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Class Nearly kahler Manifold of W – Projective Curvature Tensor

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ABSTRACT

The current study deals with new three classes of the nk ("nearly kahler") manifold of w – projective curvature tensor. The aim of this paper to calculate differential - geometrical and topological properties closest for new classes $\bar{w}_1, \bar{w}_2$, and $\bar{w}_3$, through it ,an equivalence relationship was obtained between these classes and one of or more the tensor compounds and the components of curvature tensor and with adjoint G-structure space. Finally, we discover a relationship between w_1, w_2, w_3 with each other.

Introduction

The concept of AH-Almost Hermitian structures states that there is a general rule for classifying AH-structures using the second order symmetry features of the Riemann-Christoffel tensor's invariants of differential geometry. Based on the theory advanced by A. Gray and developed within a number of respective works [1] and [2]the important to understanding the differential-geometrical characteristics of Kahler manifolds is to establish the identities of them that satisfies. Gray and Hervella[3] founded that the action of the unitary group $U(n)$ on the space of all tensors of type $(3,0)$ decomposed this space in to sixteen classes. Following are the components that make up the Riemann curvature tensor:

$\bar{W}_1$ if $\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(\alpha, \beta)Q\theta, Q\gamma \rangle$;

$\bar{W}_2$ if $\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(Q\alpha, Q\beta)\theta, \gamma \rangle + \langle W(Q\alpha, \beta)Q\theta, \gamma \rangle + \langle W(Q\alpha, \beta)Q\theta, Q\gamma \rangle$;

$\bar{W}_3$ if $\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(Q\alpha, Q\beta)Q\theta, Q\gamma \rangle$.

The AH-structures belonging to the class W_i have a tensor R that fulfills the identity W_i . If AH-any subclass of H-structures is named $\cap W_i = 0$, where i is 1, 2, or 3,

In this paper, we will generalize these relationships, definitions and theories related to them for NK – Nearly Kahler manifold of W – projective curvature tensor. In 2018 Ali A. Shihab and Dhabia'a M. Ali where studied classes of almost Hermitian manifold [4]. In the study also concentrates generalized conharmonic curvature tensor of Vaisman -Gray manifold.

Preliminaries

Assume that M is a smooth manifold of dimension- $2n$; $C^\infty(M)$ is algebra of smooth functions on M ; $\alpha(M)$ the module of smooth vector fields on M ; and that $g = \langle \cdot, \cdot \rangle$ – Riemannian metrics; $\tilde{N}$ – Riemannian connection of the metrics g on M ; d - the operator of exterior differentiation. Additional all manifold, Tensor fields, and other objects are assumed to be of class C^∞ . So Almost Hermitian (is shorter, AH) structure on a manifold M the pair (Q, g) , where Q -almost complex structure ($Q^2 = \text{id}$) on M , $g = \langle \cdot, \cdot \rangle$ – (pseudo) Riemannian metric on M . In this case $\langle Q\alpha, Q\beta \rangle = \langle \alpha, \beta \rangle$; $\alpha, \beta \in \alpha(M)$.

Definition 1 [5]

The manifold (W, Q, g) denotes to as manifold of a class:

- 1) $\bar{W}_1$ if $\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(\alpha, \beta)Q\theta, Q\gamma \rangle$;
- 2) $\bar{W}_2$ if $\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(Q\alpha, Q\beta)\theta, \gamma \rangle + \langle W(Q\alpha, \beta)Q\theta, \gamma \rangle + \langle W(\alpha, \beta)Q\theta, Q\gamma \rangle$;
- 3) $\bar{W}_3$ if $\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(Q\alpha, Q\beta)Q\theta, Q\gamma \rangle$.

Note 2

NK – manifold of class

$$W_0 = W_3 = W_5 = W_6$$

Which are also manifold of a class $\bar{W}_3$.

It is most clear when the curvature identities are expressed in terms of a spectrum. Generalized projective curvature tensor.

Theorem 3

Assume that $Z = (Q, g = \langle \cdot, \cdot \rangle)$ is NK ("Nearly Kahler") structure. Consequently, the following propositions are equivalent:

- (1) Z- Structure of a class $\bar{W}_3$;
- (2) $W_{(0)} = 0$; and
- (3) The identities $W_{bcd}^a = 0$ on space of the adjoint G-structure are acceptable.

proof.

Assume that Z – structure of a class $\bar{W}_3$. Clearly, it is equal to identity $W(\alpha, \beta)\theta + QW(Q\alpha, Q\beta)Q\theta = 0$; $\alpha, \beta, \theta \in \alpha(M)$.

By definition of a spectrum tensor

$$W(\alpha, \beta)\theta = W_0(\alpha, \beta)\theta + W_1(\alpha, \beta)\theta + W_2(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_4(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta + W_7(\alpha, \beta)\theta; \alpha, \beta, \theta \in \alpha(M).$$

$$\begin{aligned} Q \circ W(Q\alpha, Q\beta)Q\theta &= Q \circ W_0(Q\alpha, Q\beta)Q\theta + Q \circ W_1(Q\alpha, Q\beta)Q\theta + Q \circ W_2(Q\alpha, Q\beta)Q\theta + Q \circ W_3(Q\alpha, Q\beta)Q\theta + Q \circ W_4(Q\alpha, Q\beta)Q\theta + Q \circ W_5(Q\alpha, Q\beta)Q\theta + Q \circ W_6(Q\alpha, Q\beta)Q\theta + Q \circ W_7(Q\alpha, Q\beta)Q\theta \\ &= W_0(\alpha, \beta)\theta - W_1(\alpha, \beta)\theta - W_2(\alpha, \beta)\theta - W_3(\alpha, \beta)\theta - W_4(\alpha, \beta)\theta - W_5(\alpha, \beta)\theta - W_6(\alpha, \beta)\theta - W_7(\alpha, \beta)\theta; \alpha, \beta, \theta \in \alpha(M). \end{aligned}$$

Putting term by these identities, will be received:

$$W(\alpha, \beta)\theta + QW(Q\alpha, Q\beta)Q\theta = \{W_0(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta\}.$$

With means, the identity $W(\alpha, \beta)\theta + QW(Q\alpha, Q\beta)Q\theta = 0$ is equivalent to that $W_0(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta = 0$ and this identity is equivalent to identities $W_0 = W_3 = W_5 = W_6 = 0$.

the established characteristics of the adjoint G-space structure's are equal to relations:

$$W_{bcd}^a = W_{bcd}^a = W_{bcd}^a = W_{bcd}^a = W_{bcd}^a.$$

By virtue of materiality tensor W received relations which are same to relations

$$W_{bcd}^a = 0, \text{ i.e. identity } W_0(\alpha, \beta)\theta = 0.$$

Theorem 4

Assume that $Z = (Q, g = \langle \cdot, \cdot \rangle)$ be NK ("Nearly Kahler") structure, Consequently, the following propositions are equivalent

- (1) Z- Structure of a class $\bar{W}_2$;
- (2) $W_0 = W_7 = 0$; and
- (3) The identities $W_{bcd}^a = W_{bcd}^a = 0$ on space of the attached G-structure are acceptable.

Proof:

Assume that Z- structure of a class $\bar{W}_2$. We shall duplicate identity $\bar{W}_2$ in with everyone in place, this identity will be computed using the notion of a spectrum tensor as follows:

$$W(\alpha, \beta)\theta = W_0(\alpha, \beta)\theta + W_1(\alpha, \beta)\theta + W_2(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_4(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta + W_7(\alpha, \beta)\theta;$$

$$\begin{aligned} 1) W(Q\alpha, Q\beta)\theta &= W_0(Q\alpha, Q\beta)\theta + W_1(Q\alpha, Q\beta)\theta + W_2(Q\alpha, Q\beta)\theta + W_3(Q\alpha, Q\beta)\theta + W_4(Q\alpha, Q\beta)\theta + W_5(Q\alpha, Q\beta)\theta + W_6(Q\alpha, Q\beta)\theta + W_7(Q\alpha, Q\beta)\theta \\ &= -W_0(\alpha, \beta)\theta + W_1(\alpha, \beta)\theta + W_2(\alpha, \beta)\theta - W_3(\alpha, \beta)\theta - W_4(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta - W_7(\alpha, \beta)\theta; \end{aligned}$$

$$\begin{aligned} 2) W(Q\alpha, \beta)Q\theta &= W_0(Q\alpha, \beta)Q\theta + W_1(Q\alpha, \beta)Q\theta + W_2(Q\alpha, \beta)Q\theta + W_3(Q\alpha, \beta)Q\theta + W_4(Q\alpha, \beta)Q\theta + W_5(Q\alpha, \beta)Q\theta + W_6(Q\alpha, \beta)Q\theta + W_7(Q\alpha, \beta)Q\theta \\ &= -W_0(\alpha, \beta)\theta - W_1(\alpha, \beta)\theta + W_2(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_4(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta - W_6(\alpha, \beta)\theta - W_7(\alpha, \beta)\theta. \end{aligned}$$

$$\begin{aligned} 3) QW(Q\alpha, \beta)\theta &= QW_0(Q\alpha, \beta)\theta + QW_1(Q\alpha, \beta)\theta + QW_2(Q\alpha, \beta)\theta + QW_3(Q\alpha, \beta)\theta + QW_4(Q\alpha, \beta)\theta + QW_5(Q\alpha, \beta)\theta + QW_6(Q\alpha, \beta)\theta + QW_7(Q\alpha, \beta)\theta \\ &= -W_0(\alpha, \beta)\theta - W_1(\alpha, \beta)\theta + W_2(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta - W_4(\alpha, \beta)\theta - W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta + W_7(\alpha, \beta)\theta. \end{aligned}$$

Substituting these breakdown in the prior equality, we will obtain:

$$\begin{aligned} W(\alpha, \beta)\theta - W(Q\alpha, Q\beta)\theta - W(Q\alpha, \beta)Q\theta + QW(Q\alpha, \beta)\theta + QW(Q\alpha, \beta)\theta &= \\ 2\{W_0(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta + W_7(\alpha, \beta)\theta\} \end{aligned}$$

This identity is equivalent to that

$$W_0(\alpha, \beta)\theta = W_3(\alpha, \beta)\theta = W_5(\alpha, \beta)\theta = W_7(\alpha, \beta)\theta = 0.$$

Additionally, these identities on the space of the adjoint G-structure are comparable to identities.

$$W_{bcd}^a = W_{bcd}^a = W_{bcd}^a = W_{bcd}^a = W_{bcd}^a.$$

The received relations are identical to these by reason of the materiality tensor W : $W_{bcd}^a = W_{bcd}^a$, i.e. to identities $W_0(\alpha, \beta)\theta = W_0(\alpha, \beta)\theta$.

Back, let for NK- manifold identities $W_0(\alpha, \beta)\theta = W_7(\alpha, \beta)\theta = 0$ are executed.

Then we have:

$$W(\alpha, \beta)\theta - W(Q\alpha, Q\beta)\theta - W(Q\alpha, \beta)Q\theta - W(Q\alpha, \beta)Q\theta = 0$$

i.e.

$$W(\alpha, \beta)\theta = W(Q\alpha, \beta)Q\theta = W(Q\alpha, Q\beta)\theta = W(\alpha, Q\beta)\theta$$

In the received identity instead of $W(\alpha, Q\beta)\theta$ we shall put the value $\beta \rightarrow Q\beta$ and $\alpha \rightarrow Q\alpha$, i.e. $W(Q\alpha, Q\beta)Q\theta = -QW(Q\alpha, \beta)\theta$.

Then

$$W(\alpha, \beta)\theta = W(Q\alpha, Q\beta)\theta + W(Q\alpha, \beta)Q\theta - QW(Q\alpha, \beta)\theta$$

i.e.

As a result, the manifold fulfills the identity $\bar{W}_2$.

It is also proven by the next theorem.

Theorem 5

Assume that $Z = (Q, g = \langle \cdot, \cdot \rangle)$ be NK ("Nearly Kahler") structure. Consequently, the following propositions are equivalent to:

- (1) θ -structure of a class $\bar{W}_1$;
- (2) $W_0 = W_4 = W_7 = 0$;
- (3) The identities $W_{bcd}^a = W_{bcd}^a = W_{bcd}^a$ on space of the attached G-structure are acceptable.

Proof :

Assume that Z is a structure of a class $\bar{W}_1$. Clearly, it is comparable to identity

$$\langle W(\alpha, \beta)\theta, \gamma \rangle = \langle W(\alpha, \beta)Q\theta, Q\gamma \rangle$$

and we get $W(\alpha, \beta)\theta + QW(\alpha, \beta)Q\theta = 0$; $\alpha, \beta, \theta \in \alpha(M)$.

By definition of a spectrum tensor

$$1) W(\alpha, \beta)\theta = W_0(\alpha, \beta)\theta + W_1(\alpha, \beta)\theta + W_2(\alpha, \beta)\theta + W_3(\alpha, \beta)\theta + W_4(\alpha, \beta)\theta + W_5(\alpha, \beta)\theta + W_6(\alpha, \beta)\theta + W_7(\alpha, \beta)\theta; \alpha, \beta, \theta \in \alpha(M).$$

$$2) Q_0 W(\alpha, \beta)Q\theta = Q_0 W_0(\alpha, \beta)Q\theta + Q_0 W_1(\alpha, \beta)Q\theta + Q_0 W_2(\alpha, \beta)Q\theta + Q_0 W_3(\alpha, \beta)Q\theta + Q_0 W_4(\alpha, \beta)Q\theta + Q_0 W_5(\alpha, \beta)Q\theta + Q_0 W_6(\alpha, \beta)Q\theta + Q_0 W_7(\alpha, \beta)Q\theta; = -W_0(\alpha, \beta)\theta -$$

$$W_1(\alpha, \beta)\theta - W_2(\alpha, \beta)\theta - W_3(\alpha, \beta)\theta +$$

$$W_4(\alpha, \beta)\theta - W_5(\alpha, \beta)\theta - W_6(\alpha, \beta)\theta + W_7(\alpha, \beta)\theta;$$

$\alpha, \beta, \theta \in \alpha(M)$. Putting (1) and (2) in

$W(\alpha, \beta)\theta + QW(\alpha, \beta)Q\theta$ means, this identity is equivalent to that

$$W_{(0)}(\alpha, \beta)\theta + W_4(\alpha, \beta)\theta W_0(\alpha, \beta)\theta = 0.$$

And this identity is equivalent to identities $W_{(0)} = W_4 = W_7 = 0$.

The established identities in space of the adjoint G-structure are equal to relations:

$$W_{bcd}^a = W_{bcd}^a = W_{bcd}^a = 0.$$

Corollary 6

Let $Z = (Q, g = \langle \cdot, \cdot \rangle)$ be NK ("Nearly Kahler") structure. Afterward, the inclusions listed below of classes $\bar{W}_1 \subset \bar{W}_2 \subset \bar{W}_3$ are acceptable.

proof:

Let Z - structure of a class $\bar{W}_1$. Obviously, it is equivalent to $(W)_0 = (W)_4 = (W)_7 = 0$, By theorem 5.

So (By theorem 4) $(W)_0 = (W)_7 = 0$, is equivalent to class $\bar{W}_2$.

Then $\bar{W}_1 \subset \bar{W}_2$.

Also the class $\bar{W}_3$ is equivalent to $(W)_0 = 0$, that is clear from theorem 5.

Thus $\bar{W}_1 \subset \bar{W}_2 \subset \bar{W}_3$.

Conclusion

Find new classes $\bar{W}_0(N.K)$, $\bar{W}_1(N.K)$ and $\bar{W}_3(N.K)$ and proved the structure. $\bar{W}_3(N.K)$ is $\bar{W}_7(N.K) = 0$, and on space of the adjoint G-structure identities $W(N.K)_{bcd}^a = 0$ are fair.

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References

- [1] Gray A. (1976). "Curvature Identities for Hermitian and Almost Hermitian Manifold "Tohoku Math J.28, NO-4, 601-601.
- [2] Gray A., Vanhecke L.(1979). "Almost Hermitian Manifold with Constant Holomorphic Sectional curvature ", Cas.Pestov. Mat.Vol., NO-12,170-179.
- [3] Gray A ,and Hervella L.M (1980)."Sixteen classes of almost Hermitian manifold and their linear

invariants" Ann,Math .pure and Appl., Vol. 123, NO.3, 35-58

- [4] Ali A. Shihab, Dhabia'a M. Ali (2018). "Generalized Conharmonic Curvature Tensor Of Nearly Kahler Manifold". Tikrit J. Pure Science, V.23 (8).
- [5] Kirichenko V.F.(2003). Differential Geometrical Structure on smooth manifolds. -M., MSPU,495

كلاسات تنسر الاسقاط من النوع W في منطوي كوهلر التقريبي

علي خلف علي, علي عبد المجيد شهاب

قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث ندرس كلاسات جديدة للتزير الاسقاطي من نوع W لمنطوي كوهلر التقريبي مع نظريات جديدة اي اننا نجد فئات جديدة $\bar{W}_1, \bar{W}_2$ and $\bar{W}_3$, ثم قمنا بانشاء علاقة بينها وبين مكونات تنزر الانحناء الاسقاطي واخيرا نكتشف علاقة بين W_1, W_2, W_3 مع بعضها البعض.