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Pseudo Projective Tensor on Viasman Gray manifold

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Abstract.

In this work, we interested in studying the geometrical properties of Vaisman Gray manifold, we've identified these properties by using vanishing pseudo projective. Where we found the components of this tensor in Vaisman Gray. Moreover, we established a relationship between Vaisman Gray and NK-manifold and the necessary and sufficient condition in which M is an Einstein manifold are found, as well as more related results were given.

Keywords: Almost Hermitian manifold, Viasman Gray manifold, Einstein manifold, Pseudo projective tensor.

AMS Classification:53C55, 53B35.

1.Introduction

Our research dealt with one of the most important topics in "differential-geometry" which is almost Hermition manifold.Almost Hermition manifold is classified into 16 species and one of the most important varieties is Vaisman Gray manifold.Many researchers have studied Vaisman Gray manifold, in the 1979 Gray and Vanhecke [6] have studied this subject and they've got a lot of results. It was followed by several studies by Viasman [19], Kirichenko [9], Abood & Nawaf [1][2][11] and others. In our research, we studied the geometrical properties of this species through using a pseudo projective tensor, which was known in 2002 by researcher Prasad [13]. The researchers Bagewadi, Venkatesha, Prakasha and Basavara Jappa [18] [3] [4], have achieved the conditions, for "pseudo projective" flat on Kenmotsu and Lp- Sesakian manifolds. Negaraja and Somashekhara [12], proved that every “pseudo projectively” flat and pseudo projective semi symmetric Sesakian manifold is locally isomorphic to the unit sphere. Maralabhavi and Shivaprasanna [10], proved that $R(X,Y)\xi = 0$ if Lp- Sasakian manifold is “ ξpseudo projective” vanishes. Guler and Demirbag [7], studied the “pseudo projective” tensor on generalized “quasi Einstein manifold”. Eventually, the authors Shivamurthy, Prakasha and Chavan [16], studied some new outcomes on “pseudo projective” curvature tensor in a Kenmotsu manifold.

2. Preliminaries

In this section, we start with the construction of the class Vaisman Gray manifold in the adjoined G-structure space.

Definition 2.1 [5]

A Vaisman Gray structure is an AH-structure $(J, g =<.,.>)$ such that:

$$\nabla_X(F)(X,Y) = \frac{-1}{2(n-1)}\{<X,X> \delta F(Y) - <X,Y> \delta F(X) - <JX,Y> \delta F(JX)\},$$

where ∇ is the Riemannian connection of $g, F(X,Y) =<JX,Y>$ is the Kähler form, δ is a co-derivative and $X,Y \in X(M)$.

An AH-structure $(J, g =<.,.>)$ is called a structure of the class W_1 or nearly Kähler (NK-structure) if its Kähler form vanished, or equivalently,

$$\nabla_X(J) = 0;\ X \in X(M).$$

An AH-structure $(J, g =<.,.>)$ is called a structure of class W_4 or locally conformal Kähler structure (LCK-structure) if,

$$_X(F)(Y,Z) = \frac{-1}{2(n-1)}\{<X,Y> \delta F(Z) - <X,Z> \delta F(Y) - <X,JY> \delta F(JZ) + <X,JZ> F(JY)\};\ X,Y,Z \in X(M).$$

A manifold M with structure is called a Vaisman Gray manifold” (VG-manifold).

Definition 2.2 [5]

Let M be an AH-manifold, the form which is given by the following relation

$$\alpha = \frac{1}{n-1}\delta F \circ J,$$

is called a Lie form, where δ represents the coderivative.

If F is r-form, then its co-derivative is $(r-1)$-form, and its dual is a vector which is called a Lie vector.

Remark 2.1 [9]

The following table gives the classification of Gray-Hervella and the corresponding Banaru's classification for the classes that we studied:

Classification of Gray-Hervella	Classification of Banaru
Nearly Kähler manifold (W_1)	
$\nabla_X(J)(X,Y)=0$	$B^{abc}=-B^{bac}, B^{ab}{}_{c}=0$
Locally Conformal Kähler Manifold (W_4)	
$\nabla_X(F)(Y,Z)=\frac{-1}{2(n-1)}\{<X,Y> \delta F(Z) -<X,Z> \delta F(Y) -<X,JY> \delta F(JZ) +<X,JZ> F(JY)\}$	$B^{abc}=0, B^{ab}{}_{c}=\alpha^{[a}\delta_{c}^{b]}$
Vaisman-Gray manifold ($W_1 \oplus W_4$)	
$\nabla_X(F)(X,Y)=\frac{-1}{2(n-1)}\{<X,X> \delta F(Y) - <X,Y> \delta F(X) -<JX,Y> \delta F(JX)\}$	$B^{abc}=-B^{bac}, B^{ab}{}_{c}=\alpha^{[a}\delta_{c}^{b]}$

Table 2.1

The structure equations of VG-manifold provide by the following theorem.

Theorem 2.1 [8]

The collection of the structure equations of VG-manifold in the adjoined G -structure space has the following forms:

1) $d\omega^a = \omega_b^a \Lambda \omega^b + B^{ab}{}_{c}\omega^c \Lambda \omega_b + B^{abc}\omega_b \Lambda \omega_c$;

2) $d\omega_a = -\omega_a^b \Lambda \omega_b + B_{ab}{}^{c}\omega_c \Lambda \omega^b + B_{abc}\omega^b \Lambda \omega^c$;

3) $d\omega_b^a = \omega_c^a \Lambda \omega_b^c + \left(2B^{adh}B_{hbc} + A_{bc}^{ad}\right)\omega^c \Lambda \omega_d + \left(B^{ah}{}_{[c}B_{d]bh} + A_{bcd}^{a}\right)\omega^c \Lambda \omega^d + \left(B_{bh}{}^{[c}B^{d]ah} + A_{b}^{acd}\right)\omega_c \Lambda \omega_d$,

where $\{\ \ ^{i}\}$ are the components of mixture form, $\{\omega_j^i\}$ are the components of Riemannian connection of metric g, $\{A_{bcd}^{a}, A_{b}^{acd}\}$ are some functions on adjoined G-structure space and $\{A_{bc}^{ad}\}$ are system functions in the djoined G -strucure space which are symmetric by the lower and upper indices and are called components of holomorphic sectional curvature tensor.

The next theorem gives the synthesis of Riemannian curvature tensor of VG-manifold.

Theorem 2.2 [8]

In the adjoined G -structure space, the components of Riemann curvature tensor of VG -manifold are given by the following forms:

1) $R_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd})$;

2) $R_{\hat{a}bcd} = 2A_{bcd}^{a}$;

3) $R_{\hat{a}\hat{b}cd} = 2(-B^{abh}B_{hcd} + \alpha_{[c}^{[a}\delta_{d]}^{b]})$;

4) $R_{\hat{a}bc\hat{d}} = A_{bc}^{ad} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d}$,

where $\{\alpha^a{}_b, \alpha_a{}^b, \alpha_{ab}, \alpha^{ab}\}$ are some functions on adjoined G -structure space such that:

$$d\alpha_a + \alpha_b\omega_a^b = \alpha_a^b\omega_b + \alpha_{ab}\omega^b \text{ and}$$
$$d\alpha^a - \alpha^b\omega_b^a = \alpha_b^a\omega^b + \alpha^{ab}\omega_b.$$

The other components of Riemann curvature tensor R can be obtained by the property of symmetry for R .

Definition 2.3 [15]

A tensor of type$(2,0)$ which is defined as $r_{ij=}R^k_{ijk}=g^{kl}R_{kijl}$ is called a Richi tensor.

3. The Main Result

Definition 3.1 [13]

A pseudo projective tensor P_p in a Riemannian manifold is defined as follows:

$$P_p=\alpha R(X,Y,Z,W)+\beta[S(X,Z)g(Y,W)-r(Y,Z)g(X,W)]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[g(X,Z)g(Y,W)-g(Y,Z)g(X,W)],$$

where α; β are constants such that $\alpha,\beta\neq 0$, R is the Riemann curvature tensor, r is the Richi tensor and K is the scalar curvature tensor.

Remark 3.1 [14]

The pseudo projective tensor satisfies the following properties:

1) $P_p(X,Y,Z,W)=-P_p(Y,X,Z,W)$;

2) $P_p(X,Y,Z,W)=-P_p(X,Y,W,Z)$,

where $X,Y,Z,W\in T_p(M)$.

Now, we redefine the "pseudo projective" tensor on AH-manifold regarding the components as the following formula:

$$(P_p)_{ijkl}=\alpha R_{ijkl}+\beta\left[r_{ik}g_{jl}-r_{jk}g_{il}\right]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\left[g_{ik}g_{jl}-g_{jk}g_{il}\right]. \qquad (1)$$

The following theorem gives the components of "pseudo projective" tensor of VG-manifold in the adjoined G-structure space.

Remark 3.2: If $\alpha=1$ and $\beta=\frac{-1}{n-1}$, then the equation (1) becomes

$$(P_p)_{ijkl}=P_{ijkl}=R_{ijkl}-\frac{1}{n-1}\left[r_{ik}g_{jl}-r_{jk}g_{il}\right].$$

Which gives the classical projective tensor as a special case of the pseudo projective tensor.

Theorem 3.1

In the adjoined G-structure space, the components of "pseudo projective" tensor of VG-manifold are given by the following forms:

1. $(P_p)_{abcd}=2\alpha(B_{ab[cd]}+\alpha_{[a}B_{b]cd})$;
2. $(P_p)_{\hat{a}bcd}=2\alpha A^a_{bcd}-\beta r_{bc}\delta^a_d$;
3. $(P_p)_{\hat{a}\hat{b}cd}=2\alpha\left(-B^{abh}B_{hcd}+\alpha^{[a}_{[c}\delta^{b]}_{d]}\right)+\beta\left[r_{\hat{a}c}\delta^b_d-r_{\hat{b}c}\delta^a_d\right]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\left[\delta^a_c\delta^b_d-\delta^b_c\delta^a_d\right]$;
4. $(P_p)_{\hat{a}bc\hat{d}}=\alpha\left(A^{ad}_{bc}+B^{adh}B_{hbc}-B^{ah}{}_{c}B_{hb}{}^{d}\right)+\beta r_{\hat{a}c}\delta^d_b-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\delta^a_c\delta^d_b$,

and the others are conjugate to the above components.

Proof:

1) Put $i=a,\ j=b,\ k=c$ and $l=d$, then the equation (1) becomes

$$(P_p)_{abcd}=\alpha R_{abcd}+\beta[r_{ac}g_{bd}-r_{bc}g_{ad}]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[g_{ac}g_{bd}-g_{bc}g_{ad}]$$

By using Theorem 2.2, we obtain

$$(P_p)_{abcd}=2\alpha(B_{ab[cd]}+\alpha_{[a}B_{b]cd}).$$

2) Put $i=\hat{a},\ j=b,\ k=c$ and $l=d$, then the equation (1) becomes

$$(P_p)_{\hat{a}bcd}=\alpha R_{\hat{a}bcd}+\beta[r_{\hat{a}c}g_{bd}-r_{bc}g_{\hat{a}d}]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[g_{\hat{a}c}g_{bd}-g_{bc}g_{\hat{a}d}]$$

By using Theorem 2.2, we obtain

$$(P_p)_{\hat{a}bcd}=2\alpha A^{a}_{bcd}-\beta r_{bc}\delta^{a}_{d}.$$

3) Put $i=\hat{a},\ j=\hat{b},\ k=c$ and $l=d$, then the equation (1) becomes

$$(P_p)_{\hat{a}\hat{b}cd}=\alpha R_{\hat{a}\hat{b}cd}+\beta[r_{\hat{a}c}g_{\hat{b}d}-r_{\hat{b}c}g_{\hat{a}d}]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[g_{\hat{a}c}g_{\hat{b}d}-g_{\hat{b}c}g_{\hat{a}d}]$$

By using Theorem 2.2, we get

$$(P_p)_{\hat{a}\hat{b}cd}=2\alpha\left(-B^{abh}B_{hcd}+\alpha^{[a}_{[c}\delta^{b]}_{d]}\right)+\beta[r_{\hat{a}c}\delta^{b}_{d}-r_{\hat{b}c}\delta^{a}_{d}]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[\delta^{a}_{c}\delta^{b}_{d}-\delta^{b}_{c}\delta^{a}_{d}].$$

4) Put $i=\hat{a},\ j=b,\ k=c$ and $l=\hat{d}$, then the equation (1) becomes

$$(P_p)_{\hat{a}bc\hat{d}}=\alpha R_{\hat{a}bc\hat{d}}+\beta[r_{\hat{a}c}g_{b\hat{d}}-r_{bc}g_{\hat{a}\hat{d}}]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[g_{\hat{a}c}g_{b\hat{d}}-g_{bc}g_{\hat{a}\hat{d}}]$$

By using Theorem 2.2, we have

$$(P_p)_{\hat{a}bc\hat{d}}=\alpha\left(A^{ad}_{bc}+B^{adh}B_{hbc}-B^{ah}{}_{c}B_{hb}{}^{d}\right)+\beta r_{\hat{a}c}\delta^{d}_{b}-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\delta^{a}_{c}\delta^{d}_{b}.\ \square$$

Definition 3.2

An AH -manifold is called a flat (vanishing) pseudo projective tensor if the pseudo projective tensor is equal to zero.

Theorem 3.2

If M is VG -manifold of flat pseudo projective tensor with flat Richi tensor, then M is NK -manifold if and only if, $\beta=\frac{-\alpha}{n-1}$

Proof:

Suppose M that is VG-manifold of flat pseudo projective tensor, then

$$(P_p)_{ijkl}=0$$

By using Theorem 3.1, we have

$$\alpha\left(A^{ad}_{bc}+B^{adh}B_{hbc}-B^{ah}{}_{c}B_{hb}{}^{d}\right)+\beta r_{\hat{a}c}\delta^{d}_{b}-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\delta^{a}_{c}\delta^{d}_{b}=0$$

Since M is of flat Ricci tensor, it's follows that

$$\alpha\left(A^{ad}_{bc}+B^{adh}B_{hbc}-B^{ah}{}_{c}B_{hb}{}^{d}\right)-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\delta^{a}_{c}\delta^{d}_{b}=0$$

Symmetrizing and anti-symmetrizing by the indices (b,c), we get

$$\alpha\left(-B^{ah}{}_{c}B_{hb}{}^{d}\right)-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\delta^{a}_{c}\delta^{d}_{b}=0 \qquad (2)$$

Let M be NK-manifold, then

$$\beta=\frac{-\alpha}{n-1}.$$

Conversely, by using the equation (2), we have

$$\alpha\left(-B^{ah}{}_{c}B_{hb}{}^{d}\right)-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)\delta_c^a\delta_b^d=0$$

Suppose that $\beta=\frac{-\alpha}{n-1}$, then we obtain

$$B^{ah}_{c}B_{hb}^{d}=0$$

Contracting by the indices (a,b) and (d,c), we have

$$B^{ah}{}_{d}B_{ha}{}^{d}=0\Leftrightarrow B^{ah}{}_{d}\underline{B}^{ah}{}_{d}=0\Leftrightarrow$$

$$\Sigma_{a,d,h}\quad\left|B^{ah}_{d}\right|^2=0\Leftrightarrow B^{ah}_{d}=0.$$

Therefore, by Table 2.1, M is NK-manifold . □

Lemma 3.1 [17]

In the adjoined G -structure space, an AH -manifold is a manifold of class:

1) R_1 if and only if,

$$R_{abcd}=R_{\hat{a}bcd}=R_{\hat{a}\hat{b}cd}=0;$$

2) R_2 if and only if, $R_{abcd}=R_{\hat{a}bcd}=0$;

3) R_3 (RK-manifold) if and only if,

$$R_{\hat{a}bcd}=0.$$

It easy to see that $R_1\subset R_2\subset R_3$.

Definition 3.3

In the adjoined G -structure space, an almost Hermitian manifold is a manifold of class:

P_pR_1 if and only if,

$(P_p)_{abcd}=(P_p)_{\hat{a}bcd}=(P_p)_{\hat{a}\hat{b}cd}=0$;

P_pR_2 if and only if,

$(P_p)_{abcd}=(P_p)_{\hat{a}bcd}=0$;

P_pR_3 (P_pRK-manifold) if and only if, $(P_p)_{\hat{a}bcd}=0$.

Definition 3.4[17]

An AH-manifold has J-invariant Richi tensor if $J\circ r=r\circ J$.

Lemma 3.2 [17]

An AH-manifold has J-invariant Richi tensor if and only if, in the adjoined G -structure space the following equation satisfies:

$$r_b^{\hat{a}}=r_{ab}=0.$$

Theorem 3.3

Let M be a VG-manifold of class P_pR_1 with J-invariant Richi tensor, then M is a manifold of class R_1 if M is a manifold of flat Richi tensor and $\beta=\frac{-\alpha}{n-1}$.

Proof:

To prove M is a manifold of class R_1, we must prove that

$$R_{abcd}=R_{\hat{a}bcd}=R_{\hat{a}\hat{b}cd}=0.$$

Let M be a manifold of class P_pR_1, according to Definition3.3, we have

$$(P_p)_{abcd}=(P_p)_{\hat{a}bcd}=(P_p)_{\hat{a}\hat{b}cd}=0\qquad(3)$$

$$(P_p)_{abcd}=0$$

$$2\alpha\left(B_{ab[cd]}+\alpha_{[a}B_{b]cd}\right)=0$$

$$R_{abcd}=0\qquad(4)$$

By using equation (3), we get

$$(P_p)_{\hat{a}bcd}=0$$

According to Theorem 3.1, we deduce

$$\alpha R_{\hat{a}bcd}+\beta[r_{\hat{a}c}g_{bd}-r_{bc}g_{\hat{a}d}]-\frac{K}{n}\left(\frac{\alpha}{n-1}+\beta\right)[g_{\hat{a}c}g_{bd}-g_{bc}g_{\hat{a}d}]=0$$

Since M has J-invariant Richi tensor, then

$$R_{\hat{a}bcd}=0\qquad(5)$$

In addition, by the equation (3), we deduce

$$(P_p)_{\hat{a}\hat{b}cd} = 0$$

By using Theorem 3.1, we have

$$\alpha\left(A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d}\right) + \beta r_{\hat{a}c}\delta^{d}_{b} - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\delta^{a}_{c}\delta^{d}_{b} = 0$$

$$\alpha R_{\hat{a}\hat{b}cd} + \beta r_{\hat{a}c}\delta^{d}_{b} - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\delta^{a}_{c}\delta^{d}_{b} = 0$$

Suppose that M is a manifold of flat Richi tensor and $\beta = \frac{-\alpha}{n-1}$ then

$$R_{\hat{a}\hat{b}cd} = 0 \quad (6)$$

Hence, according to the equations, (4), (5) and (6), we get

$$R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0. \quad \square$$

Definition 3.5 [14]

A Riemannian manifold is called an Einstein manifold if the Richi tensor satisfies the equation $r_{ij} = eg_{ij}$, where, e is an Einstein constant.

The following theorem gives the necessary and sufficient condition in which an VG-manifold is an Einstein manifold.

Theorem 3.4

Let M be a VG-manifold of class P_pRK-manifold and J-invariant Richi tenor. Then the necessary and sufficient condition in which M is an Einstein manifold is $A^{a}_{acd} = \frac{-\beta e}{2\alpha} g_{dc}$, where e is an Einstein constant.

Proof:

Let M be a VG-manifold of class P_pRK-manifold.

According to Definition 3.1, and Theorem 3.1, we

$$2\alpha A^{a}_{bcd} - \beta r_{bc}\delta^{a}_{d} = 0$$

Contracting by the indices (a, b), we get

$$2\alpha A^{a}_{acd} - \beta r_{dc} = 0 \quad (7)$$

Let M be an Einstein manifold, then

$A^{a}_{acd} = \frac{-\beta e}{2\alpha} g_{dc}$.

Conversely, by using the equation (7), we have

$2\alpha A^{a}_{acd} - \beta r_{dc} = 0$

Since, $A^{a}_{acd} = \frac{-\beta e}{2\alpha} g_{dc}$, we deduce

$$2\alpha \frac{-\beta e}{2\alpha} g_{dc} - \beta r_{dc} = 0$$

$$r_{dc} = eg_{dc}$$

Since M has J -invariant Richi tensor, then, M is an Einstein manifold $\square$

Theorem 3. 5

Suppose that M is a VG- manifold of class P_pR_1 with J-invariant Richi tensor, then M is a an Einstein manifold if, and only if M is a manifold of class R_1.

Proof: Suppose that M is a VG-manifold of class P_pR_1. According to Definition3.3, and Theorem3.1, we have

$$\alpha R_{abcd} = \alpha R_{\hat{a}bcd} - \beta r_{bc}\delta^{a}_{d} = \alpha R_{\hat{a}\hat{b}cd} + \beta\left[r_{\hat{a}c}\delta^{b}_{d} - r_{\hat{b}c}\delta^{a}_{d}\right] - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\left[\delta^{a}_{c}\delta^{b}_{d} - \delta^{b}_{c}\delta^{a}_{d}\right] = 0$$

Symmetrizing and anti-symmetrizing by the indices (c, d), we get

$$\alpha R_{abcd} = \alpha R_{\hat{a}bcd} - \beta r_{bc}\delta_d^a =$$
$$\alpha R_{\hat{a}\hat{b}cd} - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\left[\delta_c^a\delta_d^b - \delta_c^b\delta_d^a\right] = 0 \qquad (8)$$

Let M be a VG-manifold of class R_1. According to Lemma 3.1. Then the equation becomes

$$\beta r_{bc}\delta_d^a - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\left[\delta_c^a\delta_d^b - \delta_c^b\delta_d^a\right] = 0$$

Contracting the indices (a, c), we obtain

$$r_{bd} = e\delta_d^b,$$

where $e = \frac{(n-1)K}{\beta n}\left(\frac{\alpha}{n-1} + \beta\right)$.

Since M has J -invariant Richi tensor, then, M is an Einstein manifold.

Conversely, let is an Einstein manifold. Then the equation (8) becomes

$$\alpha R_{abcd} = \alpha R_{\hat{a}bcd} - \beta e\delta_d^b\delta_d^a = \alpha R_{\hat{a}\hat{b}cd} - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\left[\delta_c^a\delta_d^b - \delta_c^b\delta_d^a\right] = 0$$

Contracting the indices (a, c), we have

$$\alpha R_{abcd} = \alpha R_{\hat{a}bcd} - \beta e g_{bd} = \alpha R_{\hat{a}\hat{b}cd} - \frac{K(n-1)}{n}\left(\frac{\alpha}{n-1} + \beta\right)\delta_d^b = 0 \qquad (9)$$

Since $e = \frac{(n-1)K}{\beta n}\left(\frac{\alpha}{n-1} + \beta\right)$, then the equation (9) becomes

$$R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0.$$

Then M is a VG-manifold of class P_pR_1.

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