Comparison Between GM (1,1) and FOGM(1,1) Models for Forecasting The Rate of Precipitation in Sulaimani

Lecturer. Shaho Tahir Ahmed
University of Sulaimani
College of Administration and Economics

Asst. Lecturer .Hindreen A. Tahir
University of Sulaimani
College of Commerce

مقارنة بين نماذج GM (1,1) و FOGM(1,1) للتبنؤ بمعدل هطول الامطار في السليمانية

شاهو طاهر احمد **هندرين عبدالله طاهر**
كلية الإدارة والاقتصاد/ جامعة السليمانية **كلية التجارة/ جامعة السليمانية**

تاريخ استلام البحث 2019/10/8 تاريخ قبول النشر 2019/12/30

Abstrac:

Forecasting for the rate of precipitation is so important because in these lately years the climate has changed that effects on the weather this caused by the pollution of environment which is increase positively with the developing of industrials sector. In our country agriculture is one of the main branches of economic specially wheat and barley that are need rain, that's why who may concern it must pay attention to collect pure data and analysing them and then forecasting the rate of precipitation to put a plan for the economic in future years, the GM(1,1) and FOGM(1,1) models have been used for forecasting the rate of precipitation, to compare and select the appropriate model the researchers depended on MAPE and precision rate for both models, according to their results the FOGM(1,1) model was the more appropriate model to represent the behaviour of precipitation rate in Iraqi-Sulaimani governorate with precision rate = 0.939297 and MAPE = 6.07029, four periods have been forecasted which tells that the precipitation rate will be between (796.926 – 801.047)mm for the next four years.

Keywords: Grey system theory, Ordinary least squares, Fourier transformation

الملخص:

يعد التنبؤ بمعدل هطول الأمطار أمرًا مهمًا للغاية لأنه في هذه السنوات الأخيرة تغيرت المناخ بشكل واضح مما أثر على الطقس وهذه التغيرات ناتجة عن تلوث البيئي الذي يزداد مع تطور قطاع الصناعي. تعتبر الزراعة القمح والشعير في بلدنا واحدة من قطاعات الرئيسية التي تؤثر على اقتصاد الدولة، ولهذا السبب يجب على من يهمه الأمر أن يهتم بجمع بيانات دقيقة لتحليلها ثم التنبؤ بمعدل هطول الأمطار لوضع خطة اقتصادية لسنوات المقبلة ، تم استخدام نماذج (1,1) GM، و FOGM (1,1)، للتنبؤ بمعدل هطول الأمطار، لمقارنة واختيار النموذج المناسب إعتمدوا الباحثون على مقياس MAPE ومعدل الدقة لكلا النموذجين، وفقًا للنتائج كان نموذج (1,1) FOGM، هو النموذج الأكثر ملائمة لتمثيل سلوك معدل هطول الأمطار في محافظة السليمانية العراقية بمعدل دقة تساوي (0.939297) و MAPE = 6.07029 ، تم التنبؤ لأربع سنوات قادمة حيث توضح النتائج أن معدل هطول الأمطار لسنوات الأربعة القادمة يتراوح ما بين (796.926 – 801.047) ملم.

الكلمات المفتاحية: نظرية النظام الرمادي ، المربعات الصغرى العادية ، تحويل فوريه.

Introduction:

In Kurdistan region the rate of precipitation is so Important, because our country has not enough main factories that be able to raise the economic, the main factor is agriculture like wheat, barley and corn that have main effect on raising the economics of Kurdistan region, therefore the seasonal rate of precipitation should be studied to forecast its rate in the future years to plan and decision making. In this study we want to forecast the seasonal rate of precipitation to plan for the future years. Deng Julong in (1982) proposed the Grey System Theory, this theory works when there exists a lack of information. Essentially, the colours were chosen to Grey System Theory. In a way that for hidden information black colour is used and for full information white colour is used. The incomplete known and incomplete unknown information for analysing is depending on Grey System Theory.

In many fields has been applied Grey System theory successfully for those cases that have lack of information. The other prediction methods, such as Time Series needs a large number of observations in another word one can say that the sample size must be greater than 100 observations, also the statistical distribution of response must be exist so as to estimate inference parameters for prediction. But grey system theory does not have the above mentioned assumptions about the data set that is why it used to predict uncertain systems.

Objective of the research

In this study firstly we want to compare GM(1,1) and FOGM(1,1) models and secondly to forecast the precipitation rate in Sulaimani governorate to plan for the future years.

2. Methodology

2.1 Grey system theory

Deng Julong in (1982) developed grey system theory to analysed the uncertainty of a system. (Deng 2012)

As far as fisher information is concerned, the systems that lack information, such as structure behaviour document, operation machinery and message are referred to as grey systems. (Huo 2012)

The grey system theory consists of five major parts which are grey prediction, relation, decision, programming and control. Grey model is the core of grey system theory, which features available information to obtain the internal regularity without using any assumptions.(Javed 2018)

The intention is to make forecasting useful for decision and policy makers who need future predictions.

2.2 Advantage of Grey models

The most important advantages of Grey Prediction Model have been shown below:

1- Grey models can be used even you have four observations.
2- To characterize an unknown system a few discrete data are sufficient.
3- The researchers can depend on the results of grey system theory in case having a suspected large sample.(Liu 2017)

2.3 Time series Data and use of Grey Model

Time series data is observing data at the equal period time respectively, to forecasting a phenomena depending on a time series model a large full information sample is needed, sometimes it is the common problem that the researchers are faced at the beginning of their study, that is why advise to use grey models instead of time series models in case lack of information.(Mao 2006)

2.4 Grey Theory Steps

The lack of information is named by grey, the Grey system provides an adequate results despite of uncompleted information or small samples under study that is because of the a cumulative generate operator which gives more weights for each observation of the sample.(Pai 2013)

2.5 Grey Model

In grey systems theory, GM (m,n) definition of lack information model, where m is the order of the difference formula and n is the number of factors. Grey models can predict the future outputs of systems with relatively high accuracy without a mathematical model of the actual system. The majority of the incisive researchers have fastened their awareness on the GM (1, 1) model in their predictions because of its computational efficiency.(Li 2014)

2.6 The GM (1, 1) Model

The GM (1, 1) is most type model that is used among the others type models of Grey System model, which determines that there is one input variable in the model. The accumulation generating operation (AGO) is the first order differential equation that adopted to match the data. (Javed 2018)
For the algorithm of GM (1, 1), the raw data sequences are presented as follow:
First of all, in order to ensure the feasibility of modelling method, it is necessary to make the necessary known data column inspection treatment. A reference data for: (Xiaolu 2017)

$X^{(0)} = (x^{(0)}(1), x^{(0)}(2) \ldots x^{(0)}(n))$, calculation sequence stage than

$$\sigma(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)} \quad , k = 2,3, \ldots .n \qquad (2.1)$$

If all the level than $\sigma(k)$ have fallen on can let cover $\left(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}}\right)$ inside, series $x^{(0)}$ can be used as a model GM (1, 1) of grey prediction data. Otherwise, need to series $x^{(0)}$ make the necessary transformation processing, make its fall into can let cover inside. Namely take appropriate constant c, for translational transformation

$y^{(0)} = x^{(0)}(k) + c \, , k = 1,2, \ldots , n$ Make series

$y^{(0)} = (y^{(0)}(1), y^{(0)}(2), , y^{(0)}(n))$ Level than $\sigma_y(k)$ fall in to can let coverage area.(Lian 2005)

In grey system theory, the GM (1, 1) model is one of the most widely used models. In general, GM (1, 1) requires a few numbers of observations to build a prediction model. Assume that is the original sequence as follows:

$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$

Where n is the total number of modeling data. The accumulated generating operation (AGO) formation of $x^{(1)}$ is defined as:

$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))$

Where $x^{(1)}(1) = x^{(0)}(1)$, and

$x^{(1)}(t) = \sum_{i=1}^{t} x^{(0)}(i)$, $t = 1, 2 \dots m$ (2.2)

The GM (1, 1) model can be built by establishing a first order differential equation for $x^{(1)}(t)$ as

$$\frac{dx^{(1)}(t)}{dt} + \alpha\, x^{(1)}(t) = b \qquad (2.3)$$

Then the discrete from of the GM (1, 1) differential equation model is expressed as:

$$x^{(0)}(t) + \alpha z^{(1)}(t) = b \qquad (2.4)$$

Where $z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n))$are called background values of $\frac{dx^{(1)}(t)}{dt}$ and calculated by

$$z^{(1)}(t) = 0.5\left(x^{(1)}(t-1) + x^{(1)}(t)\right) \quad , t = 2,3, \dots n \qquad (2.5)$$

Coefficients **α** and **b** are called the updating coefficient and grey input, respectively. In practice, coefficients **α** and **b** are not calculated directly from Eq. (2.3). Therefore, the solution of Eq. (2.5) can be obtained by using the least square method that is

$$\acute{\alpha} = [\alpha, \mathrm{b}]^T = (B^T B)^{-1} B^T Y \qquad (2.6)$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \qquad (2.7)$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)\right]^T \qquad (2.8)$$

Then, the modeling values of Eq. (2-3) are obtained as

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \qquad (2.9)$$

Applying the inverse accumulated generation operation (IAGO)

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k),\ k = 1,2, \dots, n \qquad (2.10)$$

The prediction sequence can be obtained as:

$$\hat{x}^{(0)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)(1-e^{a})e^{-ak} \;,\;\; k = 1,2,\dots,m. \qquad (2.11)$$

Where $\hat{x}^{(0)}(1) = x^{(0)}(1), \{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n)\}$ are called the

GM (1, 1) fitted values, while $\{\hat{x}^{(0)}(n+1), \hat{x}^{(0)}(n+2), \dots, \hat{x}^{(0)}(n+j)\}$ are called the GM (1, 1) predicted values.(Guo 2005)

2.7 Fourier Residual Modifications (FR)

The original idea of the Fourier model is derived from Tien. Meanwhile, the forecasting of primary series trends originally uses GM (1, 1), Then the GM (1, 1) model is adaptive using the Fourier series. The main goal behind using Fourier series is to gain a better forecasting with a high precision. In order to achieve the accuracy of forecasting models, the Fourier series has been widely and successfully applied in adapting the residuals in Grey forecasting model Fourier Residual Modification FOGM (1, 1) is:

1- According to the original sequence to build the GM(1,1), the residual time series as the difference between the real time and model fitted is obtained by $\varepsilon^{(0)} = \{\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), \dots, \varepsilon^{(0)}(t)\}$
$where \qquad \varepsilon^{(0)}(t) = x^{(0)}(t) - \hat{x}^{(0)}(t), t = 2, \dots., m$ …..(2.12)

2- Transfer $\varepsilon^{(0)}(t)$ into Fourier series from

$$\varepsilon^{(0)}(t) = 0.5\alpha^{(0)} + \sum_{i=1}^{F}\left[\alpha_i \cos\left(\frac{2\pi i}{T}(t)\right) + b_i \sin\left(\frac{2\pi i}{T}(t)\right)\right] \dots..(2.13)$$

$$where\; t = 2,3,\dots,m \quad ,T = m-1 \;and\; F = \left(\frac{m-1}{2}\right) - 1$$

3- from $\varepsilon^{(0)} = PC$ to find the value of
$$C = (P^T P)^{-1} P^T \left[\varepsilon^{(0)}\right]^T \qquad …….. (2.14)$$

Which use least square method where

$$p=\begin{bmatrix} \frac{1}{2}\cos\left(2\frac{2\pi}{T}\right)\sin\left(2\frac{2\pi}{T}\right)\cos\left(2\frac{2\pi 2}{T}\right)\sin\left(2\frac{2\pi 2}{T}\right)\ldots\cos(2\frac{2\pi F}{T})\sin\left(2\frac{2\pi F}{T}\right) \\ \frac{1}{2}\cos\left(3\frac{2\pi}{T}\right)\sin\left(3\frac{2\pi}{T}\right)\cos\left(3\frac{2\pi 2}{T}\right)\sin\left(3\frac{2\pi 2}{T}\right)\ldots\cos(3\frac{2\pi F}{T})\sin\left(3\frac{2\pi F}{T}\right) \\ \vdots \\ \frac{1}{2}\cos\left(m\frac{2\pi}{T}\right)\sin\left(m\frac{2\pi}{T}\right)\cos\left(m\frac{2\pi 2}{T}\right)\sin\left(m\frac{2\pi 2}{T}\right)\ldots\cos(m\frac{2\pi F}{T})\sin\left(m\frac{2\pi F}{T}\right) \end{bmatrix}$$

$$C=[\alpha_{(0)},\alpha_{(1)},b_{(1)},\alpha_{(2)},b_{(2)},\ldots,\alpha_{(F)},b_{(F)}$$

4- substitute all data into the Fourier series to get the value of $\hat{\varepsilon}^{(0)}(t)$

$$\hat{\varepsilon}^{(0)}(t)=0.5\alpha^{(0)}+\sum_{i=1}^{F}\left[\alpha_i\cos\left(\frac{2\pi i}{T}(t)\right)+b_i\sin\left(\frac{2\pi i}{T}(t)\right)\right]\ldots\ldots\ldots..(2.15)$$

5- the final prediction value is show in equation (2.16)

$$\hat{x}_f^{(0)}(1)=\hat{x}^{(0)}(1)$$
$$\hat{x}_f^{(0)}(t)=\hat{x}^{(0)}(t)+\hat{\varepsilon}^{(0)}(t),t=(2,3,4,\ldots,m)\qquad\ldots\ldots\ldots.(2.16)$$

2.8 Evaluate Precision of forecasting Models

Some statistical tests and measurements used to test the accuracy and the performance of the proposed model, including residual test, MAPE, precision Rate and Posterior Error Ratio (c).

2.8.1 Residual test

Step1: compute the relative error and absolute percentage error presented as follow:

$$\Delta^{(0)}(i)=\left|x^{(0)}(i)-\hat{x}^{(0)}(i)\right|\quad i=1,2,\ldots,n\qquad(2.17)$$
$$\emptyset(i)=\frac{\Delta^{(0)}(i)}{x^{(0)}(i)}\times 100\%\quad i=1,2,\ldots,n\qquad(2.18)$$

Step2: compute the two –step minimum and maximum of the relative error:

$$\Delta_{min}=min\left|\Delta^{(0)}(i)\right|\qquad(2.19)$$
$$\Delta_{max}=max\left|\Delta^{(0)}(i)\right|\qquad(2.20)$$

Step3: compute coefficients of grey incidence

$$\gamma\left(\hat{x}^{(0)}(k),x^{(0)}(k)\right)=\frac{\Delta_{min}+p.\Delta_{max}}{\Delta_{0i}(k)+p.\Delta_{max}}\qquad(2.21)$$

Of which the distinguishing coefficient p is 0.5

Step4: compute the degree of grey incidence

$$\gamma\left(\hat{x}^{(0)}, x^{(0)}\right) = \frac{1}{n}\sum_{i=1}^{n} \gamma(\hat{x}^{(0)}(i), x^{(0)}(i)) \qquad (2.22)$$

According to experience, the GM model is qualified if $\gamma\left(\hat{x}^{(0)}, x^{(0)}\right) > 0.6\,, when\ p = 0.5$

2.8.2 Mean Absolute Percentage Error (MAPE)

Means Absolute Percentage Error (MAPE) is used to test the accuracy of forecasting model, there is an index used to evaluate the performance and reliability of forecasting technique. It is defined as follows: (Christopher 2000)

$$MAPE = \frac{1}{n}\sum_{k=2}^{n} \left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \qquad (2.23)$$

Where $x^{(0)}(k)$:The actual value in time period k

$\hat{x}^{(0)}(k)$: The forecast value in time period k

And the grade of MAPE is divided into four grades:

Precision rank	MAPE
Highly accurate	MAPE ≤ 0.01
Good	MAPE ≤ 0.05
Reasonable	MAPE ≤ 0.1
Inaccurate	MAPE ≤ 0.15

The lower the MAPE, the higher precision the forecasting model can achieve.in general the MAPE below 0.01 is an accurate model and the MAPE in between 0.01 and 0.05 is a good model with acceptable accuracy.(Wu 2005)

2.8.3 Precision Rate (p)

Precision Rate, which measures the level of the closeness of the statement of forecast quantity and the actual value, p is defined as follows:

$$p = 1 - MAPE \qquad (2.24)$$

precision rank	Precision Rate
Highly accurate	$p \geq 0.99$
Good	$p \geq 0.95$
Reasonable	$p \geq 0.90$
Inaccurate	$p \leq 0.90$

The higher the precision rate, the higher precision the forecasting model can achieve. In general, the precision rate greater than 0.99 is an accurate model and the precision rate in between 0.99 and 0.95 is a good model with acceptable accuracy.(Chih 2003)

3.Applications

3.1 Data Description:

The data were collected from the directorate of meteorology and seismology of Slemani during the years (2000-2001 to 2018-2019) in seasonal as (19 consecutive observations). The data is measured in mm.

3.2 Model Specifications

The data that has been collected as described in previous sections are used to perform grey and fourier grey models and then comparing their results to select appropriate model for forecasting the precipitation rate.

3.3 Fitting GM(1,1) and FOGM(1,1) and Their Results

Depending on Eq (2.2), by using OLS method the parameters of GM(1,1) model have been estimated which are intercept and slop of the model with values (738.3606 and 0.0116) respectively.

Table (3-1)

Represents the model value and MAPE by using

Time	Real value	Model value	MAPE
2000/2001	523.90	523.9	0
2001/2002	782.50	644.7172641	0.17608
2002/2003	880.30	651.3921686	0.26003
2003/2004	842.70	658.13618	0.21901
2004/2005	653.50	664.9500137	0.01752
2005/2006	758.50	671.8343925	0.11426
2006/2007	607.50	678.7900469	0.11735
2007/2008	369.00	685.8177147	0.85858
2008/2009	430.30	692.9181416	0.61031
2009/2010	799.70	700.0920808	0.12456
2010/2011	568.50	707.3402935	0.24422
2011/2012	574.40	714.6635485	0.24419
2012/2013	579.40	722.0626229	0.24622
2013/2014	703.70	729.5383016	0.03672
2014/2015	699.80	737.0913776	0.05329
2015/2016	860.90	744.7226524	0.13495
2016/2017	587.50	752.4329355	0.28074
2017/2018	850.00	760.223045	0.10562
2018/2019	1137.80	768.0938072	0.32493

Table (3-2)

Represents the model value and MAPE by using

Time	Real value	Model value	MAPE
2000/2001	523.90	523.9	0
2001/2002	782.50	743.2302885	0.05018
2002/2003	880.30	824.7868164	0.06306
2003/2004	842.70	862.3170372	0.02328
2004/2005	653.50	611.4962192	0.06428
2005/2006	758.50	717.7108425	0.05378
2006/2007	607.50	583.7309627	0.03913
2007/2008	369.00	340.5766071	0.07703
2008/2009	430.30	464.8806362	0.08036
2009/2010	799.70	741.6716248	0.07256
2010/2011	568.50	603.225445	0.06108
2011/2012	574.40	539.8930887	0.06007
2012/2013	579.40	543.7679539	0.0615
2013/2014	703.70	798.8315909	0.13519
2014/2015	699.80	656.7257273	0.06155
2015/2016	860.90	800.2071504	0.0705
2016/2017	587.50	638.3265169	0.08651
2017/2018	850.00	797.6894802	0.06154
2018/2019	1137.80	1101.67662	0.03175

Table (3-1) represents the model value and MPAE by GM (1,1) model depending on Eq(2.2) and table (3-2) represents the model value and MPAE by FOGM (1,1) model depending on Eq(2.16) it is clear seen that the MAPE values of FOGM (1,1) are less than MAPE values of GM (1,1) it is means FOGM (1,1) model values more accurate than GM (1,1) model values.

Table (3-3)

Represents the accuracy of models

test	GM(1,1) accuracy	FoGM(1,1) accuracy
c	0.304064153	0.01142
MAPE	21.93997555	6.07029
P	78.06002445	93.9297

From the above table the value of posterior error ratio (c) is less than 0.35 for both postulated models, from the value of P we can select the best model for forecasting the precipitation rate which is FoGM (1,1) with P = 93.92.

3.4 Goodness of fit of FOGM(1,1)

FOGM (1, 1) is a time series model then the goodness of fit of the model should be tested by using Box-price test that depends on the ACF of the residuals of the FOGM (1,1) the results were as follow:

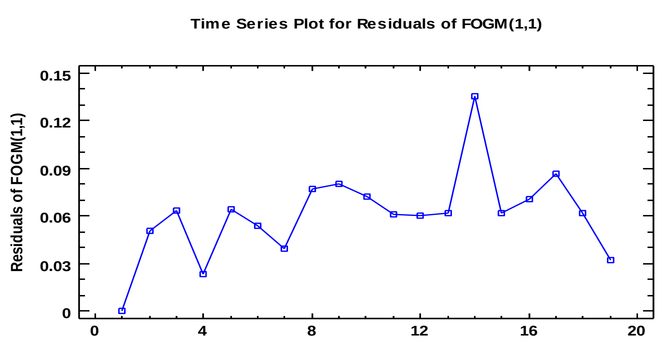

Figure (3-1)

Shows the scatter plot of residual series of FOGM (1,1) model

Box-Pierce Test

Test statistic = 2.5654

Sig.= 0.863331

The hypothesis of that the series is random cannot rejected because the sig. value is greater than 0.05.

Table (3-4)

Represents forecasting values by using GM(1,1) and FoGM(1,1) models

Period	GM(1,1)forecasting value	FoGM(1,1)forecasting value
2019 - 2020	776.0460572	798.091
2020 - 2021	784.0806386	800.169
2021 - 2022	792.1984039	796.926
2022 - 2023	800.4002143	801.047

4.1 Conclusions

From the results we conclude that:

1- From the results it can be conclude that the FOGM(1,1) is better than GM(1,1) to represent the behaviour of precipitation rate in sulaimani governorate.

2- From the forecasted values one can conclude that the precipitation rate will fall between (796.9 to 801.041) mm which is good for agriculture.

4.2 Recommendations

1- We suggest that the directorate of meteorology and seismology of Sulaimani governorate to form and organize these data in a standard database so as to the researchers be able to access them easily.

2- We proposed to use this model with the other climate variables (Temperature, humidity and wind speed).

References

1. Christopher Chatfield (2000), Time-series forecasting, Chapman & Hall/CRC, 2000, Boca Raton, ch. 6, 8.
2. Deng, L. R., Hu, Z. Y., Yang, S. G., & Yan, Y. Y. (2012). Improved Non-equidistant Grey Model GM(1,1) Applied to the Stock Market. Journal of Grey System, 15(4), 189-194.
3. Guo, Z., Song, X., & Ye, J. (2005). A Verhulst model on time series error corrected for port throughput forecasting. Journal of the Eastern Asia Society for Transportation Studies, 6, 881–891.
4. Huo, H. Y., & Zhan, S. B. (2012). Improved Grey Model GM (1, 1) and ts Application based on Genetic Algorithm. *International Journal of Digital Content Technology and its Applications*, *6*(9), 361-368.

5. Javed, Saad Ahmed; Liu, Sifeng (2018-10-08). "Evaluation of outpatient satisfaction and service quality of Pakistani healthcare projects". Grey Systems: Theory and Application. 8(4): 462–480. doi:10.1108/gs-04-2018-0018. ISSN 2043-9377.
6. Javed, Saad Ahmed; Liu, Sifeng (2018), "Predicting the research output/growth of selected countries: application of Even GM (1, 1) and NDGM models", Scientometrics, 115: 395–413, doi:10.1007/s11192-017-2586-5
7. Li, G. D., Masuda, S., & Nagai, M. (2014c). Predictor design using an improved grey model in control systems. *International Journal ofComputer Integrated Manufacturing*, 1-10.
8. Liu, Sifeng; Yang, Yingjie; Forrest, Jeffrey (2017). Grey Data Analysis. Methods, Models and Applications. Singapore: Springer. ISBN 978-981-10-1841-1.
9. Mao, M. & Chirwa, E.C. (2006), Application of grey model GM (1,1) to vehicle fatality risk estimation. Technological Forecasting and Social Change, 73, 588-605.
10. Pai, Tzu-Yi; Hanaki, Keisuke; Chiou, Ren-Jie (2013-03-27). "Forecasting Hourly Roadside Particulate Matter in Taipei County of Taiwan Based on First-Order and One-Variable Grey Model". CLEAN - Soil, Air, Water. 41 (8): 737–742. doi:10.1002/clen.201000402.
11. R. J. Lian, B. F. Lin, et al. (2005), A grey prediction fuzzy controller for constant cutting force in turning,International Journal of Machine Tools and Manufacture, vol. 45, no. 9, pp. 1047-1056.
12. T. Chih-Hung, C. Ching-Liang, C. Lieh, (2003) “Applying Grey Relational Analysis to the Vendor Evaluation Model,” International Journal of the Computer, the Internet and Management, vol. 11, no. 3, pp. 45-53.
13. Wu, W.Y. & Chen, S.P. (2005). A prediction method using the grey model GMC (1, n) combined with the grey relational analysis: a case study on internet access population forecast, Applied Mathematics and Computation, 169, 198-217.
14. Xiaolu, Li; Zheng, Wenfeng; Yin, Lirong; Yin, Zhengtong; Song, Lihong; Tian, Xia (2017-08-10). "Influence of Social-economic Activities on Air Pollutants in Beijing, China". Open Geosciences. 9 (1): 314–321. doi:10.1515/geo-2017-0026.