

A Generalization of A Contra Pre Semi-Open Maps

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الملخص

عرف T. Noiri مفهوم مجموعة شبه مفتوحة من النمط θ في الفضاء التبولوجي في [9, 10] سنة 1984 و 1986 . في هذا البحث نعرف و ندرس تعميم للدوال شبه مفتوحة من النمط contra Pre المقدمة من قبل Caldas و Baker في [3] ، التي تسمى الدالة شبه المفتوحة من النمط θ contra Pre . وهي الدوال التي تكون صور المجموعات شبه المفتوحة من النمط θ ، شبه المغلقة من النمط θ ، كما نعرف و ندرس نمطاً جديداً من الدوال المغلقة تسمى الدوال شبه مغلقة من النمط θ contra Pre ، وهذه الدوال أقوى من الدوال شبه المغلقة من النمط contra المقدمة من قبل (caldas) في [2] ، وهي الدوال التي تكون صور المجموعات شبه المغلقة من النمط θ ، شبه مفتوحة من النمط θ .

ABSTRACT

The concept of θ -semi-open sets in topological spaces was introduced in 1984 and 1986 by T. Noiri [9, 10]. In this paper we introduce and study a generalization of a contra pre semi-open maps due to (Caldas and Baker) [3], it is called contra pre θ s-open maps, the maps whose images of a θ -semi-open sets is θ -semi-closed. Also, we introduce and study a new type of closed maps called contra pre θ s-closed maps, which is stronger than contra pre semi-closed due to Caldas [2], the maps whose image of a θ -semi-closed sets is θ -semi-open. 1991 Math. Subject Classification: 54 C10, 54 D 10.

Keywords and Phrases: θ -semi-open sets, Contra pre θ s-open and Contra pre θ s-closed maps.

1. Introduction

The concept of θ -semi-open set in topological spaces was introduced in 1984 and 1986 by T. Noiri [9, 10], which depends on semi-open sets due to N. Levine [8]. When semi-open sets are replaced by θ -semi-open sets, new results are obtained. M. Caldas and C. Baker defined and studied the concept of contra pre semi-open maps [3], where the maps whose images of semi-open sets are semi-closed.

In this direction we shall define the concept of Pre θ s-open maps. In this paper we introduce two new types of open and closed maps called contra pre θ s-open and contra pre θ s-closed maps via the concept of θ -semi-open sets and study some of their basic properties. We also establish relationships among these maps with other types of continuity, openness and closedness.

2. Preliminaries

Throughout the present paper, spaces always mean topological spaces on which no separation axiom is assumed unless explicitly stated. Let S be a subset of a space X . The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively. A subset S is said to be regular open (resp. semi-open [8]) if $S = Int(Cl(S))$ (resp. $S \subset Cl(Int(S))$). A subset S is said to be θ -semi-open [9] if for each $x \in S$, there exists a semi-open set U in X such that $x \in U \subset Cl(U) \subset S$. The complement of each regular open (resp. semi-open and θ -semi-open) set is called regular closed (resp. semi-closed and θ -semi-closed). The family of all semi-open (resp. semi-closed, θ -semi-open and θ -semi-closed) sets of X is denoted by $SO(X)$ (resp. $SC(X)$, $\theta SO(X)$ and $\theta SC(X)$). A point x is said to be in the θ -semi-closure [10] of S , denoted by $sCl_{\theta}(S)$, if $S \cap Cl(U) \neq \emptyset$ for each $U \in SO(X)$ containing x . If $S = sCl_{\theta}(S)$, then S is called θ -semi-closed. A point x is said to be in the θ -semi-interior [10] of S denoted by $sInt_{\theta}(S)$, if $Cl(U) \subset S$ for some $U \in SO(X)$ containing x . If $S = sInt_{\theta}(S)$, then S is called θ -semi-open. For each $U \in SO(X)$, $Cl(U)$ is θ -semi-open and hence every regular closed set is θ -semi-open. Therefore, $x \in sCl_{\theta}(S)$ if and only if $S \cap A \neq \emptyset$ for each θ -semi-open set A containing x . A function $f: X \rightarrow Y$ is said to be contra pre semi-open [3] (resp. contra pre semi-closed [2]) if for each semi-open (resp. semi-closed) set U of X , $f(U) \in SC(Y)$ (resp. $f(U) \in SO(Y)$).

3. Contra pre θ s-open and contra pre θ s-closed maps

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map from a topological space (X, τ) into a topological space (Y, σ) .

Definition 3.1: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be contra pre θ s-open (resp. contra pre θ s-closed) if $f(A)$ is θ -semi-closed (resp. θ -semi-open) in (Y, σ) , for each set $A \in \theta SO(X, \tau)$ (resp. $A \in \theta SC(X, \tau)$).

The proof of the following two lemmas follows directly from their definitions and, therefore, they are omitted.

Lemma 3.1: Every contra pre semi-open map is contra pre θ s-open.

Lemma 3.2: Every contra pre θ s-closed map is contra pre semi-closed.

The converse of the above lemmas is not true in general as it is shown by the following examples.

Example 3.1: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then the family of all semi-open subsets of X with respect to τ is:

$SO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and the family of all θ -semi-open subsets of X with respect to τ is $\theta SO(X) = \{\phi, X, \{b\}, \{a, c\}\}$. The identity map

$f : (X, \tau) \rightarrow (X, \tau)$ is contra pre θ s-open map, but it is not contra pre semi-open maps.

Example 3.2: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then, the family of all semi-open subsets of X with respect to τ is:

$SO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and the family of all θ -semi-open subsets of X with respect to τ is :

$\theta SO(X) = \{\phi, X, \{a, c\}, \{b, c\}\}$. Define a function

$f : (X, \tau) \rightarrow (X, \tau)$ as follows:

$f(a) = b, f(b) = f(c) = a$. Then f is contra pre semi-closed, but it is not contra pre θ s-closed.

Remark 3.1: Contra pre θ s-openness and contra pre θ s-closedness are equivalent if the map is bijective.

Theorem 3.1: For a map $f : X \rightarrow Y$ the following are equivalent:

- i) f is contra pre θ s-open;
- ii) for every subset D of Y and for every θ -semi-closed subset G of X with $f^{-1}(D) \subset G$, there exists a θ -semi-open subset B of Y with $D \subset B$ and $f^{-1}(B) \subset G$;
- iii) for every $y \in Y$ and for every θ -semi-closed subset G of X with

$f^{-1}(y) \subset G$, there exists a θ -semi-open subset B of Y with $y \in B$ and $f^{-1}(B) \subset G$.

Proof: (i) $\Rightarrow$ (ii). Let D be a subset of Y and let G be a θ -semi-closed subset of X with $f^{-1}(D) \subset G$. Set, $B = Y \setminus f(X \setminus G)$. Since f is contra pre θ -open, then B is a θ -semi-open set of Y and since $f^{-1}(D) \subset G$ we have $f(X \setminus G) \subset Y \setminus D$ and hence $D \subset B$. Also,
 $f^{-1}(B) = X \setminus [f^{-1}(f(X \setminus G))] \subset X \setminus (X \setminus G) = G$.

(ii) $\Rightarrow$ (iii). It is sufficient, set $D = \{y\}$, we get the result.

(iii) $\Rightarrow$ (i). Let A be a θ -semi-open subset of X with $y \in Y \setminus f(A)$ and let $G = X \setminus A$. By (iii), there exists a θ -semi-open subset B_y of Y with $y \in B_y$ and $f^{-1}(B_y) \subset G$. Then, $y \in B_y \subset Y \setminus f(A)$.

Hence $Y \setminus f(A) = \cup \{B_y : y \in Y \setminus f(A)\}$. Therefore, by [6, Lemma 2.2] that $Y \setminus f(A)$ is θ -semi-open. Thus, $f(A)$ is a θ -semi-closed subset in Y .

Theorem 3.2: For a map $f : X \rightarrow Y$ the following are equivalent:

- i) f is contra pre θ s-closed;
- ii) for every subset D of Y and for every θ -semi-open subset A of X with $f^{-1}(D) \subset A$, there exists a θ -semi-closed subset H of Y with $D \subset H$ and $f^{-1}(H) \subset A$.

Proof: (i) $\Rightarrow$ (ii). Let D be a subset of Y and let A be a θ -semi-open subset of X with $f^{-1}(D) \subset A$. Set, $H = Y \setminus f(X \setminus A)$. Since f is contra pre θ s-closed, therefore, H is a θ -semi-closed set of Y and since $f^{-1}(D) \subset A$, we have $f(X \setminus A) \subset Y \setminus D$ and hence $D \subset H$. Also, $f^{-1}(H) \subset A$.

(ii) $\Rightarrow$ (i). Let G be a θ -semi-closed subset of X . Set, $D = Y \setminus f(G)$ and let $A = X \setminus G$.

Hence $f^{-1}(D) = f^{-1}(Y \setminus f(G)) = X \setminus f^{-1}(f(G)) \subset X \setminus G = A$. By assumption, there exists a θ -semi-closed set $H \subset Y$ for which $D \subset H$ and $f^{-1}(H) \subset A$. It follows that $D = H$. If $y \in H$ and $y \notin D$, then $y \in f(G)$.

therefore, $y = f(x)$ for some $x \in G$ and we have $x \in f^{-1}(H) \subset A = X \setminus G$ which is a contradiction. Since $D = H$, that is, $Y \setminus f(G) = H$, which implies that $f(G)$ is θ -semi-open and hence f is contra pre θ s-closed.

Taking the set D in Theorem 3.2 to be $\{y\}$ for $y \in Y$ we obtain the following result.

Corollary 3.1: If $f : X \rightarrow Y$ is contra pre θ s-closed map, then for every $y \in Y$ and every θ -semi-open subset A of X with $f^{-1}(y) \subset A$, there exists a θ -semi-closed subset H of Y with $y \in H$ and $f^{-1}(H) \subset A$.

Theorem 3.3: A map $f : X \rightarrow Y$ is contra pre θ s-open if and only if for each $x \in X$ and each semi-open set S in X containing x , there exists a θ -semi-closed set H in Y containing $f(x)$ such that $H \subset f(\text{Cl}(S))$.

Corollary 3.2: A map $f : X \rightarrow Y$ is contra pre θ s-open if and only if for each $x \in X$ and each θ -semi-open subset A of X containing x , there exists a θ -semi-closed subset H of Y containing $f(x)$ such that $H \subset f(A)$.

Corollary 3.3: A map $f : X \rightarrow Y$ is contra pre θ s-open, then for each $x \in X$ and each regular closed subset R of X containing x , there exists a θ -semi-closed subset H of Y containing $f(x)$ such that $H \subset f(R)$.

Theorem 3.4: A map $f : X \rightarrow Y$ is contra pre θ s-closed if and only if for each $x \in X$ and each θ -semi-closed subset G of X containing x , there exists a semi-open subset W of Y containing $f(x)$ such that $\text{Cl}(W) \subset f(G)$.

Corollary 3.4: A map $f : X \rightarrow Y$ is contra pre θ s-closed if and only if for each $x \in X$ and each θ -semi-closed subset G of X containing x , there exists a θ -semi-open subset B of Y containing $f(x)$ such that $B \subset f(G)$.

Theorem 3.5: For a map $f : X \rightarrow Y$, the following are equivalent:

- a) f is contra pre θ s-open;
- b) $f(\text{sInt}_\theta(A)) \subset \text{sCl}_\theta(f(A))$ for each subset A of X ;

- c) $sInt_{\theta} (f^{-1}(B)) \subset f^{-1}(sCl_{\theta}(B))$ for each subset B of Y;
d) $f^{-1}(sInt_{\theta}(B)) \subset sCl_{\theta}(f^{-1}(B))$ for each subset B of Y.

Proof: (a) $\Rightarrow$ (b). Suppose f is contra pre θ s-open maps and $A \subset X$. Since $sInt_{\theta}(A) \subset A$, $f(sInt_{\theta}(A)) \subset f(A)$ and hence $f(sInt_{\theta}(A)) \subset sCl_{\theta}(f(A))$.

(b) $\Rightarrow$ (c). Let B be any subset of Y. Then $f^{-1}(B) \subset X$. Therefore, we apply (b), we obtain $f(sInt_{\theta}(f^{-1}(B))) \subset sCl_{\theta}(f(f^{-1}(B))) \subset sCl_{\theta}(B)$. Thus, $sInt_{\theta}(f^{-1}(B)) \subset f^{-1}(sCl_{\theta}(B))$.

(c) $\Rightarrow$ (d). In (c), we take $Y \setminus B$ instead of B, we get $sInt_{\theta}(f^{-1}(Y \setminus B)) \subset f^{-1}(sCl_{\theta}(Y \setminus B))$. Then, $sInt_{\theta}(X \setminus f^{-1}(B)) \subset f^{-1}(Y \setminus sCl_{\theta}(B))$, which implies that $X \setminus sCl_{\theta}(f^{-1}(B)) \subset X \setminus f^{-1}(sInt_{\theta}(B))$. Hence $f^{-1}(sInt_{\theta}(B)) \subset sCl_{\theta}(f^{-1}(B))$.

(d) $\Rightarrow$ (a). Let A be any θ -semi-open subset of X and set $B = Y \setminus f(A) = f(X \setminus A)$. By (d), $f^{-1}(sInt_{\theta}(f(X \setminus A))) \subset sCl_{\theta}(f^{-1}(f(X \setminus A))) = sCl_{\theta}(X \setminus A) = X \setminus A$. Therefore, $f(X \setminus A) = Y \setminus f(A)$ is θ -semi-open and hence $f(A)$ is θ -semi-closed subset of Y. Thus, f is contra pre θ s-open map.

The proof of the following theorem is similar to the above theorem for the contra pre θ s-closed maps.

Theorem 3.6: For a map $f : X \rightarrow Y$, the following are equivalent:

- a) f is contra pre θ s-closed;
b) $f(sCl_{\theta}(A)) \subset sInt_{\theta}(f(A))$ for each subset A of X;
c) $sCl_{\theta}(f^{-1}(B)) \subset f^{-1}(sInt_{\theta}(B))$ for each subset B of Y;
d) $f^{-1}(sCl_{\theta}(B)) \subset sInt_{\theta}(f^{-1}(B))$ for each subset B of Y.

Theorem 3.7: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then,

- i) If f is contra pre θ s-open, then $sCl_{\theta}(f(A)) \subset f(sCl_{\theta}(A))$ for every θ -semi-open subset A of X.
ii) If f is contra pre θ s-closed, then $f(A) \subset sInt_{\theta}(f(sCl_{\theta}(A)))$ for every subset A of X.

Proof: i) Since f is contra pre θ s-open, then $sCl_{\theta}(f(A)) = f(A) \subset f(sCl_{\theta}(A))$ for every $A \in \theta SO(X, \tau)$.

ii) Since f is contra pre θ s-closed and since $sCl_{\theta}(A)$ is θ -semi-closed, then $f(A) \subset f(sCl_{\theta}(A)) = sInt_{\theta}(f(sCl_{\theta}(A)))$ for every subset A of X.

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be pre θ s-open, if $f(A)$ is θ -semi-open in (Y, σ) , for every $A \in \theta SO(X, \tau)$.

Recall, that a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called S-closed [4] if $sCl_\theta (f(A)) \subset f(sCl_\theta (A))$ for every subset A of X.

Theorem 3.8: For a map $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties hold,
i) f is S-closed, whenever f is contra pre θ s-closed and $sCl_\theta (sInt_\theta (f(A))) \subset f(A)$ for every θ -semi-closed set A of X.
ii) f is pre θ s-open, whenever f is contra pre θ s-open and $f(A) \subset sInt_\theta (sCl_\theta (f(A)))$ for every θ -semi-open set A of X.

Proof: i) Let G be a θ -semi-closed subset of X. Since $sCl_\theta (sInt_\theta (f(G))) \subset f(G)$ and $f(G)$ is θ -semi-open, then $sCl_\theta (sInt_\theta (f(G))) = sCl_\theta (f(G)) \subset f(G)$. So, by [1, Remark 1.2.6], $f(G)$ is θ -semi-closed. Therefore, by [10, Theorem 3.1], f is S-closed map.

ii) Let A be a θ -semi-open subset of X. But $f(A) \subset sInt_\theta (sCl_\theta (f(A)))$ and $f(A)$ is θ -semi-closed, then $sInt_\theta (sCl_\theta (f(A))) = sInt_\theta (f(A))$ and hence $f(A) \subset sInt_\theta (f(A))$. Therefore, $f(A) = sInt_\theta (f(A))$. So, by [1, Proposition 1.2.2(4)], $f(A)$ is θ -semi-open.

Lemma 3.3[7]: If Y is a regular closed subset of a space X and $A \subset Y$, then A is θ -semi-open in X if and only if A is θ -semi-open in Y.

Regarding the restriction $f|_R$ of a map $f : (X, \tau) \rightarrow (Y, \sigma)$ to a subset R of X we have the following:

Theorem 3.9: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra pre θ s-open and R is a regular closed set of (X, τ) , then the map $f|_R : (R, \tau_R) \rightarrow (Y, \sigma)$ is also contra pre θ s-open.

Proof: Let A be a θ -semi-open set in the subspace R. Since R is regular closed in X, then by Lemma 3.3, A is θ -semi-open set in X. Since f is contra pre θ s-open. Therefore, $f(A)$ is θ -semi-closed in Y. Thus, $f|_R$ is contra pre θ s-open map.

The proof of the following result is not hard, therefore, it is omitted.

Theorem 3.10: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$. Then,

- a) $g \circ f$ is contra pre θ s-open, if f is pre θ s-open and g is contra pre θ s-open.
- b) $g \circ f$ is contra pre θ s-open, if f is contra pre θ s-open and g is S-closed.

- c) $g \circ f$ is contra pre θ s-closed, if f is S-closed and g is contra pre θ s-closed.
d) $g \circ f$ is contra pre θ s-closed, if f is contra pre θ s-closed and g is pre θ s-open.

Recall, that a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is S-continuous [10], if and only if for each θ -semi-open subset A of Y , $f^{-1}(A)$ is θ -semi-open in X .

Theorem 3.11: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \gamma)$ be two maps such that $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ is contra pre θ s-closed.

- a) If f is S-continuous surjection, then g is contra pre θ s-closed.
b) If g is S-continuous injection, then f is contra pre θ s-closed.

Proof: a) Suppose G is any arbitrary θ -semi-closed set in Y . Since f is S-continuous. Therefore, by [10, Theorem 1.1], $f^{-1}(G)$ is θ -semi-closed in X . Since $g \circ f$ is contra pre θ s-closed and f is surjective $(g \circ f)(f^{-1}(G)) = g(G)$ is θ -semi-open in Z . This implies that g is a contra pre θ s-closed map.
b) Suppose G is any arbitrary θ -semi-closed set in X . Since $g \circ f$ is contra pre θ s-closed, $(g \circ f)(G)$ is θ -semi-open in Z . Since g is S-continuous injection, $g^{-1}((g \circ f)(G)) = f(G)$ is θ -semi-open in Y . This implies that f is a contra pre θ s-closed map.

Arguing as in the proof of Theorem 3.11, we obtain the following result.

Theorem 3.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \gamma)$ be two maps such that $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ is contra pre θ s-open.

- a) If f is S-continuous surjection, then g is contra pre θ s-open.
b) If g is S-continuous injection, then f is contra pre θ s-open.

Lemma 3.4[10]: Let (X, τ) be a topological space and D be a subset of X . Then $x \in sCl_{\theta}(D)$ if and only if for every θ -semi-open A of x such that $A \cap D \neq \emptyset$.

Definition 3.2[5]: A subset D of a topological space (X, τ) is called θ -semi-dense if $sCl_{\theta}(D) = X$.

Theorem 3.13: For a map $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties hold:

- a) If f is contra pre θ s-open and $B \subset Y$ has the property that B is not contained in proper θ -semi-open sets, then $f^{-1}(B)$ is θ -semi-dense in X .
- b) If f is contra pre θ s-closed and A is θ -semi-dense subset of Y , then $f^{-1}(A)$ is not contained in a proper θ -semi-dense set.

Proof: a) Let $x \in X$ and let A be a θ -semi-open subset of X containing x . Then $f(A)$ is θ -semi-closed and $Y \setminus f(A)$ is a proper θ -semi-open subset of Y . Thus, $B \not\subset Y \setminus f(A)$ and hence there exists $y \in B$ such that $y \in f(A)$. Let $z \in A$ for which $y = f(z)$. Then $z \in A \cap f^{-1}(B)$. Hence $A \cap f^{-1}(B) \neq \emptyset$ and thus by Lemma 3.4, $x \in sCl_{\theta}(f^{-1}(B))$. Hence $f^{-1}(B)$ is θ -semi-dense in X .

b) Assume that $f^{-1}(A) \subset O$ where O is a proper θ -semi-open subset of X . Then, we have that $f(X \setminus O)$ is a non-empty θ -semi-open set such that $f(X \setminus O) \cap A = \emptyset$, which contradicts the fact that A is θ -semi-dense.

Lemma 3.5[6]: Let X_1 and X_2 be two topological spaces and $X = X_1 \times X_2$. Let $A_i \in \theta SO(X_i)$ for $i = 1, 2$, then $A_1 \times A_2 \in \theta SO(X_1 \times X_2)$.

Definition 3.3[7]: A space X is said to be strongly semi- T_2 if and only if for each two distinct points x and y in X , there exists two disjoint θ -semi-open sets A and B in X containing x and y , respectively.

Theorem 3.14: If X is a strongly semi- T_2 space and $f : X \rightarrow Y$ is contra pre θ s-open map, then the set $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$ is θ -semi-closed in the product space $X \times X$.

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