{D_2(T)}{D_1(T)}}}\,erfc\left(\frac{|z|}{2\sqrt{D_2(T)t}}\right)$$

(17)

The concentration profiles results equation (16) and (17) are applied to two different quantum wells, and the results are presented with suitable conditions.

3.3. Analytical Solution for the Diffusion Equation in Double (QW) heterostructure

We assume a multi-quantum well, consisting of three layers heterogeneous M_1, M_2 and

M_3 , the equations in each segment can be written as equation (10) for $i = 1,2,3$ and the solutions can be given as,

$$C_1(z\,,t) = A_1 + B_1 \operatorname{erfc}\frac{z}{2\sqrt{D_1(T)t}}$$

$$0 < z < \ell_1 \qquad (18)$$

$$C_2(z\,,t) = A_2 + B_2 \operatorname{erfc}\frac{z}{2\sqrt{D_2(T)t}}$$

$$\ell_1 < z < (\ell_1 + \ell_2) \qquad (19)$$

$$C_3(z\,,t) = A_3 + B_3 \operatorname{erfc}\frac{z}{2\sqrt{D_3(T)t}}$$

$$(\ell_1 + \ell_2) < z < (\ell_1 + \ell_2 + \ell_3) \qquad (20)$$

Where, A_1 , B_1, A_2, B_2 , A_3, B_3 are constant values.

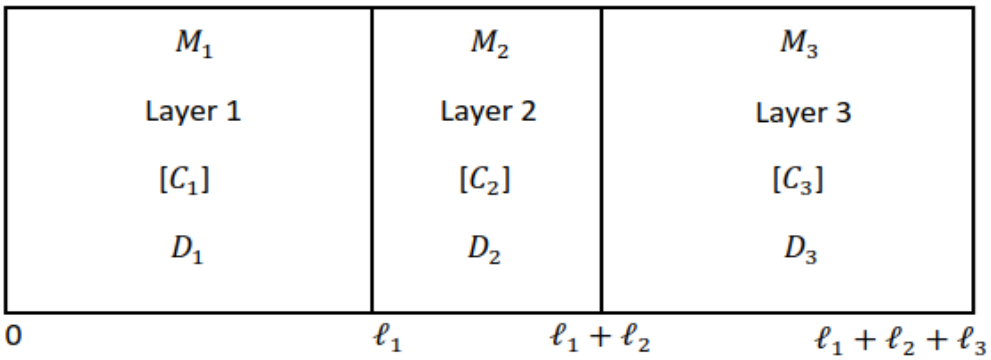


Figure (6): Schematic diagram for three-layer of deferent phases.

The initial conditions are defined, at time $t = 0$ the concentration of impurities at the surface C_o while it is zero throughout the structures, the initial conditions are given:

$$\left.\begin{aligned} C_1(z\,,0) &= C_o \qquad z < \ell_1 \\ C_i(z\,,0) &= 0 \qquad z > \ell_1 \end{aligned}\right\}$$

(21)

The boundary conditions are

$$\left.\begin{aligned} C_2(z,t) &= k_1\, C_1(z,t) \quad ,at \quad z = \ell_1 \\ D_1(T)\frac{dC_1(z,t)}{dz}\Big|_{z=\ell_1} &= D_2(T)\frac{dC_2(z,t)}{dz}\Big|_{z=\ell_1} \\ C_3(z,t) &= k_2\, C_2(z,t) \quad ,at \quad z = \ell_1 + \ell_2 \\ D_2(T)\frac{dC_2(z,t)}{dz}\Big|_{z=\ell_1+\ell_2} &= D_3(T)\frac{dC_3(z,t)}{dz}\Big|_{z=\ell_1+\ell_2} \end{aligned}\right\}$$

(22)

Where k_1, k_2 are the segregation coefficient between the first and second layers, second and third layers respectively.

The boundary conditions (2), (4) in equation (22), are for the case where ions gas accumulation is absent at the inner boundary at the $z = \ell_1$ and $z = \ell_1 + \ell_1$ between the regions M_1 and M_2, M_2 and M_3 respectively. The integral constants are found by applying the initial and boundary condition in equations (18), (19), and (20), getting a system of linear equations consisting of six equations that can be solved by algebraic methods.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -k_1 & 1 & 0 & -k_1 erf\left(\frac{\ell_1}{2\sqrt{D_1(T)\,t}}\right) & erf\left(\frac{\ell_1}{2\sqrt{D_2(T)\,t}}\right) & 0 \\ 0 & 0 & 0 & \frac{D_1(T)\,exp\left(\frac{-\ell_1^2}{4\,D_1(T)\,t}\right)}{\sqrt{\pi}\,\sqrt{D_1(T)\,t}} & \frac{-D_2(T)\,exp\left(\frac{-\ell_1^2}{4\,D_2(T)\,t}\right)}{\sqrt{\pi}\,\sqrt{D_2(T)t}} & 0 \\ 0 & -k_2 & 1 & 0 & -k_2 erf\left(\frac{\ell_1+\ell_2}{2\sqrt{D_2(T)\,t}}\right) & erf\left(\frac{\ell_1+\ell_2}{2\sqrt{D_2(T)\,t}}\right) \\ 0 & 0 & 0 & 0 & \frac{D_2(T)\,exp\left(\frac{-(\ell_1+\ell_2)^2}{4\,D_2(T)\,t}\right)}{\sqrt{\pi}\,\sqrt{D_2(T)\,t}} & \frac{-D_3(T)\,exp\left(\frac{-(\ell_1+\ell_2)^2}{4\,D_3(T)\,t}\right)}{\sqrt{\pi}\,\sqrt{D_3(T)\,t}} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} C_o \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (23)$$

By solving the above linear system, and using the same method used to solve the equation (15), the values of the constants column are obtained and by substituted in equations (18), (19), and (20), getting the concentration equations showing the distribution of dopants diffusion in double quantum well. So, in the same way, one can investigate the dopant diffusion in multi-quantum will heterostructure.

4. The Results

The diffusion of Zn into InP is a critical practical step in the manufacture of the high quality planar $InGaAs/InP$ detectors and LED diodes [13]. Where the vast amount of optical information transmitted by optical fibers, which makes it useful for building an electronic system has great reliability in dealing with that information. Accordantly, the (QW) diode laser is the basic building block for that system.

The $InGaAs/InP$ diode made of a thin film $InGaAs$ $(50\,nm)$ and substrate InP $(200nm)$, this composition has tainted Zinc (Zn) impurities, the spread of Zn impurities in $InGaAs/InP$ is an important process for manufacturing high-quality reagents and optical diodes, the amount of impurity ions $C_o = 1 \times 10^{18}\,\frac{ions}{cm^3}$, and the diffusion coefficient in the substrate $D_{InP} = 2.9 \times 10^{-12}\,\frac{cm^2}{sec}$ and the Diffusion coefficient in the $D_{InGaAs} = 2 \times 10^{-13}\,\frac{cm^2}{sec}$ during $t = 5\,min$, $10\,min$, the segregation coefficient $k = 0.98$, [3-14].

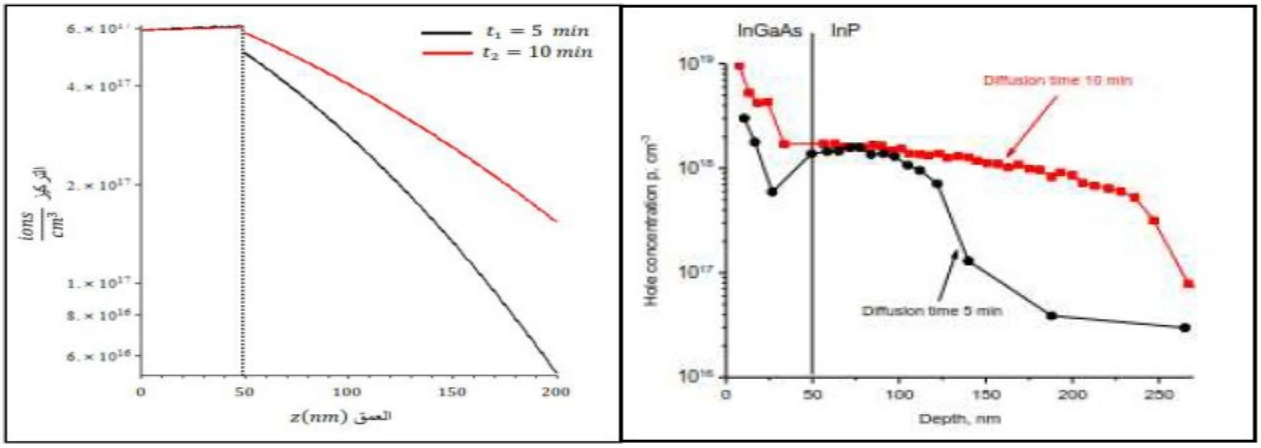


Figure (3): Diffusion profile of the quantum well heterostructures in the case of the Zn diffusion in $InGaAs/InP$ diode. **(a)** our model calculations, **(b)** experimental results [12]

Figure. (3) shows the convergence between our treatment and the practical results of the diffusion of impurities in the quantum well devices despite the approximations adopted in the model to simplify the analytical treatment. Time of diffusion and temperature as well as the segregation coefficient were adopted approached values to the application model.

The rising in the temperature value causes an increase in the diffusion coefficient value as in equation (2), and therefore the diffusion of impurities inside the material will increase.

Table (1):The Diffusion coefficient of Zinc in InP and $InGaAs$ as a function of temperature [15-16].

The diffusion coefficient is directly related to the temperature, the effect of the temperature on the material causes the movement of impurities throughout the solid due to the breakage and formation of bonds, the diffusion of most species does not occur until the temperature is employed.

$D_{\mathrm{InP}}\ (\frac{cm^2}{sec})$	$D_{\mathrm{InGaAs}}\ (\frac{cm^2}{sec})$	T (C^o)
2.9×10^{-12}	9×10^{-13}	500
1.3×10^{-11}	8×10^{-12}	550
6×10^{-11}	4×10^{-11}	600

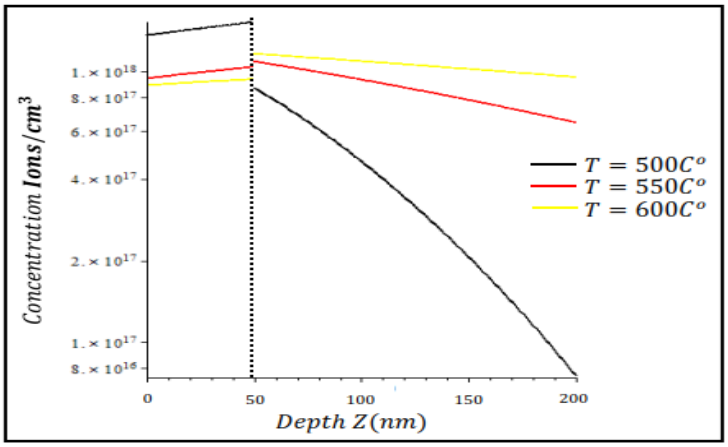


Figure (4): The concentration profile of Zinc ions in $InGaAs/InP$ diode for three different temperatures, with $k = 0.65$, $t = 60\ Sec$ and $C_o = 2.9 \times 10^{18} \frac{ions}{cm^3}$.

The segregation coefficient causes a difference in the electronic and optical properties of the (QW) devices. In the system under study, for the process of dopant diffusion in a two-layer system, the segregation coefficients are denoted by k (between M1 and M2) as in the figure (5).

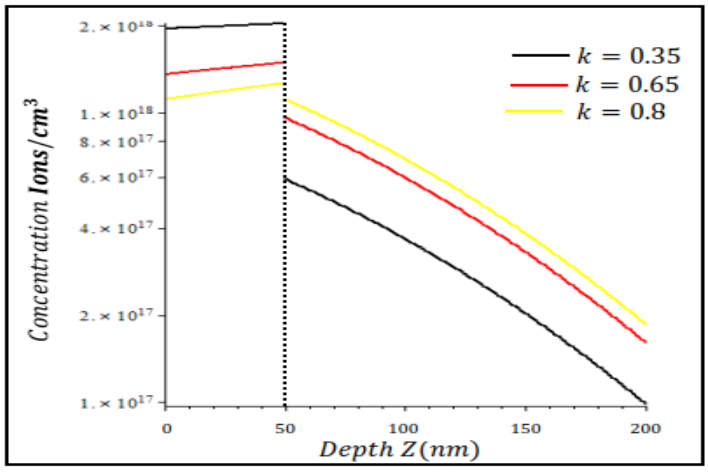


Figure (5): The concentration profile of Zinc ions in $InGaAs/InP$ diode for the three different values of segregation coefficient at T=400 C^o and $t = 15\ min$.

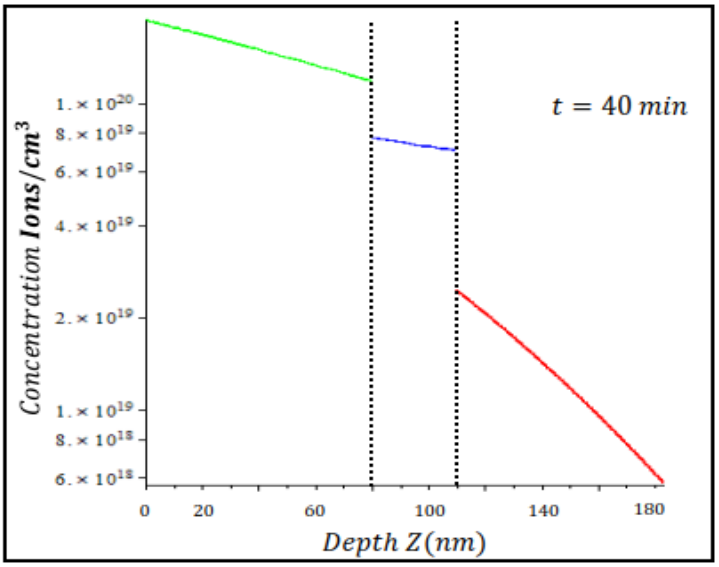
The transistor pnp $InP/In_{0.53}Ga_{0.47}As/InP$, Ohmic contacts to p-type $In_{0.53}Ga_{0.74}As$ have been extensively analyzed because of their important applications as base contacts for InP DHBTs. The InP transistors offer fast diffusive as shown in the table (2) and ballistic electron transport. This makes them particularly attractive for ultra-high-speed applications such as optical ICs operating at over 100 Gbit/s and high data-rate wireless

communications at frequencies beyond 100GHz.

The amount of impurity ions is $C_o = 7 \times 10^{19} \frac{ions}{cm^3}$ and the segregation coefficient $k_1 = 0.65$ and $k_2 = 0.35$, during $t = 40\ min$ [4-17]. The concentration profile of the Si diffusion in $InP/\ In_{0.53}Ga_{0.47}As/InP$ is presented in figure (7).

Table (1): The Diffusion coefficient of Silicon in $InP/\ In_{0.53}Ga_{0.47}As$.

Layers	$InP(80\ nm)$	$In_{0.53}Ga_{0.47}As(30nm)$	$InP(73\ nm)$
Diffusion coefficient	$2 \times 10^{-14} \frac{cm^2}{sec}$	$5.7 \times 10^{-14} \frac{cm^2}{sec}$	$2 \times 10^{-14} \frac{cm^2}{sec}$



Figure(7): The concentration profiles of the Si diffusion in $InP/In_{0.53}Ga_{0.47}As/InP$

The model gave encouraging results for its development in the requirements of the manufacture nanodevices. Our future work will be to adopt the diffusion coefficient as a function of impurities concentration.

5. Conclusion

A theoretical model for dopant diffusion in multi-quantum well heterostructure was introduced by using the system of traditional homogeneous partial differential equations for dopant diffusion. This model is associated with a segregation coefficient for interface multilayer semiconductors and temperature dependant diffusion coefficient. Real manufactured devices were handled to test the model such as the (QW) diode laser and

pnp-transistor that gave a closer approach in terms of the practical results .In future work ، the model will be developed to accommodate the more properties of the dopant diffusion in the manufacture of quantum well devices and deals with numerical or semi-numerical technics in the solution.

References

[1] V. Arivazhagan, Investigation ofQuantum Confinement Effect in pbse znse multiple Quantum Well Structures Prepared by Thermal Evaporation technique, CH.1, (2014).

[2] A. Tapriya, Ultra-shallow phosphorous diffusion in silicon using molecular mono-layer doping, technology Rochester New York, (2017).

[3] I. Yun, and Kyung-Sook Hyun, Zinc diffusion process investigation of InP-based test structures for high-speed avalanche photodiode fabrication, Micro-electronics journal 31.8 :635-639, (2000).

[4] M. Braham, et al, Thermally stable iridium contacts to highly doped p-In0:53Gas:47 As for indium phosphide double hetero-junction bipolar transistors, Microelectronic Engineering 215:111017, (2019).

[5] P. Harrison, Quantum wells wires and quantum dots, University of Leeds, UK, (2005).

[6] E. Arduca, and M. Perego, Doping of silicon nanocrystals, Materials Science in Semiconductor Processing 62, 156-170, (2017).

[7] Fatima H. Saeed, Shaker I. Easa, Jenan M. Al-Mukh, Theoretical Study in Doping Process: Ion-Implantation and Diffusion, LAB LAMPERT Academic Publishing, (2012).

[8] L. Yiheng, Base doping profile control for SiGe PNP HBTs, Diss, University of British Columbia, (2016).

[9] J. Crank, The mathematics of diffusion. Oxford university press, (1979).

[10] A. Aliefendioglu, Fabrication of interdigitated back-contact (ibc) Solar Cell with Screen- Printed Boron Doped Emitters, Diss Middle east Technical University, (2020).

[11] Minke, Mary Ann, The diffusion and segregation coefficient of Germanium in silica, The university of Arizona, (2002).

[12] H. Ceric, et al, Modeling of segregation on material interfaces by means of the finite elementmethod, na, (2003).

[13] J. S. Huang, et al, Defect diffusion model of InGaAs/InP Semiconductor laser degradation, Appl. Phys. Res, 8.1149-157, (2016).

[14] A. Marchevsky, J. Mark, and K. Yvind, Doping technologies For InP membranes on silicon for nano lasers, Novel Enplane Semiconductor Lasers XVIII.Vol.10939. International Society for optics and Photonics, (2019).

[15] R. Jakieła, A. Barcz, E. Wegner, & A. Zagojski, Diffusion and activation of Zn implanted into InP: S, *Vacuum*, *78*(2-4), 417-422, (2005).

[16] P. Ambrée, A. Halbleiter, M. Pilkuhn, & K. H. Wandel, Acceptor diffusion across

InGaAs/InP heterointerfaces, Applied physics letters, *56*(10), 931-933, (1990).
[17] A. Dadgar, et al, Ruthenium: A superior compensator of InP, Applied Physics letters 73.26 :3878-3880, (1998).

دراسة نظرية لانتشار الشوائب المطعمة في التراكيب المغايرة لأجهزة الابار الكمية المتعددة

فاطمة حسين سعيد **محمد حسن خلف**

Email[a]: mohammadhkh335@gmile.com , Email[b]: fatima.saeed@uobasrah.edu.iq

جامعة البصرة- كلية التربية للعلوم الصرف- قسم الفيزياء

المستخلص: في هذه الدراسة تم بناء انموذج نظري لدراسة انتشار الشوائب المطعمة في التراكيب المغايرة وتطبيقة على أجهزة ابار الجهد الكمية باستخدام معادلة قانون فيك الثاني للانتشار، باستخدام دالة الانتشار المعتمدة على الحرارة ومعامل الفصل في حالة الاتزان عند السطح الفاصل بين طبقات أجهزة التراكب المغايرة. اعتمد الانموذج طرق رياضية تحليلية باستخدام تحويلات لابلاس لحل معادلة الانتشار في ضروف مختلفة. تم تطبيق الانموذج على تراكيب عملية مثل كواشف $\boldsymbol{InGaAs/InP}$ وترانزستورات $\boldsymbol{InP/In_{0.53}Ga_{0.47}As/InP}$. النتائج تشجع على تطوير الانموذج واستيعاب أنواع معقدة من أجهزة التراكيب المغايرة. قد يكون الانموذج مرجع مفيد في العمليات المختبرية للسيطرة على انتشار الشوائب المطعمة في أجهزة الابار الكمية بعد تطويرة في المستقبل.

الكلمات المفتاحية: بئر الجهد الكمي ، التركيب المغاير، الانتشار الحراري، أشباه الموصلات، معامل الفصل، معادلة فيك..