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# Towards a new curvature produced by the tangent of a circle and an ellipse: The Nada's curve

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**ABSTRACT:** A new curve is produced and graphically studied. The name of my daughter, Nada, has been given to this curve; in other words, Nada is used to describe this curve whenever it is used in this paper. "Nada's curve" is a form of closed curve that is constructed when the circle's diameter and the ellipse's minor axis share the same length, and they are tangent by a point from a drawn line passing through the circumstances of the circle and ellipse. Then, from these two intersection points, the point of intersection of the vertical and horizontal lines is selected to determine a point of Nada's curve. By sharing two vertically lined points on both the circle's diameter and the ellipse's minor axis, the parametric equation can be simplified and calculated easily, starting with an equation of the circle: x2 +(y-1)2 =1. Cases of Nada's curve were graphically investigated in this study. The surface generated by the revolution of Nada's curve around its axis is called "Nadaoid."

**Keywords:** Nada; Curve; Curvature; Circle; Ellipse

## 1. INTRODUCTION

A curved line is a locus of straight-line segments, whereas a straight line is a locus of points that lie evenly and can be classified. Curves are natural formulations of small line segments and have a radius of a curvature. The chicken egg, as a modeled oviform object, has been the focus of advanced description in mathematical terms [1, 2]. In the course of history, many individuals have been honored and remembered primarily for their research contributions, such as the precise mathematical descriptions of the egg curve via the models of Hügelschäffer [3, 4] and Granville [5]. In this study, a new form of ptheadlocked's curve has been produced and graphically investigated. The name of my daughter, Nada, has been given to this curve; therefore, Nada describes this curve whenever it is used in this paper. "Nada's curve" is a new form of curve produced from the locus of points led by a line from the tangent point, intersecting the circumstance of the circle and an ellipse. Nada's curve is a curvature of a circle and an ellipse relative to one of its points and the diametrically opposite tangent from this point. In some cases, such as this case where [6]it is therefore a new special case of Granville's egg curve [7, 8]. Nevertheless, Nada's curve is produced by a tangent circle and an ellipse.

By drawing a circle and an ellipse (or any convenient radios and minor axis) that are tangent by the points of the circle's diameter and the ellipse's minor axis, the features of Nada's curve can be constructed. A common definition of Nada's curve is the locus of points P that is determined by selecting a point as the bottom of the circle and drawing from it a line to intersect and cross the circumstances of the circle and the ellipse. Then, on the basis of the intersection point of these horizontal and vertical lines, point P is designated as a point of Nada's curve. To graphically create Nada's curve, a new construction method is developed in this work as a means of determining the real positions of the curve points. Nada's curve can be produced by initially supposing that the length of a circle's diameter and an ellipse's minor axis are equal and then used as a constant. A new series for this curve is developed and investigated in this study. The proposed approach can be better adapted for use in constructing experimental curves.

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In this method, the position of the points of Nada's curve was drawn step by step in AutoCAD. All graphical figures and obtained dimensions were measured to examine the precise distance and location of points on the curve.

### 1.1 CONSTRUCTION METHOD FOR NADA'S CURVE

In the graphical method by using the (-axis), the first step is to derive the horizontal axis and vertical lengths of Nada's curve whose segments represent the circle's diameter and the ellipse's minor axis, respectively; these elements were selected in this study as an example of using the (-axis). To create Nada's curve graphically, first, draw a circle and an ellipse that are tangent at the y- and x-axes on points (A) and (B), respectively. The following steps describe the procedure of constructing Nada's curve by using this method:

- Start with a vertical line and regard point (A) as the origin.
- Then, construct a circle and an ellipse in that they are tangent on two points (i.e., the circle's diameter and the ellipse's minor axis line).
- From point (A) at the bottom of the circle, select a point along the circle's circumstance (labeled as "An" in Figure 1), and extend the segment so that it intersects the ellipse's circumstance at point (Pn).
- Horizontally mark a line segment from point (An) and vertically intersect it from another line segment from point (Pn). Denote the intersection point as Bn, which is a point of Nada's curve.
- Perform the previous steps to create a variety of (BnAn) and (AnPn) values. Mark out a set of points (Bn) on the curve known as Nada's curve.

The curve does not have an asymptotic behavior because it is a closed curve and tends to move to the +y-axis value of zero at point (B), which is the origin point. Moreover, its horizontal length is near-equal to the length of the major axis of the ellipse. In general, there are only four points of Nada's curve, and they are always located at the circumstance of both the circle and ellipse, while two of these points are the intersection points of the diameter and the minor axis (Figure 1).

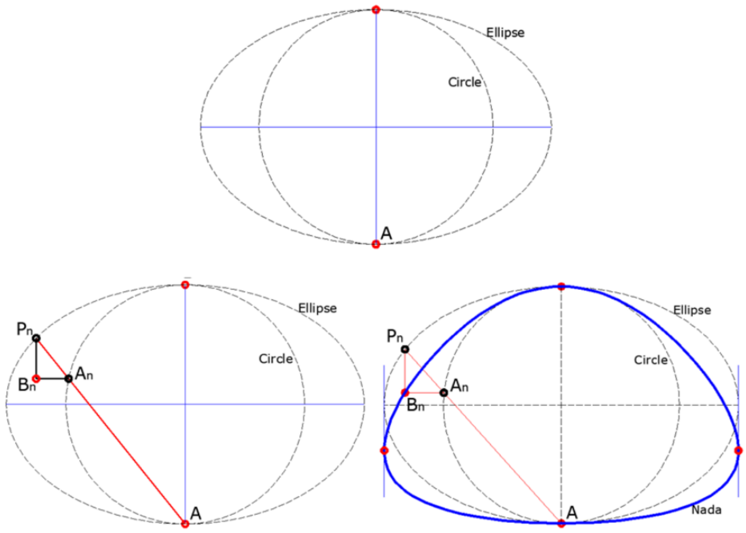


**FIGURE 1. Construction method of Nada's curve (illustrative example): the ellipse's minor axis (a) is approximately (0.65) of the major axis (b).**

## 2. DIVERSITY OF NADA'S CURVE

Different shapes of Nada's curve were studied in this research. By fixing the length of the circle's diameter and the ellipse's minor axis, the variety originates from the length of the major axis of the ellipse; thus, the shape of Nada's curve can either be or not long-drawn-out along the major axis. In general, the expansion of the curve occurs along the ellipse's major axis, and therefore, this major axis can be used as a parameter to determine the shape of Nada's curve. Eight cases were drawn in this research to check the variety of Nada's curve, primarily by considering the position of point (A) related to the aforementioned major axis. Furthermore, two cases in which the major axis lies on the x-axis or y-axis were investigated. In this approach, Nada's curve was drawn according to the position in which the circle and ellipse are tangent on point (A).

## 2.1 THE MINOR AXIS OF THE ELLIPSE IS LOCATED ON THE Y-AXIS

In this case, the ellipse is tangent to the circle at two points, namely, the circle's diameter points, in which (A) is the origin point. As the minor axis lies on the y-axis, the major axis determines the horizontal expansion of Nada's curve that is parallel to the x-axis. Different samples of Nada's curve were drawn according to this case. Figure 2 illustrates the first case in which the ellipse's major axis length is randomly chosen as (0.83) of the diameter of the circle. The circumstance length of Nada's curve is equal to that of the ellipse, but it is longer by (1.0773%) than that of the circle.

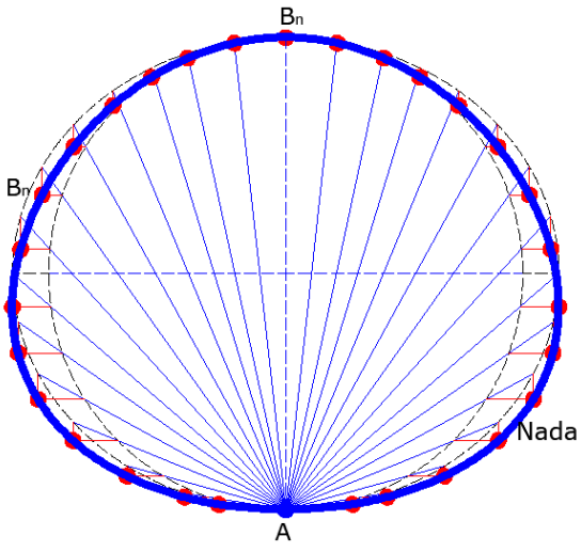


**FIGURE 2.** **Case no. 1: The length of the ellipse's major axis is randomly chosen as (0.83) of the circle's diameter. The circumstance length of Nada's curve (bold blue) is equal to that of the ellipse.**

By fixing the minor axis of the ellipse (i.e., it is equal to the diameter of the circle) on the y-axis, the shape and the circumstance length of Nada's curve in this case are regularly altered according to the major axis length of the ellipse. In this study, five cases of Nada's curve were studied. The details are given in Figure 3 and Table 1.

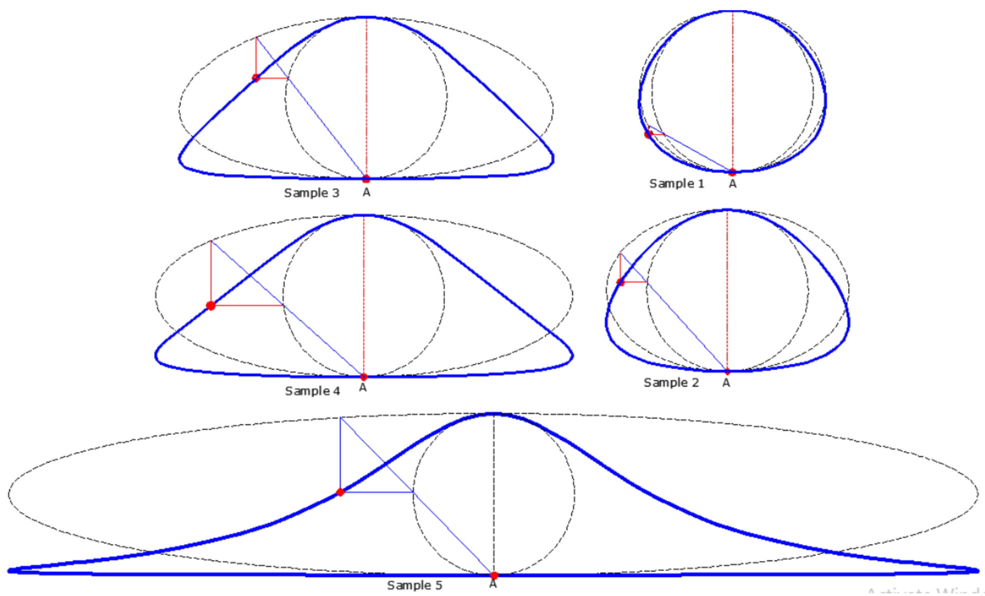


**FIGURE 3.** **The minor axis of the ellipse (i.e., it is equal to the diameter of the circle) is fixed on the y-axis. The shape and the circumstance length of Nada's curve are regularly altered according to the major axis length of the ellipse.**

AutoCAD was used to draw all five samples of Nada's curve (Figure 3), and the list of measurements was checked. Notably, the circumstance of Nada's curve is near-equal to the length of the ellipse's circumstance.

**Table 1. By fixing the circle radius, (r=0.5), circumstance of the ellipse and Nada in 5 samples when the major axis of the ellipse is located on x-axis**

| Case | Major axis | Ellipse | Nada | Difference |
|---|---|---|---|---|
| 1 | 1.1523 | 3.3851 | 3.3844 | 0.0007 |
| 2 | 1.5000 | 3.9663 | 3.9563 | 0.0100 |
| 3 | 2.3138 | 5.4119 | 5.3434 | 0.0584 |
| 4 | 2.5811 | 5.9028 | 5.8215 | 0.0813 |
| 5 | 5.9999 | 12.4498 | 12.4092 | 0.0406 |

### 2.2 THE MAJOR AXIS OF THE ELLIPSE IS LOCATED ON THE Y-AXIS:

In this case, the ellipse is tangent to the circle at a single point (i.e., point (A)), which is the origin point. As the major axis lies on the +y axis, the minor axis determines the horizontal expansion of Nada's curve along this axis. Seven samples were investigated in this study (Figure 4). AutoCAD was similarly used for drawing all seven samples of Nada's curve (Figures 4 and 5), and the list of measurements were checked. The obtained measurements indicate that the circumstance of Nada's curve does not match the length of the ellipse's circumstance. According to the step in which the major axis turns towards the x-axis, the difference generally increases. The expansion of Nada's curve also tends to move towards the x-axis. The details are presented in Table 2.

**Table 2. By fixing the circle of unit, (r=0.5), circumstance of the ellipse and Nada in 7 samples when the major axis of the ellipse is located on +y-axis**

| Case | Ellipse | Nada | Difference |
|---|---|---|---|
| 1 | 4.0716 | 3.4218 | 0.6498 |
| 2 | 4.2140 | 3.3626 | 0.8514 |
| 3 | 4.5494 | 3.4078 | 1.1416 |
| 4 | 5.4864 | 3.9133 | 1.5731 |
| 5 | 6.7837 | 5.4068 | 1.3769 |
| 6 | 9.6043 | 8.4068 | 1.1975 |
| 7 | 13.1311 | 12.0815 | 1.0496 |

## 3. GENERAL EQUATION OF NADA'S CURVE

As mentioned in the previous sections, the Nada's curve can be produced according to two main cases. The first case is when the ellipse and circle are tangent on two points (i.e., the diameter and the minor axis points, respectively). The second case is when the circle and ellipse are tangent on one point (i.e., point (A)). The curve can be conveniently produced using the second case. However, as points (Cn) and (P) move around the circumstances of the circle and ellipse, point (B) can also be traced to derive a point of Nada's curve. On this basis, the general equation of the curve can be determined according to the curvature principles of the ellipse, the circle, and (u), as shown in Figure 7:

Let an ellipse and a circle share the same center point, where the circle's radius is the minor axis of the ellipse (a). By drawing a segment line from point (A) that passes through the circle (Cn), this segment line can be extended to intersect the ellipse on point (P). Hence, this line produces an angle (u) from point (A). Then, construct the vertical and horizontal segments from points (P) and (A), respectively. Notably, regardless of the value of angle (u), the midpoint (E) of the horizontal segment (DA) passes through the midpoint of segment (PA). The general equation of Nada's curve can be summarized as follows.

For the vertical segments (ECn) = B(y),

$$\boldsymbol{B}(\boldsymbol{y}) = 2\boldsymbol{a}\cos u \ \sin u \ . \tag{1}$$

since; ; $\tan u = \frac{x}{y}$, by referring to the value of ( ) in Equation (1) and simple trigonometry, we have

$$\tan u = \frac{x - 2a\cos^2 u}{y - 2a\sin^2 u} \tag{2}$$

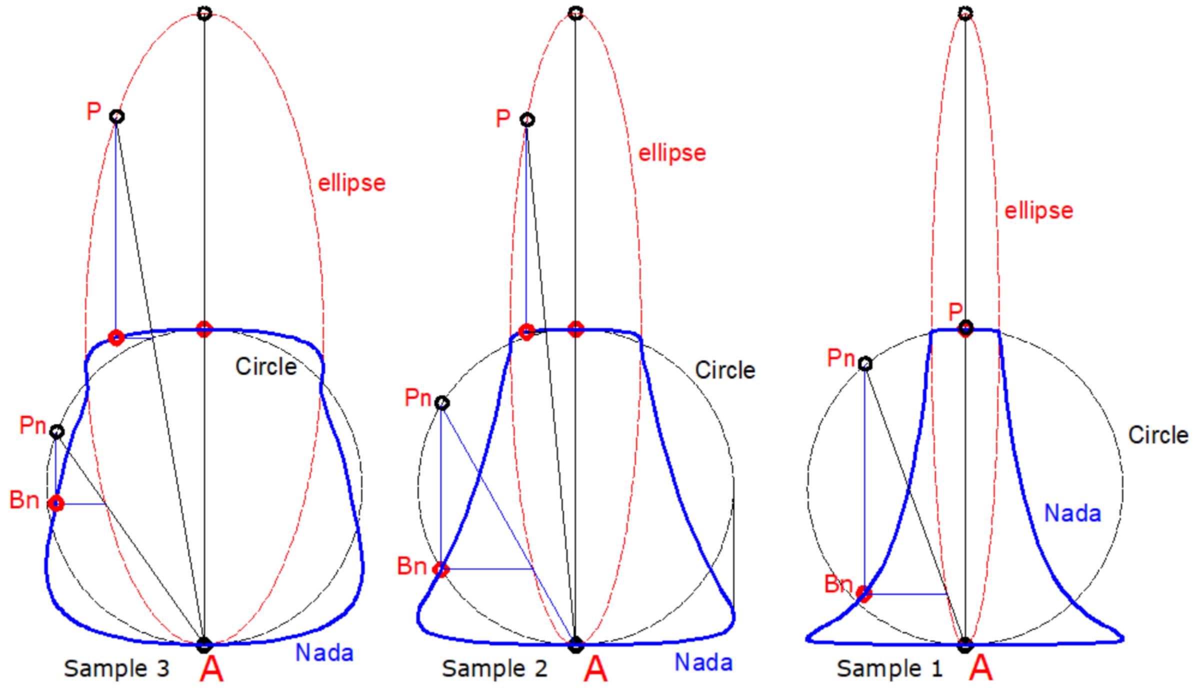


**FIGURE 4.** By fixing the length of the minor axis of the ellipse, 3 cases of Nada's curve which are drawn by this method, when the major axis of the ellipse is located on y-axis, the origin point is (A).

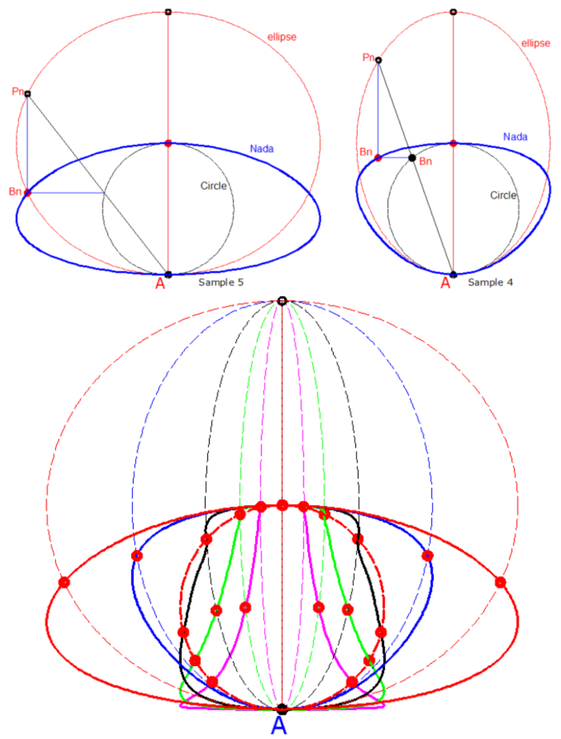


**FIGURE 5.** By fixing the length of the minor axis of the ellipse, all these nominated 7 cases of Nada's curve which are drawn by this method, when the major axis of the ellipse is located on y-axis, the origin point is (A).

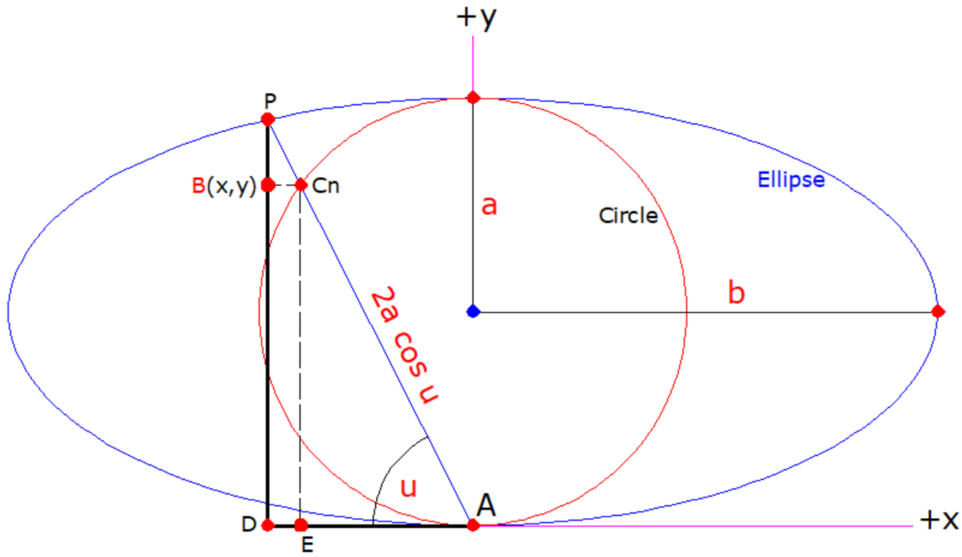


**FIGURE 6.** **Geometric construction of Nada's curve by considering the tangent of the ellipse and circle.**

Hence, the points on Nada's curve are located at *B(x, y)*.

Then, from the equation (1), the value can be re-formed as follows:

$$\boldsymbol{B}(\boldsymbol{x}) = \tan \mathrm{u}\left(\boldsymbol{y} - 2\mathrm{a}\sin^2 u\right) + 2\boldsymbol{a}\cos^2 u, \tag{3}$$

However, according to the curvature plot, the major axis of the ellipse is located on the y-axis. Thus the origin point is (A), and the tangent point. In this study, the ellipse axes (a and b), circle radius (a or b), and the angle (u) are jointly used as the parameters for the points of Nada's curve. Therefore, a set of data was obtained by equations (1) and (3). The details are given in Tables A and B and Figure (7).

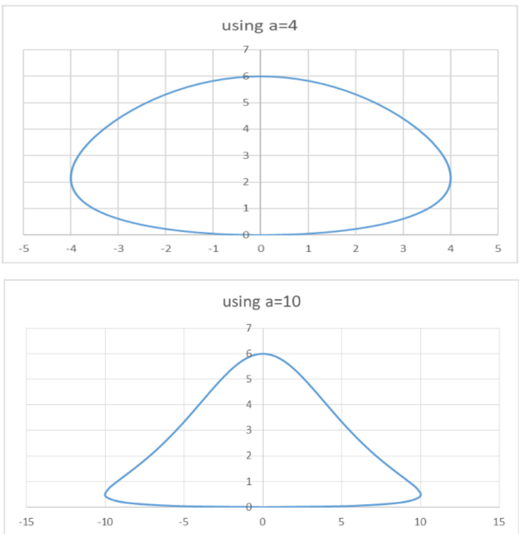


**FIGURE 7.** **Curvature plot of two cases, (a=4) and (a=10).**

**Table A.** **Quadratic formula of Nada's curve**

| Degree | sin | r=3 | Circle | | Circle | |
|---|---|---|---|---|---|---|
| | | | y | x | x | y |
| 0 | 0 | 3 | 0 | 0 | 0 | 0 |
| 5 | 0.087156 | 3 | 0.045577 | 0.520945 | 0.520945 | 0.045577 |
| 10 | 0.173648 | 3 | 0.180922 | 1.02606 | 1.02606 | 0.180922 |
| 15 | 0.258819 | 3 | 0.401924 | 1.5 | 1.5 | 0.401924 |
| 20 | 0.34202 | 3 | 0.701867 | 1.928363 | 1.928363 | 0.701867 |
| 25 | 0.422618 | 3 | 1.071637 | 2.298133 | 2.298133 | 1.071637 |
| 30 | 0.5 | 3 | 1.5 | 2.598076 | 2.598076 | 1.5 |
| 35 | 0.573576 | 3 | 1.97394 | 2.819078 | 2.819078 | 1.97394 |
| 40 | 0.642788 | 3 | 2.479055 | 2.954423 | 2.954423 | 2.479055 |
| 45 | 0.707107 | 3 | 3 | 3 | 3 | 3 |
| 50 | 0.766044 | 3 | 3.520945 | 2.954423 | 2.954423 | 3.520945 |
| 55 | 0.819152 | 3 | 4.02606 | 2.819078 | 2.819078 | 4.02606 |
| 60 | 0.866025 | 3 | 4.5 | 2.598076 | 2.598076 | 4.5 |
| 65 | 0.906308 | 3 | 4.928363 | 2.298133 | 2.298133 | 4.928363 |
| 70 | 0.939693 | 3 | 5.298133 | 1.928363 | 1.928363 | 5.298133 |
| 75 | 0.965926 | 3 | 5.598076 | 1.5 | 1.5 | 5.598076 |
| 80 | 0.984808 | 3 | 5.819078 | 1.02606 | 1.02606 | 5.819078 |
| 85 | 0.996195 | 3 | 5.954423 | 0.520945 | 0.520945 | 5.954423 |
| 90 | 1 | 3 | 6 | 0 | 0 | 6 |
| 95 | 0.996195 | 3 | 5.954423 | 0.520945 | 0.520945 | 5.954423 |
| 100 | 0.984808 | 3 | 5.819078 | 1.02606 | 1.02606 | 5.819078 |
| 105 | 0.965926 | 3 | 5.598076 | 1.5 | 1.5 | 5.598076 |
| 110 | 0.939693 | 3 | 5.298133 | 1.928363 | 1.928363 | 5.298133 |
| 115 | 0.906308 | 3 | 4.928363 | 2.298133 | 2.298133 | 4.928363 |
| 120 | 0.866025 | 3 | 4.5 | 2.598076 | 2.598076 | 4.5 |
| 125 | 0.819152 | 3 | 4.02606 | 2.819078 | 2.819078 | 4.02606 |
| 130 | 0.766044 | 3 | 3.520945 | 2.954423 | 2.954423 | 3.520945 |
| 135 | 0.707107 | 3 | 3 | 3 | 3 | 3 |
| 140 | 0.642788 | 3 | 2.479055 | 2.954423 | 2.954423 | 2.479055 |
| 145 | 0.573576 | 3 | 1.97394 | 2.819078 | 2.819078 | 1.97394 |
| 150 | 0.5 | 3 | 1.5 | 2.598076 | 2.598076 | 1.5 |
| 155 | 0.422618 | 3 | 1.071637 | 2.298133 | 2.298133 | 1.071637 |
| 160 | 0.34202 | 3 | 0.701867 | 1.928363 | 1.928363 | 0.701867 |

**Table B.** **Quadratic formula of Nada's curve**

| Degree | tan | a | b | x | y |
|---|---|---|---|---|---|
| 0 | 0 | 10 | 3 | 0 | 0 |
| 5 | 0.087489 | 10 | 3 | 5.375413 | 0.045577 |
| 10 | 0.176327 | 10 | 3 | 8.736901 | 0.180922 |
| 15 | 0.267949 | 10 | 3 | 9.93651 | 0.401924 |
| 20 | 0.36397 | 10 | 3 | 9.81606 | 0.701867 |
| 25 | 0.466308 | 10 | 3 | 9.100378 | 1.071637 |
| 30 | 0.57735 | 10 | 3 | 8.182917 | 1.5 |
| 35 | 0.700208 | 10 | 3 | 7.239899 | 1.97394 |
| 40 | 0.8391 | 10 | 3 | 6.340099 | 2.479055 |
| 45 | 1 | 10 | 3 | 5.504587 | 3 |
| 50 | 1.191754 | 10 | 3 | 4.734577 | 3.520945 |
| 55 | 1.428148 | 10 | 3 | 4.023695 | 4.02606 |
| 60 | 1.732051 | 10 | 3 | 3.363205 | 4.5 |
| 65 | 2.144507 | 10 | 3 | 2.744143 | 4.928363 |
| 70 | 2.747477 | 10 | 3 | 2.158091 | 5.298133 |
| 75 | 3.732051 | 10 | 3 | 1.597373 | 5.598076 |
| 80 | 5.671282 | 10 | 3 | 1.05501 | 5.819078 |
| 85 | 11.43005 | 10 | 3 | 0.524571 | 5.954423 |
| 90 | 1.63E+16 | 10 | 3 | 3.68E−16 | 6 |
| 95 | −11.4301 | 10 | 3 | −0.52457 | 5.954423 |
| 100 | −5.67128 | 10 | 3 | −1.05501 | 5.819078 |
| 105 | −3.73205 | 10 | 3 | −1.59737 | 5.598076 |
| 110 | −2.74748 | 10 | 3 | −2.15809 | 5.298133 |
| 115 | −2.14451 | 10 | 3 | −2.74414 | 4.928363 |
| 120 | −1.73205 | 10 | 3 | −3.36321 | 4.5 |
| 125 | −1.42815 | 10 | 3 | −4.0237 | 4.02606 |
| 130 | −1.19175 | 10 | 3 | −4.73458 | 3.520945 |
| 135 | −1 | 10 | 3 | −5.50459 | 3 |
| 140 | −0.8391 | 10 | 3 | −6.3401 | 2.479055 |
| 145 | −0.70021 | 10 | 3 | −7.2399 | 1.97394 |
| 150 | −0.57735 | 10 | 3 | −8.18292 | 1.5 |
| 155 | −0.46631 | 10 | 3 | −9.10038 | 1.071637 |
| 160 | −0.36397 | 10 | 3 | −9.81606 | 0.701867 |
| 165 | −0.26795 | 10 | 3 | −9.93651 | 0.401924 |
| 170 | −0.17633 | 10 | 3 | −8.7369 | 0.180922 |
| 175 | −0.08749 | 10 | 3 | −5.37541 | 0.045577 |
| 180 | −1.2E−16 | 10 | 3 | −8.2E−15 | 9.01E−32 |

## 4. CONCLUSIONS

A new form of curve called Nada's curve is presented in this study, and its construction method involves determining the points on the curve. Here, two cases were considered in designing the construction method. Moreover, a new form of surface, which is a result of the revolved shape surface, is identified. The surface generated by the revolution of Nada's curve around its axis is called Nadaoid. Thus, the volume of the solid of revolution is generated by rotating Nada's curve around its asymptote. In this study, the ellipse's axes (a and b), the circle's radius (a or b), and the angle (u) are jointly as the parameters for the points of Nada's curve. Thus, a Cartesian parameterization of the Nada's curve is offered.

This research may be regarded as a starting point of further investigation, such as finding the exact plane(s) intersecting the Nadaoid or with Granville's or Hügelschäffer's egg curve. This author hopes to attract the attention of mathematicians, engineers, and biologists interested in analyzing the ellipse–egg curves.

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## AUTHOR'S DECLARATION

### CONFLICT OF INTEREST:

**None**.

I hereby confirm that all figures, tables, and measurements in the manuscript are mine. All obtained figures and geometrical aspects in this study have been drawn and measured via AutoCAD ver. 2007 as a way of increasing the correctness of the results.

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