Forecasting Life-Expectancy in Iraq During the Period (2022-2035) Using Fuzzy Markov Chain

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Abstract

Life expectancy, estimate of the average number of additional years that a person of a given age can expect to live. The most common measure of life expectancy is life expectancy at birth. Life expectancy is a hypothetical measure. It assumes that the age-specific death rates for the year in question will apply throughout the lifetime of individuals born in that year. The estimate, in effect, projects the age-specific mortality (death) rates for a given period over the entire lifetime of the population born (or alive) during that time. The measure differs considerably by sex, age, race, and geographic location. Therefore, life expectancy is commonly given for specific categories, rather than for the population in general. The aim of this research is a comparison between the utilization of Markov chains with fuzzy and non-fuzzy data and studied the average life expectancy using the fuzzy Markov chain during the period (2022-2035). In this study, used a sample of average life expectancy in Iraq during 1950–2021. The results show that fuzzy Markov chains give us better results, and the average life expectancy in Iraq using this model in the years (2022-2035) will be between 65 and 66, this is due to the psychological pressures that led to sudden illnesses and deaths.

Keywords: Stochastic Process, Transition Probability Matrix, Markov Chain, Fuzzy Logic.

1 Introduction

Life expectancy is an important vital indicator. It is a statistical measure of the average number of years that the population of a given country is expected to live. Its rise or fall is linked to many factors, economic, social, environmental, health, cultural, political and legislative. Coupled with these influencing and decisive factors, the index has a significant impact, positively or negatively, on all aspects of social and cultural life in the country.

1-1 Literature Review

Fuzzy Markov Chains (FMCs) are an extension of traditional Markov Chains that allow for a more flexible modeling of uncertainty. FMCs have been used in various fields of study such as finance, control theory, decision-making, and image processing. In this literature review, we will discuss some of the significant studies conducted on FMCs, focusing on their applications, theoretical developments, and limitations. Fuzzy Markov Chains have been used in finance to model stock prices and predict market trends. For instance, in a study by Li et al. (2014), FMCs were used to model the behavior of stock prices in the Chinese stock market. They found that the model was able to capture the dynamics of the market accurately and provide reliable predictions of future prices. Similarly, in a study by Kao et al. (2019), FMCs were used to model the behavior of cryptocurrencies. They found that the model was able to capture the complex interactions between different cryptocurrencies and predict their future prices with high accuracy. In control theory, FMCs have been used to model uncertain systems and design robust control strategies. For instance, in a study by Wang (2016), FMCs were used to model a robotic arm's motion under uncertainty. They found that the model was able to capture the uncertainties in the system and design a robust controller that could adapt to changing conditions. Similarly, in a study by Wang et al. (2019), FMCs were used to model the behavior of a wind turbine under uncertainty. They found that the model was able to capture the uncertain factors affecting the system and design a robust controller that could minimize the turbine's energy loss. In decision-making, FMCs have been used to model and analyze uncertain situations. For instance, in a study by Wang et al. (2016), FMCs were used to model the decision-making process for emergency response in the event of a natural disaster. They found that the model was able to capture the uncertainties involved in the decision-making process and provide decision-makers with valuable information for making informed decisions.

1-2 Research Problem

Population studies are important for determining the dimensions of economic and social development and sustainable health, the human element is one of the basic inputs in planning either at the country level or at the level of administrative units such as governorates, districts, and sub-districts to indicate the economic, population concentration, and labor force, and the relationship of per capita gross national income as well as household consumption expenditures, as well as housing, education, the need for food, clean water, and justice Social.

1-3 Research Hypothesis

According to the research hypothesis, fuzzy Markov chains can be used to predict Iraq's average life expectancy for the foreseeable future.

1-4 Research aims

The aim of this research is:

- First, a comparison between the utilization of Markov chains with fuzzy and non-fuzzy data.
- Second: The average life expectancy for the time period was studied using the fuzzy Markov chain (2022-2035).

2 The concept of random process [6]

A random process is an indexed collection (or ensemble) of random variables, defined on a state space S i.e.,$\{Z(t); t \in T\}$, where T is some index set. We often interpret t as time

A random process is called a process if time is continuous T= $\{\ (0, \infty)$ or $(-\infty\ .\ \infty)\}$, while is called a chain if time is discrete $T = \{0,1, ..\}$

2-1 Law of Random process [8]

One of the main topics in random processes is prediction. For example, suppose that Z_n is the performance type of the company in the n year. In this case, the random process is of discrete time type. Now, the information related to the performance of the company until now is available and based on this information, we plan to estimate the performance of the company in the future period. This problem is a kind of prediction in random processes. Strictly speaking, the set of information up to time s is available as a set $\{Z(t): t \leq s\}$, want to estimate $Z(t)$ for $t \geq s$. For this purpose, let $\{Z_n: n = 1,2,\ldots\}$be a discrete random process and its state space be $S = \{a_0, a_1, \ldots\}$. Now, we intend to perform the act of prediction, which means that the process is, for example, observed up to m time, and with these observations, the value of the variable $(n \geq m)Z_n$is estimated. In other words, knowing the value of the variables $Z_0, Z_1, \ldots, Z_m$ we want to obtain the distribution of Z_n. That is, in principle, the purpose is to calculate the following probability value:

$$P(Z_n = y|Z_0 = z_0, Z_1 = z_1, \ldots, Z_m = z_m) \qquad (1)$$

The degree of our success in finding the above conditional distribution depends to a large extent on the type of relationship that exists between the variables in terms of probabilities.[5]

2-2 Markov Process

Suppose we observe the state of a process as follows: $\{ t_0 \to t_k \}$so that $t_0 \leq t_1 \leq \ldots \leq t_k \leq t_{k+1}$ .In addition, suppose that the value of z_k at time t_k “is the current state" of this process and $\{z_0, z_1, \ldots, z_{k-1}\}$ is the past history of the process." Then $\{z_{k+1}, z_{k+2}, \ldots\}$ as future are unknown. In independent processes, the future is depending only on the present state not on the past state. This property is called memoryless, because we don't need the past history to predict the future probability, but only the current state.[1],[2]

A random process is called a Markov-process if we have $t_0 \leq t_1 \leq \ldots \leq t_k \leq t_{k+1}$:

$$P_{k,k+1} = P(Z_{k+1} = z_{k+1}|Z_0 = z_0, Z_1 = z_1, \ldots, Z_k = z_k) = P(Z_{k+1} = z_{k+1}|Z_k = z_k) \quad (2)$$

This memoryless property is also called Markovian property.

2-3 Markov chain

A discrete-parameter Markov chain $\{Z(n), n \geq 0\}$with a discrete state space S = {0, 1, 2, . . .}, where this set may be finite or infinite.[11]

$$P_{zy} = P\{Z_{n+1} = y|Z_0 = z_0, \ldots, Z_1 = z_1, \ldots, Z_n = z\}$$
$$= P\{Z_{n+1} = y|Z_n = z\} \ , z \geq 0 \ , y \geq 0 \qquad (3)$$

P_{zy} is known as a 1-step transition. The process is represented to as a homogeneous Markov chain and is stated to have stationary transition probability if it is time independent. If it is not, the process is denoted as a non-homogeneous Markov chain.

2-4 Transition Probability Matrix (T.P.M.) [1],[2],[5]

Let $S = \{0, 1,2, . . .\}$be the discrete infinite state space for the homogeneous Markov chain $\{Z(n) , n \geq 0\}$. Then

$$P_{zy} = P(Z_{n+1} = y|Z_n = z) \qquad (4)$$

Regardless of the value of n. T.P.M. of $\{Z(n) , n \geq 0\}$ is defined by

$$P = [P_{ij}] = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0j} & \cdots \\ \vdots & \vdots & \cdots & \vdots & \cdots \\ p_{i0} & p_{i1} & \cdots & p_{ij} & \cdots \\ \vdots & \vdots & \cdots & \vdots & \cdots \end{bmatrix} \qquad (5)$$

Where the elements satisfy:

(a) $P = [P_{zy}]$ is square matrix

(b) For each $z. y \in S$ then $P_{zy} \geq 0$

(c) For each $z. y \in S$ then $P_{zy} \geq 0$

2-5 Multi-stage transition probability matrix

Assume that the discrete Markov chain $\{Z_n; n = 0,1,2, \dots\}$ has a state space and a transition probability matrix $P = [P_{zy}^n]$. The probabilities for n, m ≥ 1are:

$$P_{zy}^n = P(Z_{n+m} = y|Z_m = z) \qquad (6)$$

We call n-step transition probabilities and denote them by P_{zy}^n . The matrix of these probabilities is called n-stage transition probability matrix and is denoted by p_{zy}^n. Regarding p_{zy}^n, the following conditions always satisfied:[11],[12]

(a) For each $z. y \in S$ then $p_{zy}^n \geq 0$, (b)

For each $z \in S$ then $\sum_{y \in E} p_{zy}^n = 1$

2-6 Initial probability distribution

Suppose that the probability that a structure is in state A_i at a random moment is equal to 1. This possibility indicates as a probability vector $= (\pi_{1,} \pi_2, \dots , \pi_k)$, which it is the probability distribution at the moment. The probability distribution $\pi^0 = (\pi_1^0, \pi_2^0, \dots , \pi_k^0)$ is the initial probability distribution, and $\pi^n = (\pi_1^n, \pi_2^n, \dots , \pi_k^n)$ is the probability distribution after n steps.[1],[2]

3 Introduction to Fuzzy Logic

3-1 Terms of fuzzy logic:

In fuzzy logic, a procedure of many-valued logic, variables' reality values can be any real number between [0,1]. Nevertheless, variables in Boolean logic can only have truth values of 0 or 1. Fuzzy logic has been broadened to account for the idea of imperfect reality, where the reality value could range from totally true to completely untrue.[6]

In (1965), Lotfi A. Zadeh proposed fuzzy set theory, the term fuzzy logic was first used. Many domains, including control theory and artificial intelligence, have used fuzzy logic.

3-2 Fuzzy Logic's basic design:

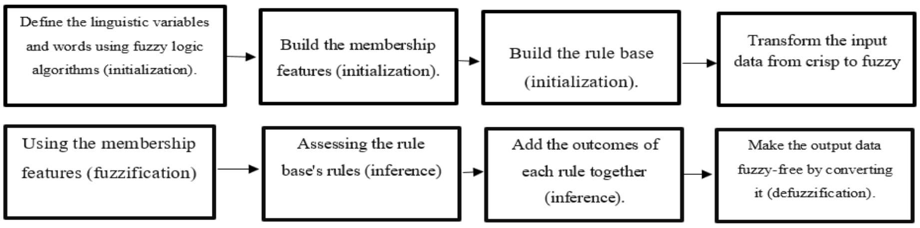


Figure (1): Fuzzy Logic's basic design [3]

3-3 Fuzzy logic's characteristics:[6],[7]

- Understanding: Fuzzy logic relies on straightforward mathematical ideas.
- Flexibility: Any fuzzy system can have a set of extra functions added to it without having to completely rebuild it.
- Efficiency in handling imprecise data: In the actual world, everything seems imperfect, even when tested properly. Fuzzy logic is known for its prowess in handling imprecision and producing excellent results even with it.
- Fuzzy logic depends on natural language: Fuzzy logic is simple to use since it relies on the everyday language that people use to communicate.
- With the advent of powerful new technologies like (ANFIS) and systems that combine fuzzy logic with artificial neural networks, fuzzy logic is now capable of simulating nonlinear functions with high complexity.
- Fuzzy logic can imitate non-linear functions with difficulty; it can create fuzzy systems that are similar to any set of input and output, especially with the advent of potent new technologies like artificial neural network systems and (ANFIS).

3-4 Classical sets and fuzzy sets:

The Greek philosopher Aristotle's notion of excluded middle, which asserts that an element z either belongs to a group A or it does not, is the cornerstone of conventional set theory. As a result, one of the following will probably be used to convey membership in an item and group membership: This is transformed into 0 and 1 that is two-valued and does not allow: (yes, no), (true, false),[1, 0].Things are frequently expressed differently in the real world. For instance, a person might use phrases like "long, short, too long, too short, and middle height" to describe their height, but a binary logic value couldn't possibly encompass all of these spectra. Due to the binary logic value's constrained possibilities, which limit its practical utility to only the most basic applications, fuzzy set theory was needed as a successor.

The traditional set theory's tight limitations were made less severe by the fuzzy set theory when an item's membership can be expressed in a variety of ways, including:[13]

- The set's constituent parts were certain.
- Element is not at all a part of the group.
- To a certain extent, or in part, an element is a part of the group.

3-5 Linguistic Variables:

The pursuit of human imitation in behavior and activity led to fuzzy logic being a technique to account using words rather than numbers. The brain of a human can explain imprecise and incomplete information offered by the senses held. As Professor Lotfi Zadeh proposed the idea of linguistic variables in 1973, you might use phrases like "extremely cold," "cold," "very hot," and "warm" to describe the heat. There are numerous reasons to use words instead of numbers, despite the fact that they are less precise.[6]

- The closest thing to human perception is useful.
- Using ward in computation rather than numbers allows for effective handling of levels of imprecision, which lowers the complexity of the solution. Instead of having numerical values, linguistic variables are input or output variables of the system whose values are words or phrases from a natural language. Typically, a linguistic variable is broken down into a group of linguistic terms. The following characteristics apply to all linguistic variables:

3-6 Universe of discourse:

The space of discourse in which fuzzy sets are defined is the domain of the system's input and output variables. The real inputs to the fuzzy system are referred to as the "universe of discourse," or UOD.[14]

3-7 Fuzzy Value:

Each of the parts and sub-ranges that make up the discourse universe has a name that conveys the characteristics of language use, such as large, medium, and negative. These names are sometimes referred to as linguistic values, labels, or sets.[13]

3-8 Membership Functions:

The mapping of each point in the input space to a membership value (or degree of membership) between [0,1] is defined by a curve known as a membership function (MF). The universe of discourse is another term for the input space, which is just another fancy way of saying the basic idea. In the fuzzification and defuzzification stages of a FLS, membership functions are employed to translate the non-fuzzy input values to fuzzy linguistic terms and vice versa. A linguistic term is quantified by a membership function. [3],[13],[14]

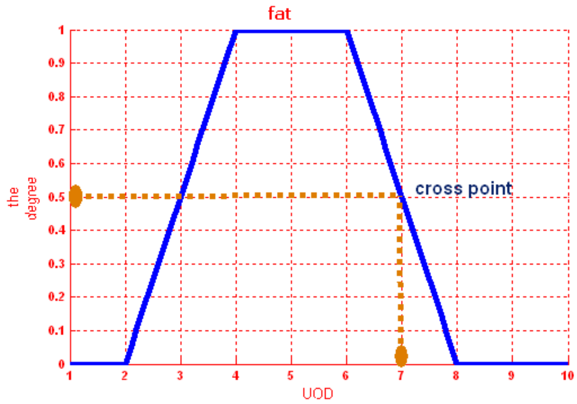


Figure (2): Computation of Membership Degree [4]

The following characteristics of the fuzzy variable are illustrated in this figure:

- Range of conversation from 1 to 10.
- One fuzzy value known as fat.

The value 7 crosses the membership function at the y-coordinate point at 1/2, indicating that the membership degree of 7 is 1/2.

3-9 Approximation method and Construction of Fuzzy Membership Functions

The indicator function of traditional sets is an extension of the membership function of a fuzzy set. The degree of truth is represented in fuzzy logic as an extension of valuation. Although conceptually separate, degrees of truth and probabilities are sometimes conflated because fuzzy truth refers to membership in ill-defined sets rather than the possibility of an event or condition. In the first publication on fuzzy sets, Zadeh proposed membership functions (1965).[4],[13]

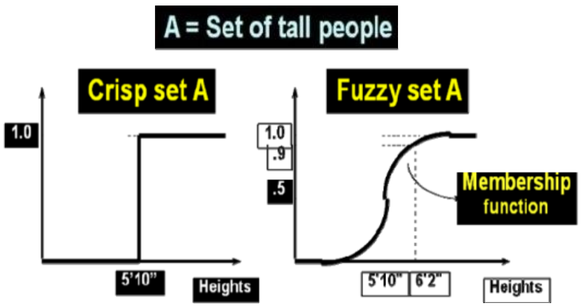


Figure (3): Membership criteria for fuzzy versus crisp sets [4]

3-10 Membership function Definition [7],[14]

Consider the fuzzy set $A, A = \{(x, \gamma A(x)) | \; x \in X\}$ where $\gamma A(x)$ the membership function for the fuzzy set is $A. X$ denoted to as the universe of discourse. The membership function associates each element $x \in X$ with a value in the interval$[0, 1]$.

In fuzzy sets, each element is mapped to $[0,1]$ by its membership function. That is $\gamma A : X \in [0, 1]$.Consequently, a fuzzy set is a ‘vague boundary set’ compared with a crisp set.

The fuzzy set A can be alternatively denoted as follows:

$$X = \left\{ \begin{array}{lll} \text{Discrete} & \text{then} & \sum \gamma_A(x_i)/x_i \\ \text{Contiuous} & \text{then} & \int \gamma_A(x_i)/\, x_i \end{array} \right\} \qquad (7)$$

The term "membership degree of membership" refers to the degree to which members of a fuzzy set are members. The grade for being a member of the fuzzy set is the membership score. The degree of membership increases as the number in [0, 1] increases. Fuzzification is the translation from x to $\gamma_A(x_i)$ where $X \rightarrow [0, 1]$

$\gamma_A(x_i) = \text{One}$ if x is totally in A;
$\gamma_A(x_i) = \text{Two}$ if x is not in A; (8)
$0 < \gamma_A(x_i) < 1$ if x is partly in A.

This set permits a range of possible choices. For any element x of universe X, membership function $\gamma_A(x)$ equals the degree to which x is an element of set A. This degree, a value between [0,1], denotes the degree of membership, also called membership value, of element x in setA.

3-11 The Basic Fuzzy Membership Functions (FMF):[4],[13]

The Support of fuzzy set $(\mathbf{A}) = \{(x, \gamma_A(x_i)) | \gamma_A(x_i) > 0)\}$

The core of fuzzy set$(\mathbf{A}) = \{x \in X | \gamma_A(x_i) = 1\}$

Crossover of fuzzy set$(\mathbf{A}) = \{x \in X | \gamma_A(x_i) = 0.5\}$

Normality: If, $\mathbf{core(A) is\ not\ equal\ to}\ \emptyset \rightarrow$ A is a normal fuzzy set.

$\mathbf{Normality\ of\ fuzzy\ set(A) = 1}$ if $\gamma_A(x_i) = 1$, for all $x \in X$ and $(x, \gamma_A(x_i)) \in A$

Fuzzy singleton: If support is single point in x with $\gamma_A(x_i) = 1$ a fuzzy set A is called fuzzy singleton.

$|A| = | \{(x, \gamma_A(x_i) = 1) | \gamma_A(x_i) = 1 \}|$ (9)

$\alpha - \mathbf{cut}$: $A\alpha = \{x \in X | \gamma_A(x_i) > \alpha\}$

Strong $\alpha - \mathbf{cut}$: $A\alpha = \{x \in X | \gamma_A(x_i) > \alpha\}$. A: is defined as crisp set for this case '

Convex shape of Fuzzy Sets: A fuzzy set A is convex if and only if for any $x_1, x_2 \in X$ and there exists $x = [0,1]$ such that

$$\mu_A\,(l\lambda x_1 + (1 - \lambda\, l)x_2)^3 \min(\mu_A(x_1), \gamma_A(x_2)) \quad (10)$$

Otherwise, if all its a-cuts are convex the A called convex.

Bandwidths: Is the distance between two unique crossoverpoints

$\mathbf{Bandwidth}\ (\mathbf{A}) =$ Absoult vale$(x_2 - x_1)$ where $\gamma_A(x_1) = \gamma_A(x_2) = 1/2$

Symmetry: If MF around a certain point x = c, satisfies the following criteria i.e.,

$\gamma_A(x + c) = \gamma_A(c - x)$ $\forall x \in X$. A fuzzy set A is called symmetric

Open left, Open Right and Closed:

Fuzzy set of A is open

$$\begin{bmatrix} \text{Left} \quad A \leftrightarrow \lim_{x\to-\infty} \gamma_A(x) = 1 \text{ and } \lim_{x\to+\infty} \gamma_A(x) = 0 \\ \text{Right} \quad A \leftrightarrow \lim_{x\to-\infty} \gamma_A(x) = 0 \text{ and } \lim_{x\to+\infty} \gamma_A(x) = 1 \\ \text{closed} \quad A \leftrightarrow \lim_{x\to-\infty} \gamma_A(x) = \lim_{x\to+\infty} \gamma_A(x) = 0 \end{bmatrix} \qquad (11)$$

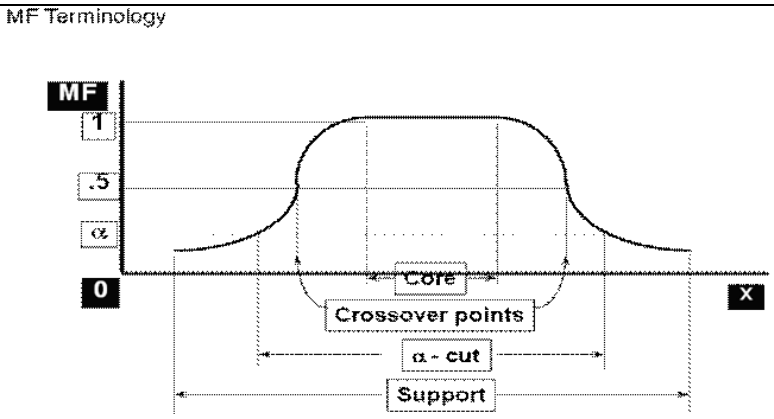


Figure (4): A terminological breakdown of the fuzzy membership function [4]

3-12 Functions of membership: parameterization and formulation:[4],[14]

1. Triangle-shaped membership criteria
2. MF trapezoidal
3. MF Gaussian
4. Broadened bell MF
5. Membership function for sigmoid

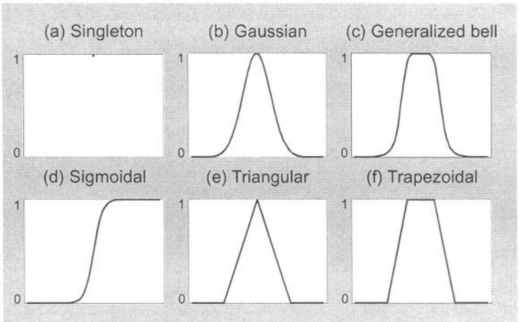


Figure (5): A variety of fuzzy membership functions [4]

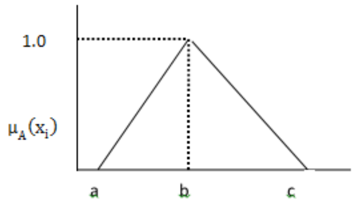
3-12-1 Triangular Membership function:[4],[13]

Let a, b and c show the x coordinates of the three vertices of $\gamma_A(x_i)$ in a fuzzy set A

a: Lower boundary

c: Upper boundary where membership degree is zero,
b: The center where membershipdegree is one

$$\mu_A(x_i) = \begin{bmatrix} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{bmatrix} \quad (12)$$



3-12-2 Trapezoidal membership function:[6],[14]

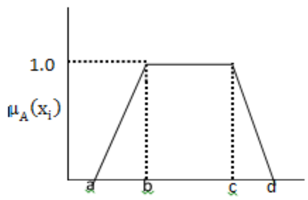
Let a, b, c and d represents the x coordinates of the membership function. then

$\text{Trapezoid}(x;\ a, b, c, d) =$

$0 \quad \text{if} \quad x \le a\,;$

$= (x-a)/\,(b-a) \quad \text{if} \quad a \le x \le b$

$= 1 \quad \text{if} \quad b \le x \le c;$



µ Trapezoidal=max(min((x-a)/(b-a),1,(d-x)/(d-c),0) (13)

3-12-3 Gaussian membership function:

The typical representation of the normal membership function is normal(x:c,s), where c, s stands for the $\bar{X}$ and S.[4],[8]

$$\gamma_A(x, c, s, m) = \exp\left[-1/2\ |(x-c)/S|^m\right] \qquad (14)$$

Here, c stands for the center, S for the breadth, and m for the fuzzification factor.

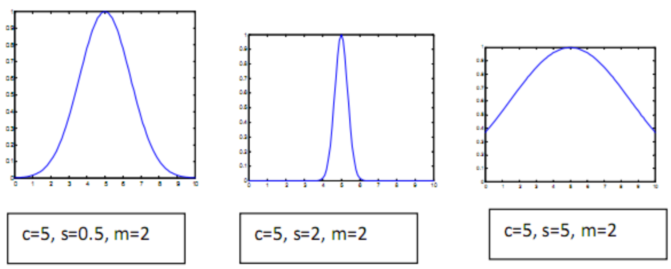


Figure (6): Multiple Normal MF forms with various S and m values Function of Sigmoid Membership [4]

Two variables compose a sigmoidal membership function: a, which determines the slope at the crossover point x=c. The sigmoid function's membership is denoted by the symbol Sigmf(x;a,b,c), and it is

$$\text{Sigmf}(x; a, b, c) = \frac{1}{1+e^{-a(x-c)}} \qquad (15)$$

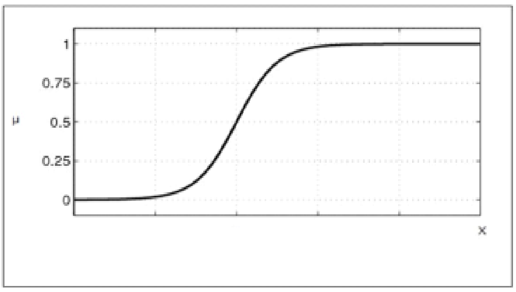


Figure (7): A general constructions of sigmoid MF[4]

Given that a sigmoid MF is naturally open to the right or left, it can be used to represent ideas like "extremely enormous" or "very negative," for example. The majority of artificial neural networks' activation functions use sigmoidal MF (NN). In order to imitate the behavior of a fuzzy inference system, a NN should create a close MF.[4]

Left-Right (LR) MF

$$\mathrm{Left} - \mathrm{Right}(x; c, \propto, \beta) = \begin{Bmatrix} F_{\mathrm{Left}}\left(\frac{c-x}{\alpha}\right), x < c \\ F_{\mathrm{Right}}\left(\frac{x-c}{\beta}\right), x \geq c \end{Bmatrix} \qquad (16)$$

3-13 Mean Square Error (MSE)

The mean square error, is the difference between the estimated values and the actual value, is measured by the mean squared error (MSE) of an estimator (of a process for estimating an unobserved variable). The MSE is a measure of an estimator's performance.[17]

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \widehat{Y}_i)^2 \qquad (17)$$

4 Results and Discussion

In this paper, used the time series data of the life expectancy in Iraq during the year (1950–2021). The data were obtained from the website (Life expectancy at birth-Iraq), https://data.worldbank.org/indicator/SP.DYN. LE00.IN?locations=IQ.Fuzzy Markov Chain will be relied upon to predict the probabilities and transitions of Life expectancy.

Table (1): Illustrates Iraq's life expectancy at birth from 1950 to 2021.

Years	Life expectancy	Years	Life expectancy	Years	Life expectancy	Years	Life expectancy
1950	39	1969	59.3	1988	57.5	2007	63.6
1951	40.6	1970	60.2	1989	58.2	2008	64.9
1952	42.2	1971	60.9	1990	58.4	2009	66.4
1953	43.7	1972	61.4	1991	62.5	2010	67.1
1954	45.2	1973	62	1992	66.7	2011	67.7
1955	46.6	1974	60.9	1993	66.9	2012	68
1956	47.9	1975	61.5	1994	66.7	2013	68.3
1957	49.2	1976	63.7	1995	66.5	2013	68.3
1958	50.4	1977	64.2	1996	66.4	2014	68.9
1959	51	1978	64.4	1997	65.8	2015	69.4
1960	52.6	1979	64.8	1998	65.5	2016	69
1961	53.4	1980	61.5	1999	66.2	2017	70.4
1962	54.3	1981	59.3	2000	66.8	2018	71.5
1963	54.9	1982	59.7	2001	67	2019	71.6
1964	55.6	1983	60.1	2002	67.1	2020	69.1
1965	56.6	1984	60.2	2003	65.6	2021	70.4
1966	57.3	1985	60.7	2004	65		
1967	58.4	1986	60.9	2005	64.8		
1968	58.8	1987	61.2	2006	63.6		

Source: The table was prepared by the researchers based on website (Life expectancy at birth - Iraq).

4-1 Analysis of Non-Fuzzy Markov Chain:

Step#1: The highest life expectancy was attained (71.60) and the lowest life expectancy was (39.00) in data collected for 72 years for the period (1950–2021); these two values were then subtracted to yield an output of (32.600), which was then divided by the number of states (7) Due to the approximately (4.6) number of possible states depicted, the classification of the states is as follows:

Table (2): Illustrate the classification of states

Class (From → To)	States
39-43.5	1
43.6-48.1	2
48.2-52.7	3
52.8-57.3	4
57.4-61.9	5
62-66.5	6
66.6-71.7	7

Source: The table was prepared by the researchers based on Excel.

Table (3): Distribution of annual data on life expectancy by state (1950-2021)

Years	Life expectancy	State	Years	Life expectancy	State	Years	Life expectancy	State	Years	Life expectancy	State
1950	39	1	1969	59.3	5	1988	57.5	5	2007	63.6	6
1951	40.6	1	1970	60.2	5	1989	58.2	5	2008	64.9	6
1952	42.2	2	1971	60.9	5	1990	58.4	6	2009	66.4	7
1953	43.7	2	1972	61.4	6	1991	62.5	7	2010	67.1	7
1954	45.2	2	1973	62	5	1992	66.7	7	2011	67.7	7
1955	46.6	2	1974	60.9	5	1993	66.9	7	2012	68	7
1956	47.9	3	1975	61.5	6	1994	66.7	6	2013	68.3	7
1957	49.2	3	1976	63.7	6	1995	66.5	6	2013	68.3	7
1958	50.4	3	1977	64.2	6	1996	66.4	6	2014	68.9	7
1959	51	3	1978	64.4	6	1997	65.8	6	2015	69.4	7
1960	52.6	4	1979	64.8	5	1998	65.5	6	2016	69	7
1961	53.4	4	1980	61.5	5	1999	66.2	7	2017	70.4	7
1962	54.3	4	1981	59.3	5	2000	66.8	7	2018	71.5	7
1963	54.9	4	1982	59.7	5	2001	67	7	2019	71.6	7
1964	55.6	4	1983	60.1	5	2002	67.1	6	2020	69.1	7
1965	56.6	4	1984	60.2	5	2003	65.6	6	2021	70.4	7
1966	57.3	5	1985	60.7	5	2004	65	6	Max	71.6	
1967	58.4	5	1986	60.9	5	2005	64.8	6	Min	39	
1968	58.8	5	1987	61.2	5	2006	63.6	6	Width	4.6	Range 32.6

Source: The table was prepared by the researchers based on Excel.

Step#2:

Calculating the state-by-state average productivity values. Each of the seven states had its values collected, and the total was divided by the total number of values in each state. The following were the state-specific average values:

Table (4): Distribution the average values for each state

States	1	2	3	4	5	6	7	Total
# of states	3	4	4	6	19	18	18	72
Average	40.6	45.85	50.8	55.35	59.95	64.77	68.48	

Source: The table was prepared by the researchers based on Excel.

Step#3:

The transition probability matrix's construction: It is clear from the earlier step that the total values (72) are the number of years, with the first state's value being 3 (40.6), the second state's value being 4 (45.85), the third state's value being 4 (50.8), the fourth state's value being 6 (55.35), the fifth state's value being 19 (59.95), the sixth state's value being 18 (64.77), and the seventh state's value being 18 (68.48).

Table (5): Frequencies State transition matrix

States	1	2	3	4	5	6	7
1	2	1	0	0	0	0	0
2	0	3	1	0	0	0	0
3	0	0	3	1	0	0	0
4	0	0	0	5	1	0	0
5	0	0	0	0	16	3	0
6	0	0	0	0	2	13	3
7	0	0	0	0	0	2	15

Source: The table was prepared by the researchers based on MATLAB.

Table (6): Transition Probability Matrix (T.P.M.)

State	1	2	3	4	5	6	7
1	0.667	0.333	0	0	0	0	0
2	0	0.750	0.250	0	0	0	0
3	0	0	0.750	0.250	0	0	0
4	0	0	0	0.833	0.167	0	0
5	0	0	0	0	0.8421	0.1579	0
6	0	0	0	0	0.111111	0.722222	0.166667
7	0	0	0	0	0	0.117647	0.882

Source: The table was prepared by the researchers based on Excel.

Step#4:

The vector is generated using the most recent value, which is the present value, where the life expectancy in (2021) attained a value of (70.4), falling in the state seven, so the vector will look like this:

Table (7): Vector is computed based on the most recent value

State	1	2	3	4	5	6	7
Vectors	0	0	0	0	0	0	1

Source: The table was prepared by the researchers based on Excel.

Step#5:

Multiply the vector by the transition probability matrix.

Table (8): Results of multiply the vector by the T.P.M

State	1	2	3	4	5	6	7
1	0.667	0.333	0	0	0	0	0
2	0	0.750	0.250	0	0	0	0
3	0	0	0.750	0.250	0	0	0
4	0	0	0	0.833	0.167	0	0
5	0	0	0	0	0.8421	0.1579	0
6	0	0	0	0	0.111111	0.722222	0.166667
7	0	0	0	0	0	0.117647	0.882

Source: The table was prepared by the researchers based on Excel.

We get:

1	2	3	4	5	6	7
0	0	0	0	0	0.117647059	0.882352941

Source: The table was prepared by the researchers based on Excel.

The result is then multiplied by the average of the seven states that were previously calculated, and they are as follows:

Table (9): Distribution of the state-specific average values

State	1	2	3	4	5	6	7	Result 2021
Average	40.6000	45.8500	50.8000	55.3500	59.9526	64.7722	68.4778	68.04183
Vector	0	0	0	0	0	0.117647059	0.882352941	
Results	0	0	0	0	0	7.620261438	60.42156863	

Source: The table was prepared by the researchers based on Excel.

In terms of life expectancy, we find that it reached a value of 70.4 in 2021, confirming the forecasting accuracy of the Markov chain. These meanings that the difference between the real and estimated life expectancy levels is 2.4% at most, as the estimated value of (68.04183) is 97.6% closer to the actual value of (70.4). The same method will be used to extract predictive values for succeeding years. For 2022, the prediction value was as follows:

Table (10): Illustration the productivity for year 2022

1	2	3	4	5	6	7
0	0	0	0	0.013071895	0.188773549	0.798154556

Source: The table was prepared by the researchers based on Excel.

After multiplying the final vector by the transition matrix, we obtain: After multiplying the final vector by the average values of the seven states, we obtain: (67.66683).

Table (11): Distribution of the state-specific average values

State	1	2	3	4	5	6	7	Result 2022
Average	40.6000	45.8500	50.8000	55.3500	59.9526	64.7722	68.4778	67.66683
Vector	0	0	0	0	0.013071895	0.188773549	0.798154556	
Result	0	0	0	0	0.783695	12.22728	54.65585	

Source: The table was prepared by the researchers based on Excel.

Step#6:

The predicted values were shown in table 10 by repeating the same processes for subsequent years:

Table (12): Forecasting values for the period (2022-2035)

State	1	2	3	4	5	6	7	Forecasting life expects
Average	40.6000	45.8500	50.8000	55.3500	59.9526	64.7722	68.4778	
Vector	0	0	0	0	0	0.117647	0.882353	68.04183
Result 2021	0	0	0	0	0	7.620261	60.42157	
Vector	0	0	0	0	0.013072	0.188774	0.798155	67.66683
Result 2022	0	0	0	0	0.783695	12.22728	54.65585	
Vector	0	0	0	0	0.031983	0.232301	0.735716	67.34432
Result 2023	0	0	0	0	1.91745	15.04665	50.38022	
Vector	0	0	0	0	0.052744	0.259378	0.687878	67.06699
Result 2024	0	0	0	0	3.162145	16.80047	47.10437	
Vector	0	0	0	0	0.073236	0.276583	0.650181	66.82854
Result 2025	0	0	0	0	4.390679	17.91491	44.52295	
Vector	0	0	0	0	0.092404	0.28781	0.619786	66.62353
Result 2026	0	0	0	0	5.539846	18.64209	42.44159	
Vector	0	0	0	0	0.109793	0.295369	0.594839	66.44727
Result 2027	0	0	0	0	6.582352	19.1317	40.73323	
Vector	0	0	0	0	0.125276	0.300639	0.574086	66.29575
Result 2028	0	0	0	0	7.510604	19.47303	39.31212	
Vector	0	0	0	0	0.1389	0.304448	0.556653	66.16549
Result 2029	0	0	0	0	8.327394	19.71976	38.11834	
Vector	0	0	0	0	0.150796	0.307299	0.541905	66.05351
Result 2030	0	0	0	0	9.040592	19.90444	37.10848	
Vector	0	0	0	0	0.16113	0.309502	0.529368	65.95724
Result 2031	0	0	0	0	9.660173	20.0471	36.24997	
Vector	0	0	0	0	0.170078	0.311249	0.518673	65.87449
Result 2032	0	0	0	0	10.1966	20.1603	35.5176	
Vector	0	0	0	0	0.177806	0.312666	0.509528	65.80335
Result 2033	0	0	0	0	10.65996	20.25206	34.89133	
Vector	0	0	0	0	0.184472	0.313833	0.501694	65.7422
Result 2034	0	0	0	0	11.0596	20.32768	34.35491	
Vector	0	0	0	0	0.190216	0.314807	0.494977	65.68963
Result 2035	0	0	0	0	11.40392	20.39078	33.89492	

Source: The table was prepared by the researchers based on Excel.

4-2 Analysis of Fuzzy-Markov Chain:

Step#1:

In this context, fuzzification is the process of identifying associations between the historical values in the dataset and the fuzzy sets defined in the previous step. Each historical value is fuzzified according to its highest degree of membership. If the highest degree of belongingness of a certain historical time variable, say $\mathrm{F}(\mathrm{t}-1)$, occurs at a fuzzy set A_{k}, then $\mathrm{F}(\mathrm{t}-1)$ it is fuzzified as A_{k}. A complete overview of fuzzified enrollments is shown in the table.

Table (13): Complete set of relationships identified

Class (From → To)	Fuzzified Enrollment	Mid-Point	Fuzzified Enrollment	FLRG's	Mid-Point FLR
39-43.5	A1	41.3	A1	A1,A2	43.55
43.6-48.1	A2	45.9	A2	A2,A3	48.15
48.2-52.7	A3	50.5	A3	A3,A4	52.75
52.8-57.3	A4	55.1	A4	A4,A5	57.35
57.4-61.9	A5	59.7	A5	A5,A6	61.95
62-66.5	A6	64.3	A6	A5,A6,A7	64.33
66.6-71.7	A7	69.1	A7	A6,A7	66.7

Source: The table was prepared by the researchers based on Excel.

Step#2:

We use the table above to find the data of the table below, as follows:

Table (14): Fuzzified historical enrollments

Years	Life expectancy	Fuzzified Enrolment	Interval Midpoints		FLRG's		
1950	39	A1	43.55	Na	Na	>	A1
1951	40.6	A1	43.55	43.55	A1	>	A1
1952	42.2	A1	43.55	43.55	A1	>	A1
1953	43.7	A2	48.15	43.55	A1	>	A2
1954	45.2	A2	48.15	48.15	A2	>	A2
1955	46.6	A2	48.15	48.15	A2	>	A2
1956	47.9	A2	48.15	48.15	A2	>	A2
1957	49.2	A3	52.75	48.15	A2	>	A3
1958	50.4	A3	52.75	52.75	A3	>	A3
1959	51	A3	52.75	52.75	A3	>	A3
1960	52.6	A3	52.75	52.75	A3	>	A3
1961	53.4	A4	57.35	52.75	A3	>	A4
1962	54.3	A4	57.35	57.35	A4	>	A4
1963	54.9	A4	57.35	57.35	A4	>	A4
1964	55.6	A4	57.35	57.35	A4	>	A4
1965	56.6	A4	57.35	57.35	A4	>	A4
1966	57.3	A4	57.35	57.35	A4	>	A4
1967	58.4	A5	61.95	57.35	A4	>	A5
1968	58.8	A5	61.95	61.95	A5	>	A5
1969	59.3	A5	61.95	61.95	A5	>	A5
1970	60.2	A5	61.95	61.95	A5	>	A5
1971	60.9	A5	61.95	61.95	A5	>	A5
1972	61.4	A5	61.95	61.95	A5	>	A5
1973	62	A6	64.33	61.95	A5	>	A6
1974	60.9	A5	61.95	64.33	A6	>	A5
1975	61.5	A5	61.95	61.95	A5	>	A5
1976	63.7	A6	64.33	61.95	A5	>	A6
1977	64.2	A6	64.33	64.33	A6	>	A6
1978	64.4	A6	64.33	64.33	A6	>	A6
1979	64.8	A6	64.33	64.33	A6	>	A6
1980	61.5	A5	61.95	64.33	A6	>	A5
1981	59.3	A5	61.95	61.95	A5	>	A5
1982	59.7	A5	61.95	61.95	A5	>	A5
1983	60.1	A5	61.95	61.95	A5	>	A5
1984	60.2	A5	61.95	61.95	A5	>	A5
1985	60.7	A5	61.95	61.95	A5	>	A5
1986	60.9	A5	61.95	61.95	A5	>	A5
1987	61.2	A5	61.95	61.95	A5	>	A5
1988	57.5	A5	61.95	61.95	A5	>	A5
1989	58.2	A5	61.95	61.95	A5	>	A5
1990	58.4	A5	61.95	61.95	A5	>	A5
1991	62.5	A6	64.33	61.95	A5	>	A6
1992	66.7	A7	66.68	64.33	A6	>	A7
1993	66.9	A7	66.68	66.68	A7	>	A7
1994	66.7	A7	66.68	66.68	A7	>	A7
1995	66.5	A6	64.33	66.68	A7	>	A6
1996	66.4	A6	64.33	64.33	A6	>	A6

Years	Life expectancy	Fuzzified Enrolment	Interval Midpoints		FLRG's		
1997	65.8	A6	64.33	64.33	A6	>	A6
1998	65.5	A6	64.33	64.33	A6	>	A6
1999	66.2	A6	64.33	64.33	A6	>	A6
2000	66.8	A7	66.68	64.33	A6	>	A7
2001	67	A7	66.68	66.68	A7	>	A7
2002	67.1	A7	66.68	66.68	A7	>	A7
2003	65.6	A6	64.33	66.68	A7	>	A6
2004	65	A6	64.33	64.33	A6	>	A6
2005	64.8	A6	64.33	64.33	A6	>	A6
2006	63.6	A6	64.33	64.33	A6	>	A6
2007	63.6	A6	64.33	64.33	A6	>	A6
2008	64.9	A6	64.33	64.33	A6	>	A6
2009	66.4	A6	64.33	64.33	A6	>	A6
2010	67.1	A7	66.68	64.33	A6	>	A7
2011	67.7	A7	66.68	66.68	A7	>	A7
2012	68	A7	66.68	66.68	A7	>	A7
2013	68.3	A7	66.68	66.68	A7	>	A7
2014	68.9	A7	66.68	66.68	A7	>	A7
2015	69.4	A7	66.68	66.68	A7	>	A7
2016	69	A7	66.68	66.68	A7	>	A7
2017	70.4	A7	66.68	66.68	A7	>	A7
2018	71.5	A7	66.68	66.68	A7	>	A7
2019	71.6	A7	66.68	66.68	A7	>	A7
2020	69.1	A7	66.68	66.68	A7	>	A7
2021	70.4	A7	66.68	66.68	A7	>	A7

Source: The table was prepared by the researchers based on Excel.

The model explored in so far is referred to as a first order FTS model. Chen later introduced its high order counterpart which incorporates n-order relationships. In the high order modified, relations of order $n \geq 2$ can be expressed as $A_{i,1}, A_{i,2}, \ldots A_{i,n} \rightarrow A_{i,n+1}$. For example, a second order relationship is denoted by $A_{i,1}, A_{i,2} \rightarrow A_{i,3}$. A third order relationship is denoted by $A_{i,1}, A_{i,2}, A_{i,3} \rightarrow A_{i,4}$.

Step#3:

Calculating the average productivity values in each of the seven states. The values were collected in each of the seven states, and the result divided by the number of values in each state. The average values for each state were as follows:

Table (15): Distribution of the average values for each state (use fuzzy data)

States	1	2	3	4	5	6	7	Total
# of states	3	4	4	6	19	18	18	72
Average	43.6	48.2	52.8	57.4	62	64.3	66.7	

Source: The table was prepared by the researchers based on Excel.

Step #4:

Creating the transition matrix: It is clear from the preceding step that the number of values in the state-1 is 3(43.6) and the number of values in the state-2 is 4(48.2) and the number of values in the state-3 is 4(52.8) and the number of values in the state-4 is 6(57.4) and the number of values in the state-5 is 19(62), the number of values in the state-6 is18(64.3) and the number of values in the state-7 is 8(66.7), and then the transition probability matrix is created:

Table (16): Frequencies Observed Data

States	1	2	3	4	5	6	7
1	2	1	0	0	0	0	0
2	0	3	1	0	0	0	0
3	0	0	3	1	0	0	0
4	0	0	0	5	1	0	0
5	0	0	0	0	16	3	0
6	0	0	0	0	2	13	3
7	0	0	0	0	0	2	15

Source: The table was prepared by the researchers based on Excel.

Table (17): Transition Probability Matrix (T.P.M.)

State	1	2	3	4	5	6	7
1	0.667	0.333	0	0	0	0	0
2	0	0.750	0.250	0	0	0	0
3	0	0	0.750	0.250	0	0	0
4	0	0	0	0.833	0.167	0	0
5	0	0	0	0	0.8421	0.1579	0
6	0	0	0	0	0.111111111	0.722222222	0.166666667
7	0	0	0	0	0	0.117647059	0.882

Source: The table was prepared by the researchers based on Excel.

Step#5:

The vector is calculated using the most recent value, which is the current value, and since the life expectancy in 2021 obtained a value of (70.4), falling within the 7-state, the vector will be as follows: (0,0,0,0,0,0,1):

Table (18): Vector is computed based on the most recent value

State	1	2	3	4	5	6	7
Vectors	0	0	0	0	0	0	1

Source: The table was prepared by the researchers based on Excel.

Step#6:

Multiply the vector by the transition probability matrix.

Table (19): Results of multiply the vector by the T.P.M

State	1	2	3	4	5	6	7
1	0.667	0.333	0	0	0	0	0
2	0	0.750	0.250	0	0	0	0
3	0	0	0.750	0.250	0	0	0
4	0	0	0	0.833	0.167	0	0
5	0	0	0	0	0.8421	0.1579	0
6	0	0	0	0	0.111111111	0.722222222	0.166666667
7	0	0	0	0	0	0.117647059	0.882

Source: The table was prepared by the researchers based on Excel.

We get:

1	2	3	4	5	6	7
0	0	0	0	0	0.117647059	0.882352941

The result is then multiplied by the average of the seven states that were previously determined, and they are:

Table (20): Distribution of the average values for each state

State	1	2	3	4	5	6	7	Result 2021
Average	43.6000	48.2000	52.8000	57.4000	62.0000	64.3000	66.7000	66.41765
Vector	0	0	0	0	0	0.117647059	0.882352941	
Results	0	0	0	0	0	7.564705882	58.85294118	

Source: The table was prepared by the researchers based on Excel.

In terms of life expectancy, we find that it reached (66,7) in 2021, confirming the forecasting accuracy of the Markov model. This means that the difference between the actual and estimated life expectancy levels is less than 0.3%, as the estimated value of (66.41765) is 99.7% closer to the actual value of (66.7) than the estimated value. The same method will be used to extract predictive values for succeeding years. For 2022, the prediction value was as follows:

Table (21): Show the productivity for year 2022

1	2	3	4	5	6	7
0	0	0	0	0.013071895	0.188773549	0.798154556

Source: The table was prepared by the researchers based on Excel.

After multiplying the final vector by the average values of the seven states, we obtain: (66.18551).

Table (22): Distribution of the average values for each state

State	1	2	3	4	5	6	7	Result 2022
Average	43.6000	48.2000	52.8000	57.4000	62.0000	64.3000	66.7000	66.18551
Vector	0	0	0	0	0.013071895	0.188773549	0.798154556	
Result	0	0	0	0	0.810457516	12.13813918	53.23690888	

Source: The table was prepared by the researchers based on Excel.

Step#7:

The predicted values were shown in table 19 by repeating the same procedures for succeeding years:

Table (23): Forecasting life expectancy for the period (2022-2035)

State	1	2	3	4	5	6	7	Forecasting life expects
Average	43.6000	48.2000	52.8000	57.4000	62.0000	64.3000	66.7000	
Vector	0	0	0	0	0	0.117647059	0.882352941	66.41765
Result 2021	0	0	0	0	0	7.564705882	58.85294118	
Vector	0	0	0	0	0.013071895	0.188773549	0.798154556	66.18551
Result 2022	0	0	0	0	0.810457516	12.13813918	53.23690888	
Vector	0	0	0	0	0.031982751	0.232300971	0.735716278	65.99216
Result 2023	0	0	0	0	1.982930542	14.93695245	49.07227575	
Vector	0	0	0	0	0.052744062	0.259377688	0.68787825	65.8296
Result 2024	0	0	0	0	3.270131826	16.67798533	45.8814793	
Vector	0	0	0	0	0.073235795	0.276583193	0.650181012	65.69199
Result 2025	0	0	0	0	4.540619294	17.7842993	43.3670735	
Vector	0	0	0	0	0.092403714	0.287809959	0.619786327	65.57496
Result 2026	0	0	0	0	5.72903029	18.50618034	41.33974802	
Vector	0	0	0	0	0.109792538	0.295368846	0.594838615	65.47509
Result 2027	0	0	0	0	6.807137386	18.99221683	39.67573562	
Vector	0	0	0	0	0.125275635	0.300638622	0.574085743	65.38967
Result 2028	0	0	0	0	7.767089384	19.33106341	38.29151903	
Vector	0	0	0	0	0.138899563	0.304447756	0.55665268	65.3165
Result 2029	0	0	0	0	8.611772914	19.57599074	37.12873379	
Vector	0	0	0	0	0.150795582	0.307298996	0.541905423	65.25374
Result 2030	0	0	0	0	9.349326062	19.75932543	36.14509169	
Vector	0	0	0	0	0.161130086	0.309501571	0.529368343	65.19988
Result 2031	0	0	0	0	9.990065322	19.90095104	35.30886846	
Vector	0	0	0	0	0.170077557	0.311249134	0.518673309	65.15364
Result 2032	0	0	0	0	10.54480852	20.0133193	34.59550974	
Vector	0	0	0	0	0.177806443	0.312665781	0.509527776	65.11391
Result 2033	0	0	0	0	11.02399946	20.10440975	33.98550264	
Vector	0	0	0	0	0.184472384	0.313833321	0.501694295	65.07978
Result 2034	0	0	0	0	11.43728779	20.17948255	33.46300948	
Vector	0	0	0	0	0.190215534	0.314807475	0.49497699	65.05045
Result 2035	0	0	0	0	11.79336312	20.24212067	33.01496526	

Source: The table was prepared by the researchers based on Excel.

Step#8:

Comparison between Markov Chin with and without fuzzy:

We make a comparison between the two methods of Markov chains (without fuzzy and fuzzy) based on the year 2021 and we depend on the error proportion in the data in both cases as follows:

Table (24): Comparison between the results fuzzy and non-fuzzy Markov chain

Markov-Chai	Ordinal data	Forecast Value	Difference %	Correct %
No-fuzzy data	70.4	68.04183	2.35817	97.64183
Fuzzy data	66.7	66.41765	0.28235	99.71765

Source: The table was prepared by the researchers based on Excel.

Through the above table, we notice that fuzzy Markov chains give us better results because the error proportion in 2021 is fewer compared to non-fuzzy Markov chains.

5 Conclusions and Recommendations

5-1 Conclusions

The researcher has the following conclusions and recommendations based on the analysis:

1. Life expectancy is an important vital indicator. It is a statistical measure of the average number of years that the population of a given country is expected to live. In this study, the average life expectancy was predicted, and through prediction we concluded that the average life expectancy began to decline, and this is due to economic, social, environmental, health, cultural, political, and legislative factors. Coupled with these influencing and decisive factors, the indicator has a significant impact, positively or negatively, on all aspects of social and cultural life in the country.
2. The results of the paper also established that the Markov chains do not need old historical data, which did not constitute a major difficulty in the explanation of the prediction results, as it is recognized that the effect of future values.
3. By comparing the non-fuzzy Markov chains model with the fuzzy Markov chains, we notice that the error value in the fuzzy Markov chains is less compared to the non-fuzzy Markov chains model, so we can choose the model that gives us the least error value.

5-2 Recommendations

Based on the conclusions, the following are some recommendations for this study.

1. Markov chains don't require a lot of data to predict, and the research suggests using them to analyze data with inconsistent precision in some of its observations.
2. Given the importance of having access to reliable modeling techniques and projections, the research suggests paying attention to the statistical component and selecting the proper statistical tools.

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