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Sadik and complex Sadik integral transforms of system of ordinary differential equations

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ABSTRACT: Integral transformations have a fundamental role in solving differential and integral equations. In this paper, the Sadiq and complex Sadiq integral transformations were discussed in detail through an applied study to solve the initial value problem, including the ODEs system, where the definition of the Sadiq integral transformation and the definition of complex Sadiq integral transformation were presented with mention of some important properties that help in finding solutions to the systems of ordinary differential equations with initial conditions, including the linear property and the derivative property, the inverse of the Sadiq integral transform and the inverse of the complex Sadiq integral transform were addressed, where the inverse of some basic functions were identified in table No. 1, and since the aim of this research is to compare between the Sadiq transform and the complex Sadiq transform to solve systems of ordinary differential equations, we presented three Examples include systems of differential equations, where it was found that the results are identical in the two transformations, and the result shows that the Sadiq integral transformation and the complex Sadiq integral transformation are closely related

Keywords: Sadik transform, complex Sadik transform, system of ODE,. Initial conditions

1. INTRODUCTION

Integral transformations are used in a lot of scientific research and continuously over the years because these transformations give accurate solutions to many complex problems and have many applications such as astronomy, radar and mechanics.

Many researchers have sought to develop several integral transformations, which we mention as follows: Laplace, Formable,Sadik, Mahgoub,Sumuda, Sawi,etc transforms .In 2018, Sadik Ali [1] introduces a novel integral transform called Sadik transform to resolve the linear ordinary differential equations (ODEs). Saed and Emad in [2] used a complex Sadik transform for solve linear higher order ODEs. Higazy and Aggarwal in [3] are solving the system of ODEs by Sawi transformation with application. Patil in [4] the boundary value problem solved by using Aboodh transform and Mahgoub transform and discuss application of system of ODEs. Saleh. et in[5] presented some the properties of Sadik transform for example time delay and integration. Also the authors application this transform in control theory for solving fractional-order dynamical systems. Adem. et in [6] observed the Sumudu transform with convolution terms to solve system ODEs . Rania and Bayan in [7] introduced Formable integral transform for resolving differential equations, both ordinary and partial. Sara ,et in [8] introduced a new transform called Emad-Sara Transform been proposed to resolve linear ODEs whose coefficients are constant.

There are many researchers who have used some types of method to solve different types of equations such as [9-14].

The aim of this work is to compare Sadik transform (ST), complex Sadik transform (SCT) for solving system of ODEs.

This paper is ordered as follows. Second section contains definitions. Properties of Sadik transform, complex Sadik transform are in third section. Fourth section is devoted to Sadik transform, complex Sadik transform of derivatives of the function. Fifth section contains inverse Sadik transform, inverse complex Sadik transform of some elementary functions, Sixth section contains Sadik transform, complex Sadik transform of some elementary functions.

Initial value problems are solved in seventh section titled applications. Last eighth section is devoted for conclusion

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2. DEFINITIONS

2.1 SADIK TRANSFORM (ST)

Definition[15]: The piecewise continuous function $f(t)$ on $(-\infty,\infty)$ disappear on the interval $(-\infty,0)$ and have exponential order δ, where $\delta \in R$ is known as Sadik transformable function.

Let $\aleph$ a collection of all Sadik transformable functions.

Definition [15]: If $f(t) \in \aleph$ then (ST) of $f(t)$ is symbolized by, $S[f(t)]$ which is defined as

$$S[f(t)] = \frac{1}{s^{\beta}} \int_0^{\infty} f(t) e^{-ts^{\alpha}} dt \qquad (1)$$

where , $\beta \in R$ with and $a \neq 0$ is a complex variable**.**

2.2 COMPLEX SADIKL TRANSFORM (CST)

Definition[2]: Analyze functions among the set C which is defined as

$$C = \{f(t) : \exists M, L_1 \, and \, L_2 \, are > 0 \ni |f(t)| < Me^{-\ell L_\iota |t|}, if \ t \in (-1)^{\ell} \times [0,\infty), \iota = 1,2\}$$

Where $\ell \in C$,and $f(t)$ has a finite number M in the set C ,although L_1, L_2 are possibly finite or infinite.

Definition[2]: The Complex Sadik Transform (CST) represents by the operator $S_a^c\{.\}$, which is defined as

$$S_a^c[f(t)] = \frac{1}{s^{\beta}} \int_0^{\infty} f(t) e^{-its^{\alpha}} dt \qquad (2)$$

Where $s \in \square, \alpha \in \square^{+}, \beta \in \square$.

3. PROPERTIES OF SADIK AND COMPLEX SADIK TRANSFORMATIONS

3.1 LINEARITY PROPERTY

A. Linearity Property of Sadik Transform:

If $S[f_1(t)] = F_1(s)$ and $S[f_2(t)] = F_2(s)$ then

$$S[af_1(t) \mp bf_2(t)] == aF_1(s) \mp bF_2(s)$$

Where a and b are arbitrary constants.

,**Proof :** the definition of ST is :

$$S[f(t)] = \frac{1}{s^{\beta}} \int_0^{\infty} f(t) e^{-ts^{\alpha}} dt \quad \text{then}$$

$$S[af_1(t) \mp bf_2(t)] = \frac{1}{s^{\beta}} \int_0^{\infty} (af_1(t) \mp bf_2(t)) e^{-ts^{\alpha}} dt$$
$$= \frac{a}{s^{\beta}} \int_0^{\infty} f_1(t) e^{-ts^{\alpha}} dt \mp \frac{b}{s^{\beta}} \int_0^{\infty} f_2(t) e^{-ts^{\alpha}} dt$$
$$= aF_1(s) \mp bF_2(s)$$

B. Linearity Property of Complex Sadik Transform:

If $S_a^c[f_1(t)] = F_1^c(s)$ and $S_a^c[f_2(t)] = F_2^c(s)$ then

$$S_a^c[af_1(t) \mp bf_2(t)] = aF_1^c(s) \mp bF_2^c(s)$$

Where a and b are arbitrary constants.

Proof : the definition of SCT is :

$$S_a^c[f(t)] = \frac{1}{s^{\beta}} \int_0^{\infty} f(t) e^{-its^{\alpha}} dt \quad \text{then}$$

$$S_a^c\left[a f_1(t) \mp b f_2(t)\right] = \frac{1}{s^\beta}\int_0^\infty \left(a f_1(t) \mp b f_2(t)\right) e^{-its^\alpha} dt$$
$$= \frac{a}{s^\beta}\int_0^\infty f_1(t) e^{-its^\alpha} dt \mp \frac{b}{s^\beta}\int_0^\infty f_2(t) e^{-its^\alpha} dt$$
$$= a F_1^c(s) \mp b F_2^c(s)$$

4. PROPERTIES OF DERIVATIVES OF FUNCTIONS

A-ST of derivatives of functions:

If $F(s)$ is ST of $f(t)$ then :

1- $s^\alpha F(s) - s^\beta f(0)$ is ST of $f'(t)$

2- $s^{2\alpha} F(s) - s^{-\beta} f'(0) - s^{\alpha-\beta} f(0)$ is ST of $f''(t)$

3- $s^{n\alpha} F(s) - \sum_{k=0}^{n-1} s^{k\alpha-\beta} f^{(n-1)-k}(0)$ is ST of $f^{(n)}(t)$

B- CST of derivatives of function:

If $F^c(s)$ is Sadik Transform of $f(t)$ then :

1. $is^\alpha F^c(s) - s^{-\beta} f(0)$ is ST of $f'(t)$

2. $\left(is^\alpha\right)^2 F^c(s) - s^{-\beta} f'(0) - is^{\alpha-\beta} f(0)$ is ST of $f''(t)$

3. $\left(is^\alpha\right)^n F^c(s) - s^{-\beta} \sum_{k=1}^{n} \left(is^\alpha\right)^{k-1} f^{(n-k)}(0)$ is ST of $f^{(n)}(t)$

5. THE INVERSE OF ST AND CST

A- If $S\left[f(t)\right] = F(s)$ is ST then $f(t) = S^{-1}\left[F(s)\right]$ is called an inverse of the ST.

B- If $S_a^c\left[f(t)\right] = F^c(s)$ is the CST then $f(t) = \left(S_a^c\right)^{-1}\left[F^c(s)\right]$ is called an inverse of the CST.

6. THE ST AND CST FOR SOME FUNDAMENTAL FUNCTIONS [2]

In this paragraph, will demonstrate ST as well as the novel SCT for some fundamental functions in the table below:

Table 1. ST and SCT for some fundamental functions

$f(t)$	**Sadik transform** $S\left[f(t)\right] = F(s)$	**Complex Sadik transform** $S_a^c\left[f(t)\right] = F^c(s)$
a,a is constant	$\frac{a}{s^{\beta+\alpha}}$	$\frac{-ia}{s^{\beta+\alpha}}$
t	$\frac{1}{s^{\beta+2\alpha}}$	$\frac{-1}{s^{\beta+2\alpha}}$
$t^n, n \in \mathrm{N}$	$\frac{n!}{s^{n\alpha+(\beta+\alpha)}}$	$(-i)^{n+1}\frac{n!}{s^{n\alpha+(\beta+\alpha)}}$

e^{at}	$\dfrac{1}{s^{\beta}\left(s^{\alpha}-a\right)}$	$\dfrac{-1}{s^{\beta}}\left[\dfrac{a}{s^{2\alpha}+a^{2}}+i\,\dfrac{s^{\alpha}}{s^{2\alpha}+a^{2}}\right]$
$\sin at$	$\dfrac{1}{s^{\beta}}\left(\dfrac{a}{s^{2\alpha}+a^{2}}\right)$	$\dfrac{1}{s^{\beta}}\left(\dfrac{-a}{s^{2\alpha}-a^{2}}\right)$
$\cos at$	$\dfrac{1}{s^{\beta}}\left(\dfrac{s^{\alpha}}{s^{2\alpha}+a^{2}}\right)$	$\dfrac{1}{s^{\beta}}\left(\dfrac{-is^{\alpha}}{s^{2\alpha}-a^{2}}\right)$
$\sinh at$	$\dfrac{1}{s^{\beta}}\left(\dfrac{a}{s^{2\alpha}-a^{2}}\right)$	$\dfrac{1}{s^{\beta}}\left(\dfrac{-a}{s^{2\alpha}+a^{2}}\right)$
$\cosh at$	$\dfrac{1}{s^{\beta}}\left(\dfrac{s^{\alpha}}{s^{2\alpha}-a^{2}}\right)$	$\dfrac{1}{s^{\beta}}\left(\dfrac{-is^{\alpha}}{s^{2\alpha}+a^{2}}\right)$

7. APPLICATION

The numerical applications to solve the initial value problem of systems of ODEs are discussed in the section by using Sadik transform and complex Sadik transform:

Let $S[x]=F_1(s), S[y]=F_2(s), S[z]=F_3(s), S_a^c[x]=F_1^c(s), S_a^c[y]=F_2^c(s), S_a^c[y]=F_3^c(s)$

Example 1: Consider the system of equations

$$x'+4y=0 \tag{3}$$

$$x+y'=t \tag{4}$$

With initial conditions : $x(0)=-1, y(0)=0$

A.Solution by ST

Take Sadik transform of both sides of (3) and (4) and by using properties we get

$$s^{\alpha}F_1(s)-s^{-\beta}x(0)+4F_2(s)=0$$

$$F_1(s)+s^{\alpha}F_2(s)-s^{-\beta}y(0)=\frac{1}{s^{\beta+2\alpha}}$$

So

$$s^{\alpha}F_1(s)+4F_2(s)=\frac{-1}{s^{\beta}} \tag{5}$$

$$F_1(s)+s^{\alpha}F_2(s)=\frac{1}{s^{\beta+2\alpha}} \tag{6}$$

Multiplying equation (6) by s^{α} and adding the obtained equations we get ,

$$F_2(s)=\frac{1}{s^{\beta}\left(s^{2\alpha}-4\right)}+\frac{1}{s^{\beta+\alpha}\left(s^{2\alpha}-4\right)}$$

So

$$F_2(s)=\frac{1}{s^{\beta}\left(s^{2\alpha}-4\right)}+\frac{1/4}{s^{\beta+\alpha}}+\frac{s^{\alpha}/4}{s^{\beta+\alpha}\left(s^{2\alpha}-4\right)} \tag{7}$$

Substations (7) in (5) we get

$$F_1(s)=\frac{1}{s^{\beta+2\alpha}}-\frac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}-4\right)}-\frac{s^{\alpha}}{s^{\beta+\alpha}\left(s^{2\alpha}-4\right)} \tag{8}$$

Applying inverse Sadik transform to eq. (7) and (8).

$$S^{-1}\left\{F_2(s)\right\}=S^{-1}\left\{\frac{1}{s^{\beta}\left(s^{2\alpha}-4\right)}\right\}+S^{-1}\left\{\frac{1/4}{s^{\beta+\alpha}}\right\}+S^{-1}\left\{\frac{s^{\alpha}/4}{s^{\beta+\alpha}\left(s^{2\alpha}-4\right)}\right\}$$

And $S^{-1}\left\{F_1(s)\right\}=S^{-1}\left\{\frac{1}{s^{\beta+2\alpha}}\right\}-S^{-1}\left\{\frac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}-4\right)}\right\}-S^{-1}\left\{\frac{s^{\alpha}}{s^{\beta+\alpha}\left(s^{2\alpha}-4\right)}\right\}$

Then

$$y=\frac{1}{2}\sinh 2t+\frac{1}{4}\cosh 2t-\frac{1}{4}, x=t-\cosh 2t+\frac{1}{2}\sinh 2t$$

B-Solution by CST

Take Complex Sadik transform of both sides of (3) and (4) and using properties we get

$$is^{\alpha}F_1^{c}(s)-s^{-\beta}x(0)+4F_2^{c}(s)=0$$

$$F_1^{c}(s)+is^{\alpha}F_2^{c}(s)-s^{-\beta}y(0)=\frac{-1}{s^{\beta+2\alpha}}$$

So

$$is^{\alpha}F_1^{c}(s)+4F_2^{c}(s)=\frac{-1}{s^{\beta}} \tag{9}$$

$$F_1^{c}(s)+is^{\alpha}F_2^{c}(s)=\frac{-1}{s^{\beta+2\alpha}} \tag{10}$$

Multiplying equation (10) by is^{α} and adding the obtained equations we get ,

$$F_2^{c}(s)=\frac{-1}{s^{\beta}\left(s^{2\alpha}+4\right)}+\frac{is^{\alpha}}{s^{\beta+2\alpha}\left(s^{2\alpha}+4\right)}$$

$$F_2^{c}(s)=\frac{-1}{s^{\beta}\left(s^{2\alpha}+4\right)}+\frac{i/4}{s^{\alpha+\beta}}+\frac{-is^{\alpha}/4}{s^{\beta+2\alpha}\left(s^{2\alpha}+4\right)} \tag{11}$$

Substations (11) in (9) we get

$$F_1^{c}(s)=\frac{-1}{s^{\beta+2\alpha}}+\frac{is^{\alpha}}{s^{\beta}\left(s^{2\alpha}+4\right)}+\frac{s^{2\alpha}}{s^{\beta+2\alpha}\left(s^{2\alpha}+4\right)} \tag{12}$$

Applying inverse Complex Sadik transform to eq (11) and (12).

$$S^{-1}\left\{F_2^{c}(s)\right\}=-S^{-1}\left\{\frac{1}{s^{\beta}\left(s^{2\alpha}+4\right)}\right\}+1/4\,S^{-1}\left\{\frac{i}{s^{\alpha+\beta}}\right\}-1/4\,S^{-1}\left\{\frac{is^{\alpha}}{s^{\beta+2\alpha}\left(s^{2\alpha}+4\right)}\right\}$$

And $S^{-1}\left\{F_1^{c}(s)\right\}=-S^{-1}\left\{\frac{-1}{s^{\beta+2\alpha}}\right\}-S^{-1}\left\{\frac{-is^{\alpha}}{s^{\beta}\left(s^{2\alpha}+4\right)}\right\}+S^{-1}\left\{\frac{s^{2\alpha}}{s^{\beta+2\alpha}\left(s^{2\alpha}+4\right)}\right\}$

Then $y=\frac{1}{2}\sinh 2t+\frac{1}{4}\cosh 2t-\frac{1}{4}, x=t-\cosh 2t+\frac{1}{2}\sinh 2t$

From A and B we notice that the values of x and y are the same

Example 2: Consider the system of equations

$$x'' + y' = \cos t \tag{13}$$

$$y'' - x = \sin t \tag{14}$$

With initial conditions : $x(0) = -1, x'(0) = -1, y(0) = 1, y'(0) = 0$

A-Solution by ST

Take Sadik transform of both sides of (13) and (14) and using properties we get :

$$s^{2\alpha}F_1(s) - s^{-\beta}x'(0) - s^{\alpha-\beta}x(0) + s^{\alpha}F_2(s) - s^{-\beta}y(0) = \frac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)}$$

,

$$s^{2\alpha}F_2(s) - s^{-\beta}y'(0) - s^{\alpha-\beta}y(0) - F_1(s) = \frac{1}{s^{\beta}\left(s^{2\alpha}+1\right)}$$

So

$$s^{2\alpha}F_1(s) + s^{\alpha}F_2(s) = \frac{-s^{3\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)} \tag{15}$$

$$s^{2\alpha}F_2(s) - F_1(s) = \frac{1+s^{3\alpha}+s^{\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)} \tag{16}$$

Multiplying equation (16) by $s^{2\alpha}$ and adding the obtained equations we get ,

$$F_2(s) = \frac{s^{5\alpha}+s^{2\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)\left(s^{4\alpha}+s^{\alpha}\right)}$$

Then $F_2(s) = \dfrac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)}$ (17)

Substations (17) in (15) we get

$$F_1(s) = \frac{-1}{s^{\beta}\left(s^{2\alpha}+1\right)} - \frac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)} \tag{18}$$

Applying inverse Sadik transform to eq (17) and (18) we get :

$$S^{-1}\{F_2(s)\} = S^{-1}\left\{\frac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)}\right\}, S^{-1}\{F_1(s)\} = -S^{-1}\left\{\frac{1}{s^{\beta}\left(s^{2\alpha}+1\right)}\right\} - S^{-1}\left\{\frac{s^{\alpha}}{s^{\beta}\left(s^{2\alpha}+1\right)}\right\}$$

So $y = \cos t, x = -\sin t - \cos t$

B-Solution by CST

Take Complex Sadik transform of both sides of (13) and (14) and by using properties we get

$$\left(is^{\alpha}\right)^2 F_1^c(s) - s^{-\beta}x'(0) - is^{\alpha-\beta}x(0) + is^{\alpha}F_2^c(s) - s^{-\beta}y(0) = \frac{-is^{\alpha}}{s^{\beta}\left(s^{2\alpha}-1\right)}$$

$$\left(is^{\alpha}\right)^2 F_2^c(s) - s^{-\beta}y'(0) - is^{\alpha-\beta}y(0) - F_1^c(s) = \frac{-1}{s^{\beta}\left(s^{2\alpha}+1\right)}$$

So

$$\left(is^{\alpha}\right)^{2}F_{1}^{c}\left(s\right)+is^{\alpha}F_{2}^{c}\left(s\right)=\frac{-is^{\alpha}}{s^{\beta}\left(s^{2\alpha}-1\right)}-is^{\alpha-\beta} \tag{19}$$

$$\left(is^{\alpha}\right)^{2}F_{2}^{c}\left(s\right)-F_{1}^{c}\left(s\right)=\frac{-1}{s^{\beta}\left(s^{2\alpha}+1\right)}+is^{\alpha-\beta} \tag{20}$$

Multiplying equation (20) by $s^{2\alpha}$ and adding the obtained equations we get ,

$$F_{2}^{c}\left(s\right)=\frac{s^{2\alpha}-is^{5\alpha}}{is^{\beta}\left(s^{2\alpha}-1\right)\left(s^{\alpha}-is^{4\alpha}\right)}$$

$$F_{2}^{c}\left(s\right)=\frac{-is^{\alpha}}{s^{\beta}\left(s^{2\alpha}-1\right)} \tag{21}$$

Substations (21) in (19) we get

$$F_{1}^{c}\left(s\right)=\frac{1}{s^{\beta}\left(s^{2\alpha}-1\right)}+\frac{is^{\alpha}}{s^{\beta}\left(s^{2\alpha}-1\right)} \tag{22}$$

Applying inverse Complex Sadik transform to eq (21) and (22).

$$S^{-1}\left\{F_{2}^{c}\left(s\right)\right\}=-S^{-1}\left\{\frac{is^{\alpha}}{s^{\beta}\left(s^{2\alpha}-1\right)}\right\}$$

$$S^{-1}\left\{F_{1}^{c}\left(s\right)\right\}=S^{-1}\left\{\frac{1}{s^{\beta}\left(s^{2\alpha}-1\right)}\right\}+S^{-1}\left\{\frac{is^{\alpha}}{s^{\beta}\left(s^{2\alpha}-1\right)}\right\}$$

Then $y=\cos t, x=-\sin t-\cos t$

So the values are itself from A and B

Example 3: The mathematical model telling the dynamics of the colonic cell population [16] is identifiable by the following SODEs:

$$x'=\left(a_{3}-a_{4}-a_{5}\right)x \tag{23}$$

$$y'=\left(a_{6}-a_{4}-a_{5}\right)y+a_{2}x \tag{24}$$

$$z'=a_{4}y-\mu z \tag{25}$$

In terms of initial conditions: $x\left(0\right)=c_{1}, y\left(0\right)=c_{2}, z\left(0\right)=c_{3}$

The above model's natural explanation of its parameters is shown in

Table 2 - The parameters' natural explanation.

Parameter	Meaning
$x(t)$	No. of stem cells
$y(t)$	No. of of cells that are semi-differentiated
$z(t)$	No. of differentiated cells
a_1	stem cell death rates
a_2	No. of stem cells that turned semi-differentiated
a_3	the rate of stem cell cellular renewal
a_4	the proportion of semi-differentiated cells that die
a_5	No. of the cells that transition from semi-differentiated

a_6	state to differentiated cells
	the semi-differentiated cells' rates of cell renewal
μ	the number of differentiated cells removed from the crypt

Let $A=(a_3-a_1-a_2), B=(a_6-a_4-a_5)$

A- Solution by ST

Take Sadik transform of both sides of (23),(24) and (25) and using properties we get :

$$s^{\alpha}F_1(s)-s^{-\beta}x(0)=AF_1(s)$$
$$s^{\alpha}F_2(s)-s^{-\beta}y(0)=BF_2(s)+F_1(s)$$
$$s^{\alpha}F_3(s)-s^{-\beta}z(0)=a_4F_2(s)-\mu F_3(s)$$

So

$$s^{\alpha}F_1(s)-AF_1(s)=\frac{c_1}{s^{\beta}} \quad (26)$$

$$s^{\alpha}F_2(s)-BF_2(s)-F_1(s)=\frac{c_2}{s^{\beta}} \quad (27)$$

$$s^{\alpha}F_3(s)-a_4F_2(s)+\mu F_3(s)=\frac{c_3}{s^{\beta}} \quad (28)$$

By eq (26) we get ,

$$F_1(s)=\frac{c_1}{s^{\beta}(s^{\alpha}-A)} \quad (29)$$

Substation (29) in (27) we get

$$F_2(s)=\frac{a_2c_1}{s^{\beta}(s^{\alpha}-A)(s^{\alpha}-B)}+\frac{c_2}{s^{\beta}(s^{\alpha}-B)} \quad (30)$$

Substation (30) in (28) we get

$$F_3(s)=\frac{c_3}{s^{\beta}(s^{\alpha}-\mu)}+\frac{a_4a_2c_1/A-B}{s^{\beta}(s^{\alpha}-A)(s^{\alpha}-\mu)}+\frac{(a_4a_2c_1/A-B)+a_4c_2}{s^{\beta}(s^{\alpha}-B)(s^{\alpha}-\mu)} \quad (31)$$

Applying inverse Sadik transform to eqs (29) ,(30)and (31).

$$S^{-1}\{F_1(s)\}=S^{-1}\left\{\frac{c_1}{s^{\beta}(s^{\alpha}-A)}\right\}, S^{-1}\{F_2(s)\}=S^{-1}\left\{\frac{a_2c_1}{s^{\beta}(s^{\alpha}-A)(s^{\alpha}-B)}\right\}+S^{-1}\left\{\frac{c_2}{s^{\beta}(s^{\alpha}-A)}\right\}$$

$$S^{-1}\{F_3(s)\}=S^{-1}\left\{\frac{c_3}{s^{\beta}(s^{\alpha}-\mu)}\right\}+S^{-1}\left\{\frac{a_4a_2c_1/A-B}{s^{\beta}(s^{\alpha}-A)(s^{\alpha}-\mu)}\right\}+S^{-1}\left\{\frac{(a_4a_2c_1/A-B)+a_4c_2}{s^{\beta}(s^{\alpha}-B)(s^{\alpha}-\mu)}\right\}$$

Then $x=c_1e^{At}, y=\frac{a_4a_2c_1}{A-B}e^{At}-\frac{a_4a_2c_1}{A-B}e^{Bt}+c_2e^{Bt}$

$$z=\left[c_3-\frac{a_4a_2c_1}{(A-B)(A-\mu)}-\frac{(a_4a_2c_1/A-B)+a_4c_2}{(B-\mu)}\right]e^{\mu t}-\frac{a_4a_2c_1}{(A-B)(A-\mu)}e^{At}-\frac{(a_4a_2c_1/A-B)+a_4c_2}{(B-\mu)}e^{Bt}$$

B-Solution by CST

Take complex Sadik transform of both sides of (23),(24) and (25) and using properties we get :

$$is^{\alpha}F_1^c(s)-s^{-\beta}x(0)=AF_1^c(s)$$
$$is^{\alpha}F_2^c(s)-s^{-\beta}y(0)=BF_2^c(s)+F_1^c(s)$$
$$is^{\alpha}F_3^c(s)-s^{-\beta}z(0)=a_4F_2^c(s)-\mu F_3^c(s)$$

$$is^{\alpha}F_1^c(s) - AF_1^c(s) = \frac{c_1}{s^{\beta}} \tag{32}$$

$$is^{\alpha}F_2^c(s) - BF_2^c(s) - F_1^c(s) = \frac{c_2}{s^{\beta}} \tag{33}$$

$$is^{\alpha}F_3^c(s) - a_4F_2^c(s) + \mu F_3^c(s) = \frac{c_3}{s^{\beta}} \tag{34}$$

By eq (32) we get ,

$$F_1^c(s) = \frac{c_1}{s^{\beta}\left(is^{\alpha} - A\right)} \tag{35}$$

Substation (35) in (33) we get

$$F_2^c(s) = \frac{-a_2c_1}{s^{\beta}\left(is^{\alpha} - A\right)\left(is^{\alpha} - B\right)} + \frac{c_2}{s^{\beta}\left(is^{\alpha} - A\right)} \tag{36}$$

Substation (36) in (34) we get

$$F_3^c(s) = \frac{c_3}{s^{\beta}\left(is^{\alpha} - \mu\right)} + \frac{a_4a_2c_1/A - B}{s^{\beta}\left(is^{\alpha} - A\right)\left(is^{\alpha} - \mu\right)} + \frac{\left(a_4a_2c_1/A - B\right) + a_4c_2}{s^{\beta}\left(is^{\alpha} - B\right)\left(is^{\alpha} - \mu\right)} \tag{37}$$

Applying inverse complex Sadik transform to eqs (35) ,(36)and (37) we get

$$S^{-1}\left\{F_1^c(s)\right\} = S^{-1}\left\{\frac{c_1}{s^{\beta}\left(is^{\alpha} - A\right)}\right\}$$

$$S^{-1}\left\{F_2^c(s)\right\} = S^{-1}\left\{\frac{-a_2c_1}{s^{\beta}\left(is^{\alpha} - A\right)\left(is^{\alpha} - B\right)}\right\} + S^{-1}\left\{\frac{c_2}{s^{\beta}\left(is^{\alpha} - A\right)}\right\}$$

$$S^{-1}\left\{F_3^c(s)\right\} = S^{-1}\left\{\frac{c_3}{s^{\beta}\left(is^{\alpha} - \mu\right)}\right\} + S^{-1}\left\{\frac{a_4a_2c_1/A - B}{s^{\beta}\left(is^{\alpha} - A\right)\left(is^{\alpha} - \mu\right)}\right\} + S^{-1}\left\{\frac{\left(a_4a_2c_1/A - B\right) + a_4c_2}{s^{\beta}\left(is^{\alpha} - B\right)\left(is^{\alpha} - \mu\right)}\right\}$$

Then $x = c_1e^{At}, y = \frac{a_4a_2c_1}{A - B}e^{At} - \frac{a_4a_2c_1}{A - B}e^{Bt} + c_2e^{Bt}$

$$z = \left[c_3 - \frac{a_4a_2c_1}{(A - B)(A - \mu)} - \frac{\left(a_4a_2c_1/A - B\right) + a_4c_2}{(B - \mu)}\right]e^{\mu t} - \frac{a_4a_2c_1}{(A - B)(A - \mu)}e^{At} - \frac{\left(a_4a_2c_1/A - B\right) + a_4c_2}{(B - \mu)}e^{Bt}$$

From A and B we notice that the values of x ,yand z are the same

8- CONCLUSION

Sadiq transformation and complex Sadiq transformation are new transformations. There are many problems in engineering and applied sciences that can be considered to be solved by the above transformations. The main objective of this paper is to make a comparison between the Sadiq transformation and the complex Sadiq transformation to solve a system of ODE. In the applications section, some numerical examples were obtained to justify our findings where we solve system ODEs using both Sadik transform and Complex Sadik transformation. Numerical applications show that both transformations are closely related to each other.

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CONFLICT OF INTEREST:

Author declare no conflict of interest

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