VARITIONAL ITERATION METHOD FOR GENERALIZED BURGERS-FISHER EQUATION

Waleed K.Jaber
College of Computer and Mathematical
Thi-Qar University.

Zeanab k. jabar
College of Computer and Mathematical
Thi-Qar University.

المستخلص

تناولنا في هذا البحث حل معادلة بورغر- فيشر المعممة باستعمال طريقة تكرار التغاير حيث تم مقارنة النتائج التي تم الحصول عليها باستعمال هذه الطريقه مع الحل الدقيق لهذه المعادله واثبتت النتائج ان هذه الطريقه متقاربة واكثر كفاءة.

Abstract

In this paper, numerical solutions of the generalized Burgers-fisher equation are obtained by using Variational Iteration Method (VIM) and compare our result with exact solution numerical results show that variational iteration method is more efficient and powerful method.

1.Introduction

Variational iteration method (VIM) is preferable over numerical methods as it is free from rounding off errors and neither requires large computer power /memory.He [1],[2]&[3] has applied this method for obtaining analytical solutions of autonomous ordinary differential equation, nonlinear partial differential equations with coefficients and integro differential equations. the variational iteration method was successfully applied to Burgers and coupled Burgers equation [4], to Schruoding-kdv,generalized kdv and shallow water equations [5], to linear Helmholtz partial differential equation[6]. Linear and nonlinear wave equations,kdv,k(2,2), Burgers,and cubic Boussinesq equations have been solved by Wazwaz[7] and [8]using the VIM. In the present paper we employ VIM method for solving Burgers-fisher equation and find the error between the approximate solution which find by this method and exact solution.

Consider the generalized Burgers-fisher equation [9]

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u(1-u^{\delta}) \qquad 0\le x\le 1, \quad t\ge 0 \tag{1}$$

With the initial condition ***u(x, 0) = f(x),*** and the exact solution is

$$u(x,t)=(\frac{1}{2}+\frac{1}{2}\tanh[\frac{-\alpha\delta}{2(\delta+1)}(x-(\frac{\alpha}{\delta+1}+\frac{\beta(\delta+1)}{\alpha})t])^{\frac{1}{\delta}} \quad (2)$$

2. BASICE IDEA OF VIM.

To illustrate its basic concepts of variational iteration method, we consider the following deferential equation:

$$Lu+Nu=g(x) \quad (3)$$

Where $\boldsymbol{Lu_n}$ is a linear operator, $\boldsymbol{Nu_n}$ a non linear operator, and g(x) is an inhomogeneous term. According to VIM, we can construct a correction as follows:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda\{Lu_n+N\tilde{u}_n-g\}dt \quad (4)$$

where $\boldsymbol{\lambda}$ is called a general Lagrange multiplier [10] and [11], which can be identified via the variational theory, the subscript n indicates the nth order approximation,$\boldsymbol{u_n}$ is considered as a restricted variation,

(i.e. $\boldsymbol{\delta u_n = 0}$ ***)***

3. Application of the VIM:

To solve equation (1) By means of the **VIM**, we substitute in equation (4) by

$$Lu=\frac{\partial u_n}{\partial t} \qquad Ru_n=\frac{-\partial^2 u_n}{\partial x^2} \qquad Nu_n=\alpha u_n^{\delta}\frac{\partial u}{\partial x}-Bu(1-u^{\delta})$$

To get

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda(\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}+\alpha u^{\delta}\frac{\partial u}{\partial x}-Bu(1-u^{\delta}))dt \quad (5)$$

$$Let\ N_n(u)=\alpha u_n^{s}\frac{\partial u}{\partial x}-B u_n(1-u_n^{s})\ non\ linear\ operator\ .$$

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda(s)\left[\frac{\partial}{\partial s}u_n(x,s)-\frac{\partial^2}{\partial x^2}\tilde{u}_n(x,s)+N_n(\tilde{u}_n(x,s))\right]ds$$

...(6)

where $\widetilde{u}(x,s)$ is considered as a restricted variation (i.e. $\delta\widetilde{u}_n(x,s)=0$)
The variation of (6) is:

$$\delta u_{n+1}(x,t)=\delta u_n(x,t)+\delta\int_0^t \lambda(s)\left[\frac{\partial}{\partial s}u_n(x,s)-\frac{\partial^2}{\partial x^2}\widetilde{u}_n(x,s)+N(\widetilde{u}_n(x,s))\right]ds$$

by using integration by parts we have

$$\delta u_{n+1}=\delta u_n+\delta\Big|_{s=t}\lambda(s)u_n(x,s)-\delta\int_0^t \lambda^{\backslash}(s)u_n(x,s)\,ds-0+0=0$$

We obtain

$$\delta u_{n+1}=1+\lambda(s)\Big|_{s=t}\delta u_n-\int_0^t \lambda^{\backslash}(s)\,\delta u_n\,ds$$

By stationary conditional:

$$1+\lambda(t)=0 \qquad \Longrightarrow \qquad \lambda(t)=-1$$

$$\lambda'(t)=0 \qquad \dots (7)$$

From this equation, we obtain ***λ = -1***, substitutes the value of ***λ*** in equation (5) we obtain:-

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^t\left(\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}+\alpha u^{\delta}\frac{\partial u}{\partial x}-Bu(1-u^{\delta})\right)dt \qquad (8)$$

The solution of these equation can be find by using matlap 6.5, we obtain:
(Take δ=1)

$$u_0 = \frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x$$

$$u_1 = \frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x + \frac{1}{2\alpha}(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x)\tanh(\frac{-\alpha}{4})t + b(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x)$$
$$(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x)t.$$

$$u_2 = \frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x + \frac{1}{2\alpha}(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x)\tanh(\frac{-\alpha}{4})t + b(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x(\frac{1}{2}$$
$$+ \frac{1}{2}\tanh(\frac{-\alpha}{4})xt + \frac{\alpha}{4}(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x\tanh(\frac{-\alpha}{4}) + b(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})x)(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})xt$$
$$-\frac{1}{4}b\tanh(\frac{-\alpha}{4})^2 t^2 + \frac{1}{3}(\frac{-\alpha}{4}(\frac{1}{2} + \frac{1}{2}\tanh(\frac{-\alpha}{4})xt.$$

.

.

.

u_n

Continue for n-iterative…

4. Numerical Results

In this paper we compare between the exact and approximate solution for 3-iterative of ***VIM*** by using the :

$$error = \left| u_{exact} - u_{approximate} \right|$$

Our results show that VIM is more efficient and power full method.

Spatial cases.

4-1. Case I: -

For computational work, we have taken ($\alpha = 1 and\ \beta = 0$) , we can find approximate solution, and make comparison between the exact solution and approximate solution.

Table (1)

x	t	Exact solution	Approx. VIM	Error VIM
0.1	0.2	0.395320	0.394619	7.018374E-04
	0.0001	0.487508	0.487760	2.511903E-04
0.5	0.5	0.395321	0.394620	7.018372E-04
	0.0005	0.438797	0.437854	9.429356E-04
0.9	0.1	0.395333	0.394640	6.938374E-04
	0.0001	0.389791	0.389366	4.246652E-04

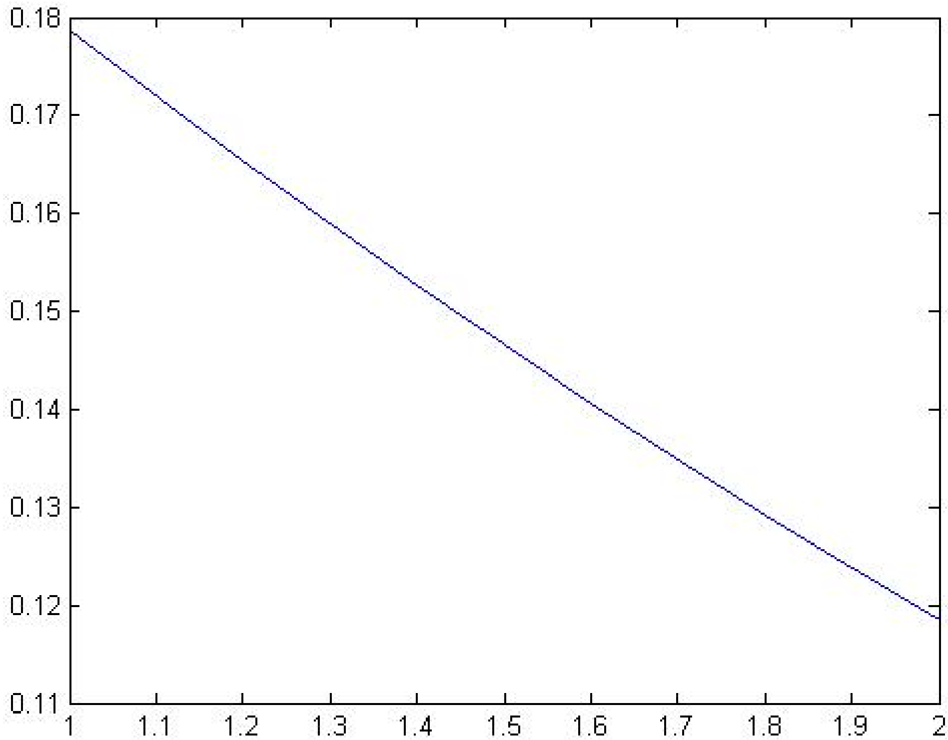


Figure (1)

Exact solution

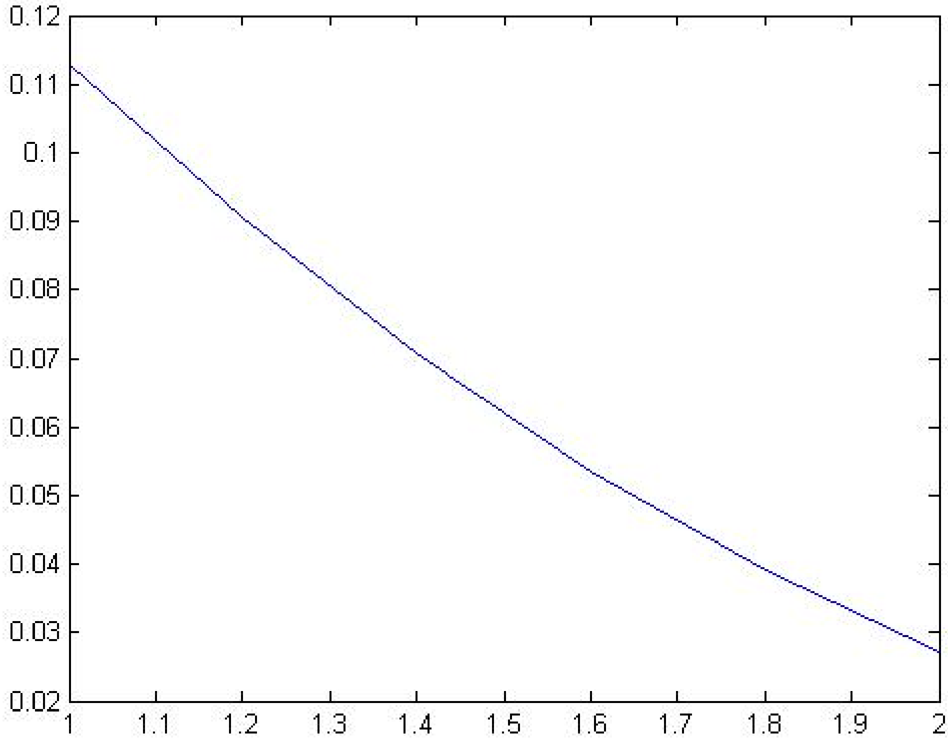


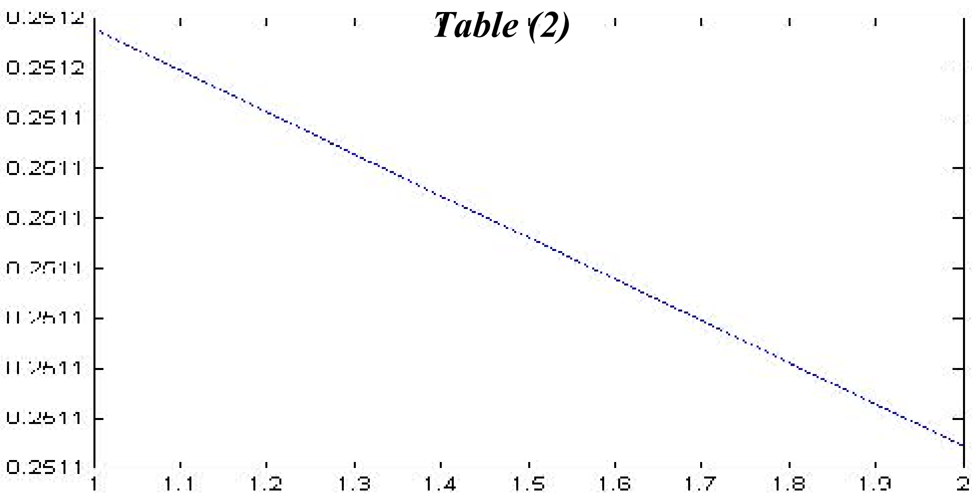
Figure (2)

Approximate solution

<u>4-2. Case II: -</u>

For computational work, we have taken (***α = 0.001*** and ***β = 0.1***) , we can find approximate solution, and make comparison between the exact solution and approximate solution.

Table (2)



x	t	Exact solution	Approx. VIM	Error VIM
0.1	0.2 0.0001	0.389955 0.499990	0.390269 0.499987	3.091890E-04 2.493756E-06
0.5	0.5 0.0005	0.468790 0.499950	0.467386 0.499937	1.404588E-03 1.246878E-05
0.9	0.1 0.0001	0.389955 0.499890	0.390269 0.499887	3.091890E-04 2.493757E-06

Figure (3)

Exact solutions

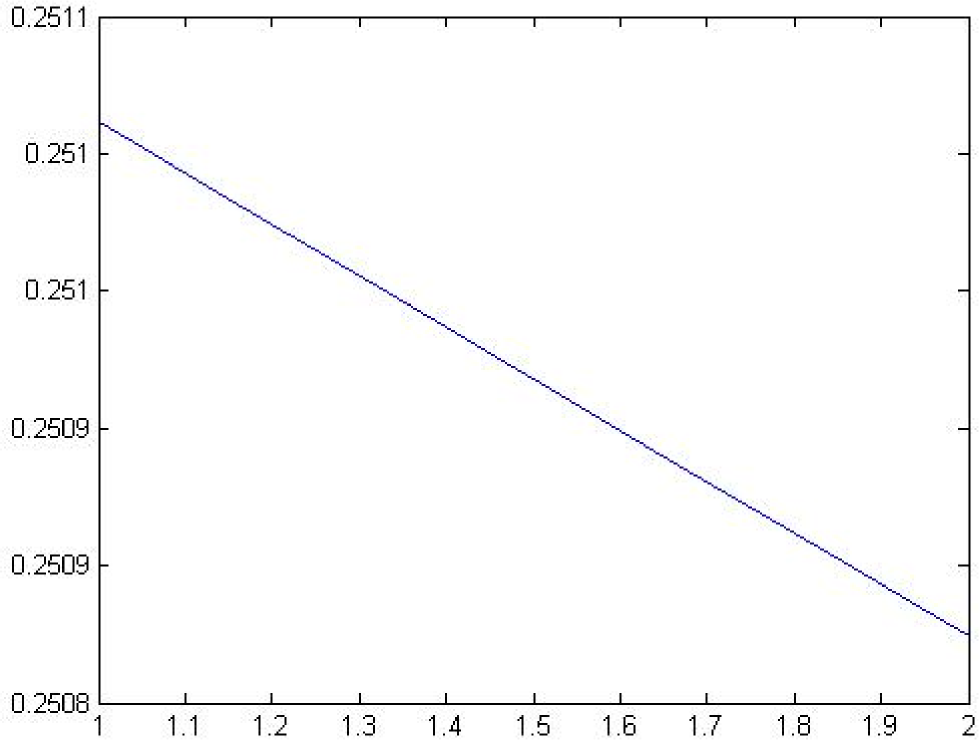


Figure (4)

Approximate solutions

Reference

[1] "Variational principles for some nonlinear partial differential equations with variable coefficients", J.H.He ,chaos,solitons and fractals,Vol.19, 847-851, (2004).

[2] "Variational iteration method for autonomous ordinary differential system" ,J.H.He , Appl.Math.Comput.,Vol. 114, 115-123, (2000)

[3] "Variational iteration method for solving integro differential equations",S.Q.wang and J.H.He, physics Letters A,(2007).

[4] "Variational iteration method for solving Burgers and coupled Burgers equation",M.A.Abdou and A.A.Soliman, J.Comput.Appl.Math., Vol. 81, 45-251, (2005)

[5] "New application of variational iteration method",M.A.Abdou and A.A.Soliman, physics D,2111-8 ,(2005)

[6] "Application of He's variational iteration method to Helmholtz equation",S.Momani and S.Abuasad, Chaocs,Solitons and Fractals,Vol. 27,1119-1123 ,(2006)

[7] "The variational iteration method:A reliable analytic tool for solving linear and non linear wave equations", A.M.Wazwaz, Compute and Mathematics with Applications,(2007)

[8] "The variational iteration method for rational solutions for KdV,A.M.Wazwaz, k(2,2),Burgers,and cubic Boussinesq equations,J.Compute.Appl. Math.,Vol. 207(1),(18-23,),(2007)

[9] "Adomain decomposition method for Burgers-Huxely and Burgers-Fisher equations" ,H.N.A.,K.R and A.A. , Department of mathematical and physical science,Benha Higher Institute of Technology, Benha 135/2,Egypt.

[10] " Solving the Fisher's Equation by Means of Variational Iteration Method, M. Matinfar and M. Ghanbari Int. J. , Contemp. Math. Sciences, Vol. 4, 2009, no. 7, 343 - 348.

[11] "Approximate solution of nonlinear differential equations with convolution product nonlinearities" , J.H.He,. Compute Math App. Mech Eng, Vol.167:69-73, (1998)