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A New Simple 6D Hyperchaotic System with Hyperbolic Equilibrium and Its Electronic Circuit

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ABSTRACT: This paper represents a sequential design for controlling a 3D chaotic system to a simple 6D hyperchaotic system using state feedback control and coupling strategies. The new 6D system has a three hyperbolic equilibrium point with four positive Lyapunov exponents (LE). Moreover, some dynamic properties are analyzed, including inclined equilibrium and their stability (dissipative, conservative, and multistable). In addition, the corresponding analogue electronic circuit is designed and implemented to verify the new simple 6D hyperchaotic system. Finally, the NI Multisim 14.2 simulation results observed from the digital oscilloscope are consistent with the mathematical simulation based on Matlab R2021a.

Keywords: Simple 6D system, hyperbolic equilibrium, Lyapunov exponents, Multistability, electronic circuit.

1. INTRODUCTION

Lorenz system is the first 3D chaotic attractor discovered in 1963 [1]. Subsequently, many 3D chaotic systems have been established in recent years, such as the Rösslor system [2], Chen system [3], Lü system [4], Liu system [5], and Sprott systems [6]. Most of these systems mentioned above have only one positive Lyapunov exponent (1+ve LE). Furthermore, to increase the complexity for some practical applications like encryption [7-11], secure communication [12], stability [13], synchronization phenomena [14-16], and Polluted Environment [17]. Low-dimension systems with one positive Lyapunov is often unsafe. These many reasons have motivated researchers and scientists to introduce high-dimensional systems with more than (1+ve LE). Recently, high-dimensional systems (4D, 5D, and 6D) are presented with two, three, and four (+ve LE), respectively [18-21]. In order to obtain an n-dimensional ($n \geq 4$) autonomous hyperchaotic system with ($n - 2$) positive Lyapunov exponents, two important conditions are necessary [22]:

- The minimal dimension of the phase space that embeds a hyperchaotic attractor should be at least n, which requires the minimum number of coupled first-order autonomous ordinary differential equations to be n.
- The number of the terms in the coupled equations giving rise to instability should be at least two, of which at least one should be nonlinear.

The Lyapunov exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories. The sensitive dependence on initial conditions of a dynamical system is characterized by the presence of a positive Lyapunov exponent. A positive Lyapunov exponent reflects a direction of stretching and folding and along with phase-space compactness indicates the presence of chaos in a dynamical system. An n-dimensional dynamical system has a spectrum of n Lyapunov exponents and the maximal Lyapunov exponent (MLE) of a chaotic system is defined as the largest positive Lyapunov exponent of the system [23].

Despite the fact that several of these systems have (1+ve LE) only such as [24-26] and Table 5 lists all six-dimensional systems available in the recent literature [27-38]. Through the intense search, it is found that the 6D systems have not received as wide attention as the 4D and 5D systems. That motivates us to introduce a new simple 6D hyperchaotic system with four positive Lyapunov exponents that belong to the category of a self-excited attractor. The proposed system contains twelve terms, four of which are nonlinear. In addition, it does not have a non-hyperbolic equilibrium point. Some dynamical properties are analyzed, such as Lyapunov exponents, Kaplan-York dimension, stability, multistability, dissipative and conservative. Finally, the existence of the hyperchaotic attractor is verified by the analog circuit realization.

The following points summarize the main contribution of this work:

- A new 6D hyperchaotic system with four positive LEs is constructed

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- it is a simple system due to its consisting of 12 terms with four nonlinear terms, so it fulfils one criterion of Sprott.
- The maximum Lyapunov exponents (MLE) of the proposed 6*D* system is greater than the MLE for another systems in Table 5.
- Kaplan-Yorke dimension D_{KY} of the new 6D is greater than the D_{KY} for other systems in Table 5, which means that the new system has more complex behavior.
- The proposed system is dissipative.
- It belongs to self-excited attractors because it has three unstable hyperbolic equilibrium points.
- Dynamical behaviors such as chaotic, hyperchaotic, and hyperchaotic with 2-torus and 3-torus are observed.
- The electronic circuit of a new system implemented via Multisim 14.2 software.

2. THE NEW 6D-HYPERCHAOTIC SYSTEM

This section adopts a new simple 6D system by adding three linear controllers to a Liu 3D system [5]. The major dynamic characteristics of the new system, such as equilibrium, dissipation, and Lyapunov exponents, are discussed.

The original 3D chaotic system is depicted as:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx - kxz \\ \dot{z} = -cz + hx^2 \end{cases} \tag{1}$$

Hereafter x, y, z are state variables, xz and x^2 are nonlinear terms, $a, b, c, k, h > 0$. The system (1) has three equilibria points with one positive Lyapunov exponent under the parameters $(a, b, c, k, h) = (10,40,2.5,1,4)$ respectively, and:

$$LE_1 = 1.6433 \quad LE_2 = 0 \quad LE_3 = -14.142$$

By using the state feedback strategy and adding the linear controller w to the first equation of the system (1), the new 4D system has became:

$$\begin{cases} \dot{x} = a(y-x) + rw \\ \dot{y} = bx - kxz \\ \dot{z} = -cz + hx^2 \\ \dot{w} = dy \end{cases} \tag{2}$$

Where $a, b, c, k,$ and h are constant parameters, r, d are the controller parameters. Now, the system (2) has a chaotic behavior under typical parameters$(a, b, c, k, h, r, d) = (10,40,2.5,1,4,6.5,2.2)$ respectively having three equilibria points, and the LE_s are:

$$LE_1 = 2.0524, \quad LE_2 = 0.0001,$$
$$LE_3 = -1.4235, \quad LE_4 = -13.1227.$$

Relying on the coupling strategy and the system (2), a simple 5D hyperchaotic system was designed as:

$$\begin{cases} \dot{x} = a(y-x) + rw \\ \dot{y} = bx - kxz \\ \dot{z} = -cz + hx^2 \\ \dot{w} = dy \\ \dot{v} = -n_1 wx + n_2 v \end{cases} \tag{3}$$

where a, b, c, k, h, r and d are positive parameters, n_1, n_2 are the coupling parameters. System (3) has a hyperchaotic attractor, under the parameters $a = 10, b = 40, c = 2.5, k = 1, h = 4, r = 6.5, d = 0.0001, n_1 = 0.5, n_2 = 8.2$. The corresponding five LE_s are:

$$LE_1 = 8.1937, \quad LE_2 = 1.6949, \quad LE_3 = 0.0054,$$
$$LE_4 = -0.0001, \quad LE_5 = -14.1814.$$

Furthermore, by introducing the third linear dynamical controller u via the coupling strategy to the fourth equation of the system (3), a new simple 6D hyperchaotic system is presented, which is described as:

$$\begin{cases} \dot{x} = a(y - x) + rw \\ \dot{y} = bx - kxz \\ \dot{z} = -cz + hx^2 \\ \dot{w} = dy \\ \dot{v} = -n_1 wx + n_2 v \\ \dot{u} = -q_2 xz + q_1 u \end{cases} \tag{4}$$

where a, b, c, k, h, r, d, n_1 and n_2 are constant parameters and q_1, q_2 are coupling parameters. The system (4) exhibits hyperchaotic attractors system with four positive Lyapunov exponents as given in (6) under the following typical parameters:

$$\begin{matrix} a = 10, & k = 1, & n_2 = 9.2, \\ b = 40, & h = 4, & q_1 = 0.81, \\ c = 2.5, & r = 6.5, & q_2 = 1.1. \\ d = 0.001, & n_1 = 0.25, & \end{matrix} \tag{5}$$

System (4) is unstable equilibrium and the seven LE_s are:

$$\begin{matrix} LE_1 = 9.1913, & LE_2 = 1.5999, \\ LE_3 = 0.0314, & LE_4 = 0.0023, \\ LE_5 = 0.0000, & LE_6 = -14.1120. \end{matrix} \tag{6}$$

The proposed system has a unique non-hyperbolic equilibrium, and it is considered a simple system because it consists of only twelve terms with four nonlinearities according to Sprott's criteria [39]. Fig.1 shows the hyperchaotic behavior for system (4) under the following initial condition:

$$(x_0, y_0, z_0, w_0, v_0, u_0) = (1,4.1,2.1,7,6.1,0.1) \tag{7}$$

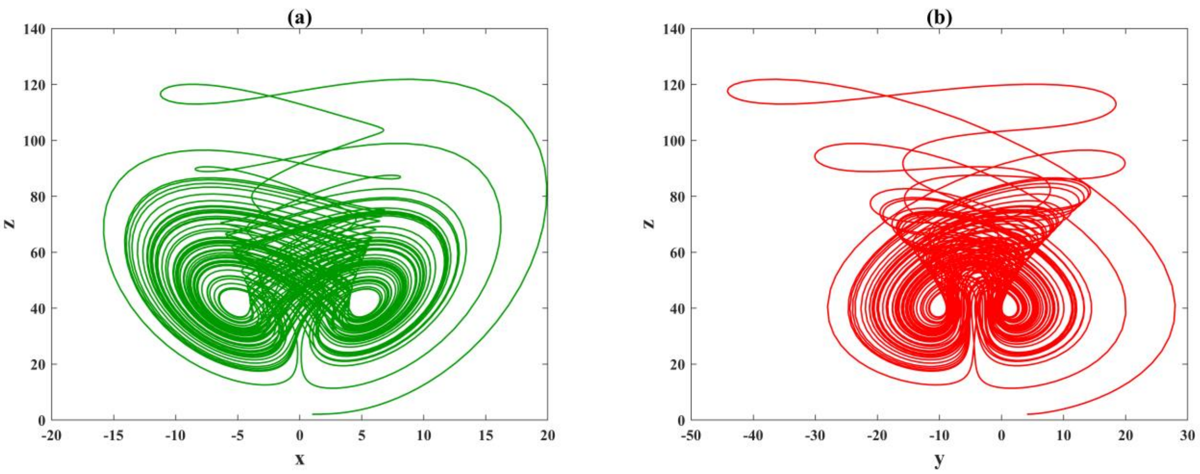

FIGURE 1. -Attractor of the system (4) in a: x-z plane, b: y-z plane.

3. ANALYSIS OF THE NEW SIMPLE 6D HYPERCHAOTIC SYSTEM

Some basic dynamical characteristics of the new 6D simple system will be discussed in this section.

3.1 Equilibrium point and stability

The equilibrium point of the system (4) is evaluated by setting $\dot{x} = \dot{y} = \dot{z} = \dot{w} = \dot{v} = \dot{u} = 0$. So, system (4) becomes as [40]:

$$\begin{cases} a(y - x) + rw = 0 \\ bx - kxz = 0 \\ -cz + hx^2 = 0 \\ dy = 0 \\ -n_1 wx + n_2 v = 0 \\ -q_2 xz + q_1 u = 0 \end{cases} \tag{8}$$

Solving Eq. (8), lead to three cases that are obvious for the system (4):

- If $b = 0$ or $c = 0$, and $h, r, k \neq 0$, system (4) has a unique (trivial) equilibrium point $E_1 = (0,0,0,0,0,0)$
- If $h, r, k = 0$, and $n_2, q_1 \neq 0$, system (4) has no explicit solution. So, the system belongs to hidden attractors.
- If all parameters are not equal zero, system (4) has two equilibrium points (nontrivial):

$$E_{2,3} = \left(\pm\sqrt{\frac{bc}{hk}}, 0, \frac{b}{k}, \pm a\frac{\sqrt{\frac{bc}{hk}}}{r}, \frac{n_1 abc}{n_2 hkr}, \pm b q_2 \frac{\sqrt{\frac{bc}{hk}}}{kq_1} \right)$$

Jacobian matrix and characteristic equation of the system (4) expressed as Eq. (9) and Eq. (10), respectively:

$$J = \begin{bmatrix} -a & a & 0 & r & 0 & 0 \\ b-kz & 0 & -kx & 0 & 0 & 0 \\ 2hx & 0 & -c & 0 & 0 & 0 \\ 0 & d & 0 & 0 & 0 & 0 \\ -n_1 w & 0 & 0 & -n_1 x & n_2 & 0 \\ -q_2 z & 0 & -qx & 0 & 0 & q_1 \end{bmatrix} \tag{9}$$

$$\lambda^6 + A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0 \tag{10}$$

Whereas:

$$\begin{cases} A = a + c - n_2 - q_1 \\ B = ac - ab - an_2 - cn_2 - aq_1 - cq_1 + n_2 q_1 + akz \\ C = 2ahkx^2 - abc + abn_2 - acn_2 + abq_1 - acq_1 - bdr + an_2 q_1 + cn_2 q_1 + ackz - an_2 kz - akq_1 z + dkrz \\ D = bdrq_1 + abcn_2 + abcq_1 - abn_2 q_1 - bcdr + acn_2 q_1 + bdn_2 r - acn_2 kz - ackq_1 z + an_2 kq_1 z + cdkrz \\ \qquad -dekrz - dkrq_1 z - 2an_2 hkx^2 - 2ahk q_1 x^2 + 2dhkrx^2 \\ E = bcdn_2 r - abcn_2 q_1 + bcdrq_1 - bdn_2 rq_1 + 2an_2 hkq_1 x^2 - 2dn_2 hkrx^2 \\ \qquad -2dhkrq_1 x^2 + acn_2 kq_1 z - cdn_2 krz - cdkrq_1 z + dn_2 krq_1 z \\ F = 2dn_2 hkrq_1 x^2 - bcdn_2 rq_1 + cdn_2 krq_1 z \end{cases}$$

For E_1, and by using typical parameters for Eq. (5) via MATLAB, the following eigenvalues are obtained:

$$\begin{cases} \lambda_1 = -2.5 \\ \lambda_2 = 0.81 \\ \lambda_3 = 9.2 \\ \lambda_4 = 15.6159 \\ \lambda_5 = -25.6153 \\ \lambda_6 = -0.0006 \end{cases}$$

This case is called a saddle equilibrium, whereas other cases are gotten (saddle-focus equilibrium points) under the points $E_{2,3}$ and the corresponding eigenvalues depict as:

$$\begin{cases} \lambda_1 = 0.81 \\ \lambda_2 = 9.2 \\ \lambda_3 = -17.5613 \\ \lambda_4 = -0.0007 \\ \lambda_{5,6} = 2.5310 \pm 10.3673i \end{cases}$$

Hence, all the three equilibrium points of system (4) are unstable.

3.2 Dissipative and Conservative system

The divergence of the system (4) can be evaluated as:

$$\nabla v = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} + \frac{\partial \dot{v}}{\partial v} + \frac{\partial \dot{u}}{\partial u} = -a - c + n_2 + q_1$$

Remark: The divergence of the system (4) can be classified as dissipative, conservative, or expanding volume, based on the coupling parameter n_2 and q_1 as:

- If $n_2 + q_1 < a + c$, then the system (4) is dissipative.
- If $n_2 + q_1 = a + c$, then the system (4) is conservative.
- If $n_2 + q_1 > a + c$, then the system (4) is expanding volume.

Under the typical parameters from Eq. (5), the system (4) has realized the first condition, and therefore system (4) is dissipative.

3.3 The Lyapunov Exponents (LEs) and Kaplan-York dimension (D_{KY})

Through the sign of the LEs, different behaviors of the system can be declared, such as stability, periodically (2-torus, 3-torus), chaotic, and hyperchaotic in dynamic system. The wolf Algorithm [41] presents many factors that affect computing the LE_s, such as the initial condition, step, observation time and parameters. It is noted that the system (4) has five positive LEs, i.e., has fulfilled the merit$(n-2) + ve\ LE_s$, so it is referred to having "hyperchaotic attractors," as depicted in Fig. 2.

$$\begin{cases} LE_1 = 9.1925695 \\ LE_2 = 1.6220390 \\ LE_3 = 0.8078124 \\ LE_4 = 0.0059941 \\ LE_5 = 0.0000707 \\ LE_6 = -14.1058610 \end{cases} \Rightarrow \sum_{i=1}^{6} LE_i = -2.4773753$$

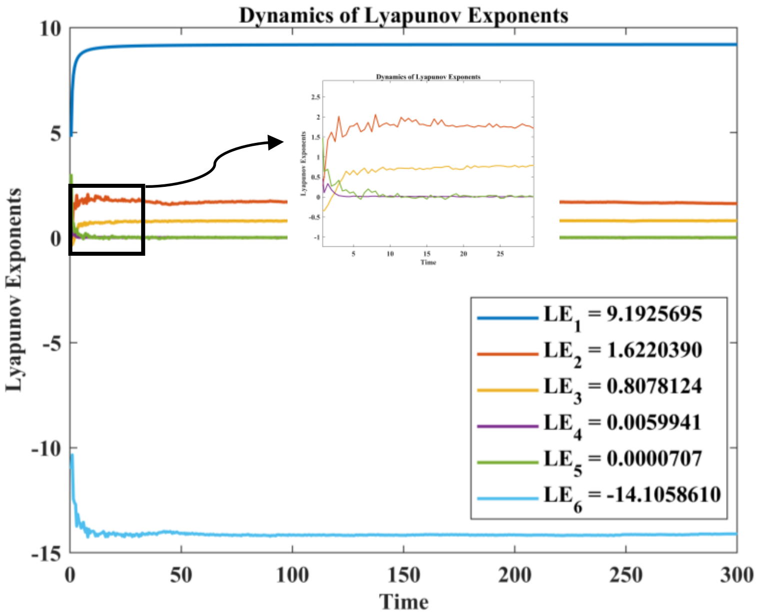

FIGURE 2. -**Lyapunov exponents for system (4).**

Remark 1: Assume that LE_i $(i = 1,2,3,4,5,6)$ are the Lyapunov exponents of the system (5), satisfying the condition. $LE_1 \geq LE_2 \geq LE_3 \geq LE_4 \geq LE_5 \geq LE_6$. The dynamical behaviors of system (5) can be classified as follows based on the Lyapunov exponents, show Table 1.

Table 1. -**Behaviors of 7D dynamical system with different sign of the LE$_i$**

LE_1	LE_2	LE_3	LE_4	LE_5	LE_6	Type of Attractors
-	-	-	-	-	-	Solution approach fixed pionts
0	-	-	-	-	-	Periodic solution (limit cycles
0	*0*	-	-	-	-	Quasi-periodic solutions (2-torus)
0	*0*	*0*	-	-	-	Quasi-periodic solutions (3-torus)
+	*0*	-	-	-	-	Chaotic attractors
+	+	*0*	*0*	-	-	Hyperchaotic (2-torus)
+	+	+	+	*0*	-	Hyperchaotic

The new 6D system has many different behaviors can obtained by changing initial condition (IC) and by changing the control parameter q_2. The parameter q_2 act as major substance in classifying system (4) either it is chaotic or hyperchaotic system, as in Eq. (7), the effect of parameter q_2 on the system is illustrated in the Tables (2-3).

TABLE 2. -**LEs with various IC**

IC	LE_1	LE_2	LE_3	LE_4	LE_5	LE_6	$\sum_{i=1}^{6} LE_i$	Attractor
$(1,1,1,1,1,1)$	9.1884	1.5936	0.8086	0.0011	-0.0003	-14.069	-2.4752	Hyperchaotic
$(0.1,0.1,0.1,0.1,0.1,0.1)$	9.1846	1.5936	0.8161	0.0039	0.0002	-14.0724	-2.4782	Hyperchaotic
$(1,0.8,1.6,2.8,1.2,3.7)$	9.1913	1.6447	0.8062	0.0014	-0.00008	-14.1209	-2.47738	Hyperchaotic
$(-2.1,-4.1,2.1,-8.1,0.07,6.1)$	9.1925	1.7133	0.8077	0.0003	0.0003	-14.1209	-2.4074	Hyperchaotic (2-torus)

TABLE 3. -**LEs with various coupling parameters q_2 at $(1,4.1,2.1,7,6.1,0.1)$**

Parameter q_2	LE_1	LE_2	LE_3	LE_4	LE_5	LE_6	$\sum_{i=1}^{6} LE_i$	Attractor
0.4	9.1926	1.6969	0.8043	0.0041	-0.0004	-14.1747	-2.4772	Hyperchaotic
0.7	9.1926	1.7060	0.8064	-0.0001	-0.0009	-14.1808	-2.4768	Hyperchaotic (2-torus)
6.002	9.1926	1.6996	0.8124	0.0002	-0.0002	-14.1818	-2.4772	Hyperchaotic (2-torus)
5.9	9.1926	1.6756	0.8034	0.0000	-0.0019	-14.1472	-2.4775	Hyperchaotic

The D_{ky} for the system (4) is given as:

$$D_{ky} = j + \frac{\sum_{i=1}^{5} LE_i}{|LE_{i+1}|} = 5 + 0.8244 = 5.8244 \tag{11}$$

Observing through Eq. (11), the Lyapunov dimension is fractal, therefore the trajectories of system (4) are strong. To illustrate the complexity of system (4), a comparison was made between the Kaplan-York dimension of the proposed system and known hyperchaotic systems. As can be seen from Table 5, the proposed system has a higher Kaplan-Yorke dimension compared to other systems, so it is more complex.

3.4 Multistability

The multistability of nonlinear dynamical systems means that a system can have more solutions under the same parameters. Table 5 illustrates two attractors under the control parameters with different initial conditions, whereas the corresponding attractors (solutions) are depicted in Figs. 4.

TABLE 4. Multistability at $(a = 5, b = 10)$ with numerous IC.

Figs.	Initial Conditions	Color
Fig.3 (a)	$(3.1,3.1,3.1,3.1,3.1,3.1)$	red
	$(3,3,3,3,3,3)$	blue
Fig.3 (b)	$(3.1,3.1,3.1,3.1,3.1,3.1)$	red
	$(5,5,5,-5,5,5)$	blue
Fig.3 (c)	$(0.07,-1.1,2.3,5.1,-6,4.2)$	red
	$(-1,1,1,1,1,1)$	blue

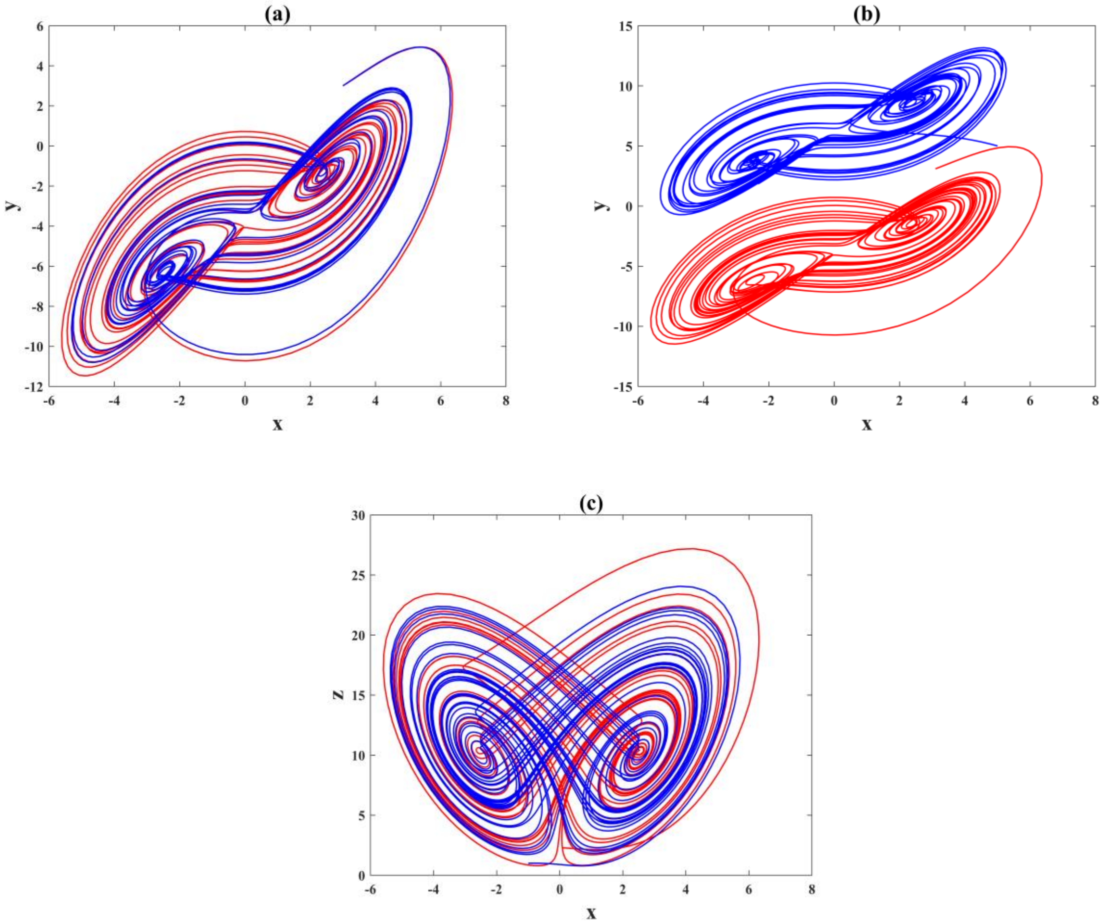

FIGURE 3. -Multistability of the two attractors with two ICs corresponding to Table 4.

4. Circuit Implementation of the new simple 6D hyperchaotic system

Due to its practical applications, the circuit implementation of hyperchaotic systems is very important in nonlinear science [42-45]. In this section, an improved modular design for the circuit is adopted to obtain the attractor of the system (4), as well as to enhance the validity of the experimental results. The circuit of system (4) is simulated via NI Multisim 14.2 software. The designed circuit consists of operational amplifiers (OP-AMP), resistors (R), capacitors (C), and multipliers as depicted in Fig. 4. The main elements of this circuit are OP-AMP TL082CD and multiplier AD633. The operation voltage of the OP-AMP is ±18 V, whereas it is saturated in ±16 V. as shown in the Fig. 4, the state variables exceed the dynamic range of ±16 V, so it is not necessary to change all variables of Eq. (4).

According to Kirchhoff's law and typical parameters (5) letting $\tau = \tau_0 \, t$, where $\tau_0 = 1000$ (τ_0 time scale transform factor), system (4) can be represented as follow:

$$\begin{cases} \dot{x} = \frac{\tau}{R_1C_1}y - \frac{\tau}{R_2C_1}x + \frac{\tau}{R_3C_1}w \\ \dot{y} = \frac{\tau}{R_4C_2}x - \frac{\tau}{10R_5C_2}xz \\ \dot{z} = -\frac{\tau}{R_6C_3}z + \frac{\tau}{10R_7C_3}x^2 \\ \dot{w} = \frac{\tau}{R_8C_4}y \\ \dot{v} = -\frac{\tau}{10R_{10}C_5}wx + \frac{\tau}{R_{11}C_5}v \\ \dot{u} = -\frac{\tau}{10R_{12}C_6}xz + \frac{\tau}{R_9C_4}u \end{cases} \quad (12)$$

where x, y, z, w, v and u denoted the voltage across the capacitors C_1, C_2, C_3, C_4, C_5 and C_6, respectively. Comparing system (4) with system (12), the following formula can be found:

$$\begin{cases} \frac{\tau}{R_1C_1} = \frac{\tau}{R_2C_1} = 10; \ \frac{\tau}{R_3C_1} = 6.5; \\ \frac{\tau}{R_4C_2} = 40; \ \frac{\tau}{10R_5C_2} = 1 \\ \frac{\tau}{R_6C_3} = 2.5; \ \frac{\tau}{10R_7C_3} = 4 \\ \frac{\tau}{R_8C_4} = 0.001; \ \frac{\tau}{R_9C_4} = 0.81 \\ \frac{\tau}{10R_{10}C_5} = 0.25; \ \frac{\tau}{R_{11}C_5} = 9.2 \\ \frac{\tau}{10R_{12}C_6} = 1.1 \end{cases} \quad (13)$$

The parameters of each component in the circuit are set as:

$$\begin{cases} R_1 = R_2 = 100k\Omega; \ R_3 = 153.84k\Omega; \\ R_4 = 25k\Omega; \ R_5 = 400k\Omega; R_6 = 400k\Omega; \\ R_7 = 1000k\Omega; R_8 = 100M\Omega; R_9 = 123.45M\Omega; \\ R_{10} = 400k\Omega; R_{11} = 108.69k\Omega; R_{12} = 909.09k\Omega; \\ R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = 100k\Omega; \\ R_{18} = R_{19} = R_{20} = R_{21} = R_{22} = 100k\Omega; \\ C_1 = C_2 = C_3 = C_4 = C_5 = 1nF, C_6 = 100\mu F. \end{cases}$$

The investigational results of hyperchaotic attractor in the circuit are depicted in Figs. 5 and 6 matched with the numerical simulation in Figs. 1(a) – 1(b). Therefore, the hyperchaotic behaviors of system (4) are performed by the physical experiment.

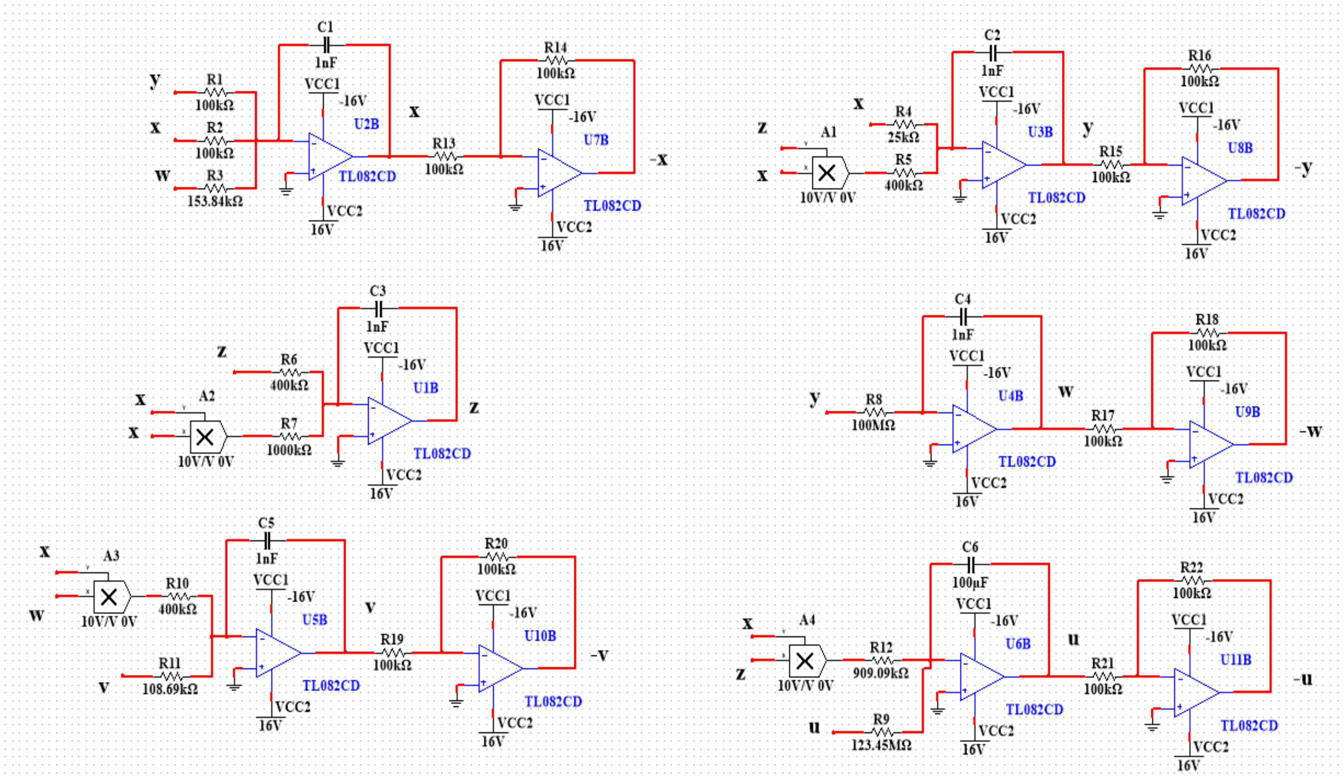

FIGURE 4.-The diagram of circuit implementation of system (12).

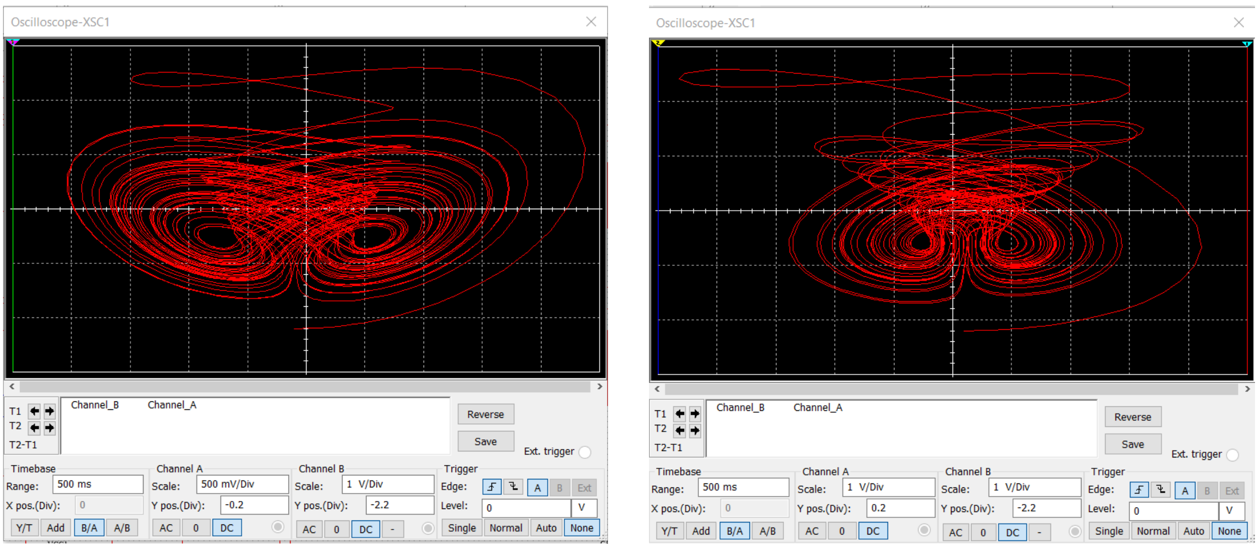

FIGURE 5. -Attractors of system (12) with an oscilloscope.

5. DISCUSSION ON THE RESULTS

In this item, some features were presented through a comparison table between the proposed 6D system and some 6D systems in the literature. It is noted that there are some advantages of the proposed system in terms of larger positive exponential Lyapunov as well as the amount of dimension.

Table 5. - Comparison of results between the proposed system and the related work.

No.	Types of attractors	No. of terms		No. of $+ve\ LEs$	D_{KY}	MLE	Reference
		Total	Nonlinear				
1	Not studied	20	3	2	---------	---------	2019 [27]
2	Self-excited	18	4	3	---------	---------	2009 [28]
3	Self-excited	14	2	4	5.5721	0.093	2019 [29]
4	Hidden	14	2	4	5.1057	1.3613	2020 [30]
5	Self-excited	15	3	4	5.0694	0.4692	2017 [31]
6	Hidden	14	3	4	5.1055	1.42048	2022 [32]
7	Hidden	14	2	4	5.0847	7.340	2020 [33]
8	Not studied	15	3	2	---------	---------	2020 [34]
9	Not studied	14	2	4	4.1057	1.3613	2021 [35]
10	Self-excited	16	6	1	4.2308	0.4480	2019 [36]
11	Hidden	13	3	2	4.7723	10.16	2020 [37]
12	Not studied	15	4	2	4.0019	0.799484	2021 [38]
13	Self-excited	**12**	4	**4**	**5.8244**	**9.1925**	**This work**

6. CONCLUSION

In this paper, a new simple 6D hyperchaotic with $(n-2)+ve$ is built up based on 3D Liu system, which has twelve terms, four of them is a quadratic. Dynamical characteristics of the designed system are investigated via equilibrium point, stability, LEs, and D_{ky}. The new 6D system is highly dynamic and a have complex hyper-chaotic behavior. In addition, the electrical circuit preformed a model of the new 6D simple system, which verified the feasibility of the new system. All results were verified by using MATLAB R2021a, and Multisim 14.2 software.

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CONFLICT OF INTEREST:

Author declare no conflict of interest

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