

CONSTRUCTION OF COMPLETE AND MAXIMAL (k, n)
ARCS IN THE PROJECTIVE PLANE $PG(2, 7)$

By Najim Abdullah Ismaeel

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Abstract

The purpose of this paper is to study the construction of complete and maximal (k, n) -arcs in the projective plane $PG(2, 7)$, $n = 2, 3, \dots, 8$.

A (k, n) -arc K in a projective plane is a set of K points such that no $n + 1$ of which are collinear. A (k, n) -arc is complete if it is not contained in a $(k + 1, n)$ -arc.

A (k, n) -arc is a maximal if and only if every line in $PG(2, p)$ is

a O -secant, or n -secant, which represented as $(k, 2)$ -arc and $(k, 8)$ -arc.

1. Introduction:

Ahmad(1999) [4] studied the complete arcs in the projective plane over Galois Field $GF(9)$, also Rashad (1999) [10] showed the complete arcs in the projective plane over Galois Field $GF(q)$ and Massa (2004) [8] studied the construction of (k, n) -arcs from (k, m) -arcs in the $PG(2, 17)$ for $2 \leq m < n$. Finally Najm (2005) [9] studied the construction of (k, n) -arcs from (k, m) -arcs in the $PG(2, 13)$ for $2 \leq m < n$. This paper deals with complete (k, n) -arc, maximal (k, n) -arc and how constructed from complete (k, m) -arc, $2 \leq m < n$.

The construction of complete (k, n) -arc, $n < k$ prepared from the union of some complete (k, m) -arc, $2 \leq m < n$. Usually the construction arc is incomplete arc and we get the complete by eliminating some points from the incomplete (k, n) -arc.

The only two maximal arcs are $(k, 2)$ -arc and $(k, 8)$ -arc which represented the whole plane since each line contains eight points.

2- Basic Definition:

2.1 Definition (K, n) – Arcs [1, 2, 6, 7] : A (k, n) -arc in the projective plane $PG(2, p)$ is a set K points such that some line meets K in n points but no line meets k in more than n points $n \geq 2$, p is prime

2.2 Definition [4, 6, 9, 10] : A (k, n) -arc is complete if it is not contained in $(k + 1, n)$ -arc.

2.3 Definition [3, 6, 8, 12] : A point p which is not on (k, n) -arc K has index i if there are exactly i $(n - \text{secant})$ through p , we denoted the numbers of point p of index i by C_i .

2.4 Definition [5, 6, 9, 11] : A (k, n) -arc K is a maximal if and only if every line in $PG(2, p)$ is a O -secant or n -secant.

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2.5 Definition $PG(2, 7)$ [1, 6, 10]: A $PG(2, 7)$ is the two – dimensional projective space which consists of points and lines with incidence relation between them and satisfying the following axioms:

- i- Any two distinct lines are intersected in a unique point.
- ii- Any two distinct points are contained in a unique line.
- iii – There exist at least four points such that no three of them are collinear .

Remark (1) [4, 5, 6] : A (k, n) – arc K is complete if and only if $C_0 = O$, we mean that C_0 is 0 (n – secant), thus K is complete if and only if every point of $PG(2, p)$ lies on some (n – secant) of K

3- The Projective Plane $PG(2, 7)$:

The projective plane $PG(2, 7)$ contains 57 points and 57 lines, every line contains 8 points and every point is on 8 lines. Any line in $PG(2, 7)$ can be constructed by means of variety v. let P_i and L_i , $i = 1, 2, \dots, 57$ be the points and lines of $PG(2, 7)$ respectively. Let i stands for the points P_i and the lines L_i , then all the points and the lines in $PG(2, 7)$ are given in the table (1)

i	P_i	L_i							
1	(1, 0, 0)	2	9	16	23	30	37	44	51
2	(0, 1, 0)	1	9	10	11	12	13	14	15
3	(1, 1, 0)	8	9	22	28	34	40	46	52
4	(2, 1, 0)	5	9	19	29	32	42	45	55
5	(3, 1, 0)	4	9	18	27	36	38	47	56
6	(4, 1, 0)	7	9	21	26	31	43	48	53
7	(5, 1, 0)	6	9	20	24	35	39	50	54
8	(6, 1, 0)	3	9	17	25	33	41	49	57
9	(0, 0, 1)	1	2	3	4	5	6	7	8
10	(1, 0, 1)	2	15	22	29	36	43	50	57
11	(2, 0, 1)	2	12	19	26	33	40	47	54
12	(3, 0, 1)	2	11	18	25	32	39	46	53
13	(4, 0, 1)	2	14	21	28	35	42	49	56
14	(5, 0, 1)	2	13	20	27	34	41	48	55
15	(6, 0, 1)	2	10	17	24	31	38	45	52
16	(0, 1, 1)	1	51	52	53	54	55	56	57
17	(1, 1, 1)	8	15	21	27	33	39	45	51
18	(2, 1, 1)	5	12	22	25	35	38	48	51
19	(3, 1, 1)	4	11	20	29	31	40	49	51
20	(4, 1, 1)	7	14	19	24	36	41	46	51
21	(5, 1, 1)	6	13	17	28	32	43	47	51
22	(6, 1, 1)	3	10	18	26	34	42	50	51
23	(0, 2, 1)	1	30	31	32	33	34	35	36
24	(1, 2, 1)	7	15	20	25	30	42	47	52
25	(2, 2, 1)	8	12	18	24	30	43	49	55

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26	(3, 2, 1)	6	11	22	26	30	41	45	56
27	(4, 2, 1)	5	14	17	27	30	40	50	53
28	(5, 2, 1)	3	13	21	29	30	38	46	54
29	(6, 2, 1)	4	10	19	28	30	39	48	57
30	(0, 3, 1)	1	23	24	25	26	27	28	29
31	(1, 3, 1)	6	15	19	23	34	38	49	53
32	(2, 3, 1)	4	12	21	23	32	41	50	52
33	(3, 3, 1)	8	11	17	23	36	42	48	54
34	(4, 3, 1)	3	14	22	23	31	39	47	55
35	(5, 3, 1)	7	13	18	23	35	40	45	57
36	(6, 3, 1)	5	10	20	23	33	43	46	56
37	(0, 4, 1)	1	44	45	46	47	48	49	50
38	(1, 4, 1)	5	15	18	28	31	41	44	54
39	(2, 4, 1)	7	12	17	29	34	39	44	56
40	(3, 4, 1)	3	11	19	27	35	43	44	52
41	(4, 4, 1)	8	14	20	26	32	38	44	57
42	(5, 4, 1)	4	13	22	24	33	42	44	53
43	(6, 4, 1)	6	10	21	25	36	40	44	55
44	(0, 5, 1)	1	37	38	39	40	41	42	43
45	(1, 5, 1)	4	15	17	26	35	37	46	55
46	(2, 5, 1)	3	12	20	28	36	37	45	53
47	(3, 5, 1)	5	11	21	24	34	37	47	57
48	(4, 5, 1)	6	14	18	29	33	37	48	52
49	(5, 5, 1)	8	13	19	25	31	37	50	56
50	(6, 5, 1)	7	10	22	27	32	37	49	54
51	(0, 6, 1)	1	16	17	18	19	20	21	22
52	(1, 6, 1)	3	15	16	24	32	40	48	56
53	(2, 6, 1)	6	12	16	27	31	42	46	57
54	(3, 6, 1)	7	11	16	28	33	38	50	55
55	(4, 6, 1)	4	14	16	25	34	43	45	54
56	(5, 6, 1)	5	13	16	26	36	39	49	52
57	(6, 6, 1)	8	10	16	29	35	41	47	53

Table (1)

(Contains 57 points and 57 lines, every line contains 8 points and every point is on 8 lines)

4- The Construction of (k, n) – Arcs in $PG(2, 7)$:

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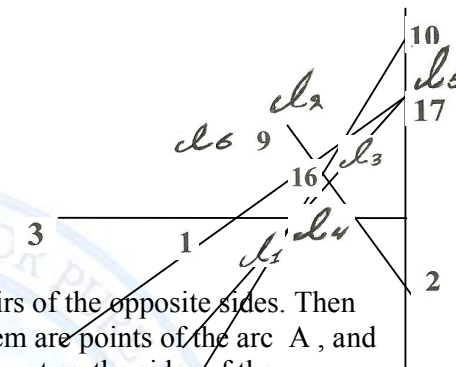
Let $A = \{1, 2, 9, 17\}$ be the set reference and unit points in the table (1) such that $1 = (1, 0, 0)$, $2 = (0, 1, 0)$, $9 = (0, 0, 1)$, $17 = (1, 1, 1)$. A is a $(4, 2)$ – arc, since no three points of A are collinear, the points of A are the vertices of a quadrangle whose side are the lines

$$\begin{aligned} l_1 &= [1, 2] = \{1, 2, 3, 4, 5, 6, 7, 8\} \\ l_2 &= [1, 9] = \{1, 9, 10, 11, 12, 13, 14, 15\} \\ l_3 &= [1, 17] = \{1, 16, 17, 18, 19, 20, 21, 22\} \\ l_4 &= [2, 9] = \{2, 9, 16, 23, 30, 37, 44, 51\} \\ l_5 &= [2, 17] = \{2, 10, 17, 24, 31, 38, 45, 52\} \\ l_6 &= [9, 17] = \{3, 9, 17, 25, 33, 41, 49, 57\} \end{aligned}$$

The diagonal points of A are the points $\{3, 10, 16\}$ where:

$$\begin{aligned} 3 &= l_1 \cap l_4 \\ 10 &= l_1 \cap l_5 \\ 16 &= l_2 \cap l_3 \end{aligned}$$

which are the intersection points of pairs of the opposite sides. Then there are 20 points on the sides of the quadrangle, four of them are points of the arc A , and three of them are diagonal points of A , so there are 20 points not on the sides of the quadrangle which are the points of index zero for A these points are: $\{24, 27, 28, 29, 32, 34, 35, 36, 39, 40, 42, 43, 46, 47, 48, 50, 53, 54, 55, 56\}$
Hence A is incomplete $(4, 2)$ – arc



**5- The Conics In PG (2, 7) Through the Reference
The general equation of conic is and Unit points**

$$a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 + a_4 x_1 x_2 + a_5 x_1 x_3 + a_6 x_2 x_3 = 0 \quad \dots\dots (1)$$

By substituting the points of the arc – A in (1), we get

$$\begin{aligned} 1 &= (1, 0, 0) \rightarrow a_1 = 0 \\ 2 &= (0, 1, 0) \rightarrow a_2 = 0 \\ 9 &= (0, 0, 1) \rightarrow a_3 = 0 \\ 17 &= (1, 1, 1) \rightarrow a_4 + a_5 + a_6 = 0 \end{aligned}$$

So equation (1) becomes

$$a_4 x_1 x_2 + a_5 x_1 x_3 + a_6 x_2 x_3 = 0 \quad \dots\dots (2)$$

If $a_4 = 0$, then $a_5 x_1 x_3 + a_6 x_2 x_3 = 0$

Hence $x_3 (a_5 x_1 + a_6 x_2) = 0$, $x_3 = 0$ or $a_5 x_1 + a_6 x_2 = 0$

Which are a pair of lines, then the conic is degenerated, therefore $a_4 \neq 0$

Similarly $a_5 \neq 0$ and $a_6 \neq 0$

Dividing equation (2) by a_4 we get

$$\begin{aligned} x_1 x_2 + \frac{a_5}{a_4} x_1 x_3 + \frac{a_6}{a_4} x_2 x_3 &= 0 \\ x_1 x_2 + \alpha x_1 x_3 + \beta x_2 x_3 &= 0 \quad \dots\dots (3) \end{aligned}$$

$$\alpha = \frac{a_5}{a_4}, \quad \beta = \frac{a_6}{a_4} \text{ then}$$

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a_4

a_4

$$1 + \alpha + \beta = 0 \pmod{7}$$

$\beta = -(1 + \alpha) + ZK$, then (3) can be written as:

$$x_1 x_2 + \alpha x_1 x_3 - (1 + \alpha) x_2 x_3 = 0 \quad \dots\dots(4)$$

Where $\alpha \neq 0$ and $\alpha \neq 6$, for if $\alpha = 0$ or $\alpha = 6$, we get degenerated conic, i.e. $\alpha = 1, 2, 3, 4, 5$

6-The Equations and the Points of the Conic of PG (2, 7) Through The Reference and Unit Points:-

For any value for α there is a unique conic containing eight points, four of them are the reference and unit points

1- If $\alpha = 1$, then the equation of the conic C_1 is

$$x_1 x_2 + x_1 x_3 + 5 x_2 x_3 = 0, \text{ the point of } C_1 \text{ are } \{1, 2, 9, 17, 29, 35, 40, 48\}$$

which is a complete (7, 2) – arc, since there are no points of index zero for C_1

2- If $\alpha = 2$, then the equation of the conic C_2 is

$$x_1 x_2 + 2 x_1 x_3 + 4 x_2 x_3 = 0$$

The points of C_2 are $\{1, 2, 9, 17, 28, 36, 39, 55\}$, which is a complete (7, 2) – arc, since there are no points of index zero for C_2

3- If $\alpha = 3$, then the equation of the conic C_3 is

$$x_1 x_2 + 3 x_1 x_3 + 3 x_2 x_3 = 0$$

The points of C_3 are $\{1, 2, 9, 17, 26, 32, 50, 56\}$, which is a complete (7, 2) – arc, since there are no points of index zero for C_3

4- If $\alpha = 4$, then the equation of the conic C_4 is

$$x_1 x_2 + 4 x_1 x_3 + 2 x_2 x_3 = 0$$

The points of C_4 are $\{1, 2, 9, 17, 27, 43, 46, 54\}$, which is a complete (7, 2) – arc since there are no points of index zero for C_4

5- If $\alpha = 5$, then the equation of the conic C_5 is

$$x_1 x_2 + 5 x_1 x_3 + x_2 x_3 = 0$$

The points of C_5 are $\{1, 2, 9, 17, 34, 42, 47, 53\}$, which is complete (7, 2) – arc, since there are no points of index zero for C_5

Thus there are five complete (7, 2) – arcs in the PG (2, 7) which are

$$C_1 = \{1, 2, 9, 17, 29, 35, 40, 48\}$$

$$C_2 = \{1, 2, 9, 17, 28, 36, 39, 55\}$$

$$C_3 = \{1, 2, 9, 17, 26, 32, 50, 56\}$$

$$C_4 = \{1, 2, 9, 17, 27, 43, 46, 54\}$$

$$C_5 = \{1, 2, 9, 17, 34, 42, 47, 53\}$$

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7-Construction of Complete $(k, 3)$ – Arcs:

We get complete $(k, 3)$ – arcs through the following steps:

We take the union of two complete $(8, 2)$ – arcs, say C_1 and C_2 denoted by D_1 .

a . Let $D_1 = C_1 \cup C_2 = \{1, 2, 9, 17, 28, 29, 35, 36, 39, 40, 48, 55\}$, we notice that D_1 is incomplete $(k, 3)$ – arc, since there exist the points $\{3, 5, 16, 18, 45, 47, 51, 53\}$ of index zero for D_1

B – We add the point $\{3\}$ from the index zero to D_1 , therefore

$D_1^1 = \{1, 2, 3, 9, 17, 28, 29, 35, 36, 39, 40, 48, 55\}$ is a complete $(13, 3)$ – arc, since there is no point of index zero i.e $C_0 = 0$.

Let $D_2 = C_1 \cup C_3 = \{1, 2, 9, 17, 26, 29, 32, 35, 40, 48, 55\}$, we notice that there are some line meet D_2 in four points, hence $(k, 3)$ is not complete. So we eliminate some points from D_2 to determine a complete $(k, 3)$ – arc as follows:

Let $D_2^1 = C_1 \cup C_3 / \{48\} = \{1, 2, 9, 17, 26, 29, 32, 35, 40, 50, 56\}$, we notice that D_2 is incomplete

$(k, 3)$ – arc, since there exist the points of index zero for D_2 which are $\{8, 10, 11, 13, 18, 38, 41, 51, 52\}$

We add $\{8, 11\}$ from the index zero to D_2 , therefore

$D_2 = \{1, 2, 8, 9, 11, 17, 26, 29, 32, 35, 40, 50, 56\}$ is a complete $(13, 3)$ – arc, since $C_0 = 0$

Let $D_3 = C_1 \cup C_4 = \{1, 2, 9, 17, 27, 29, 35, 40, 43, 46, 48, 54\}$, notice that D_3 is incomplete $(k, 3)$ – arc, since there exist points of index zero for D_3 which are $\{10, 16, 18, 32, 51, 56\}$ we add $\{56\}$ from the index zero to D_3 , therefore

$D_3^1 = \{1, 2, 9, 17, 27, 29, 35, 40, 43, 46, 48, 54, 56\}$ is a complete $(13, 3)$ – arc, since $C_0 = 0$

Let $D_4 = C_1 \cup C_5 = \{1, 2, 9, 17, 29, 34, 35, 40, 42, 47, 48, 53\}$, notice that there are some line meet D_4 in four point, hence $(k, 3)$ is not complete. So we eliminate some points from D_4 to determine a complete $(k, 3)$ – arc as follows

Let $D_4 = C_1 \cup C_5 / \{53\} = \{1, 2, 9, 17, 29, 34, 35, 40, 42, 47, 48\}$, we notice that D_4 is incomplete $(k, 3)$ – arc since there exist points of index zero for D_4 which are $\{3, 4, 15, 18, 24, 25, 51, 57\}$

We add $\{4, 25\}$ from the index zero to D_4 , therefore

$D_4^1 = \{1, 2, 4, 9, 17, 25, 29, 34, 35, 40, 42, 47, 48\}$ is a complete $(13, 3)$ – arc since $C_0 = 0$.

8-Construction of Complete $(k, 4)$ – Arcs:

Let $E_1 = D_1 \cup D_2 = \{1, 2, 3, 8, 9, 11, 17, 26, 28, 29, 32, 35, 36, 39, 40, 48, 50, 56\}$, we notice that there are some line meet E_1 in five points, and hence E_1 is not complete $(k, 4)$ – arc there for we eliminate $\{32, 48\}$ from it to determine complete $(k, 4)$ – arc as follows,

$E_1 = D_1^1 \cup D_2^1 / \{32, 48\} = \{1, 2, 3, 8, 9, 11, 17, 26, 28, 29, 35, 36, 39, 40, 50, 56\}$, E_1 is incomplete since there are points of index zero which are $\{5, 10, 13, 18, 19, 30, 33, 41, 45, 47, 51, 53\}$.

We add $\{5, 10, 31\}$ from the index zero to E_1 , therefore

$E_1^1 = \{1, 2, 3, 5, 8, 9, 10, 11, 17, 26, 28, 29, 31, 35, 36, 39, 40, 50, 56\}$, is a complete $(19, 4)$ – arc, since $C_0 = 0$.

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Let $E_2 = D_1^{-1} \cup D_3^{-1} = \{1, 2, 3, 9, 17, 27, 28, 29, 35, 36, 39, 40, 43, 46, 48, 54, 55, 56\}$, notice that E_2 is not complete $(k, 4)$ – arc, since there are points of index zero which are $\{5, 10, 31, 33, 45\}$

We add $\{5, 31\}$ from index zero to E_2 , therefore
 $E_2^{-1} = \{1, 2, 3, 5, 9, 17, 27, 29, 31, 35, 36, 39, 40, 43, 46, 48, 54, 55, 56\}$ is a complete $(19 - 4)$ – arc, since there is no point of index zero.

Let $E_3 = D_1^{-1} \cup D_4^{-1} = \{1, 2, 3, 4, 9, 17, 25, 28, 29, 34, 35, 36, 39, 40, 42, 47, 48, 55\}$. Notice that E_3 is not complete $(k, 4)$ – arc, since there are points of index zero which are $\{16, 50, 51, 53\}$

We add $\{16, 50\}$ from index zero to E_3 , therefore
 $E_3^{-1} = \{1, 2, 3, 4, 9, 16, 17, 25, 28, 29, 34, 35, 36, 39, 40, 42, 47, 48, 50, 55\}$ is complete $(20, 4)$ – arc, since there is no points of index zero.

9- Construction of Complete $(k, 5)$ – Arcs :-

Let $F_1 = E_1^{-1} \cup E_2^{-1} = \{1, 2, 3, 5, 8, 9, 10, 11, 17, 26, 27, 28, 29, 31, 35, 36, 39, 40, 43, 46, 48, 50, 54, 55, 56\}$, notice that there is a line meet F_1 in six points, hence $(k, 5)$ is not complete. So we eliminate a point $\{11\}$ from F_1 to determine a complete $(k, 5)$ – arc as follows

Let $F_1 = E_1 \cup E_2 / \{11\} =$
 $\{1, 2, 3, 5, 8, 9, 10, 17, 26, 27, 28, 29, 31, 35, 36, 39, 40, 43, 46, 48, 50, 54, 55, 56\}$. Notice that F_1 is incomplete since there exist points of index zero which are $\{12, 13, 16, 18, 19, 32, 38, 41, 44, 45, 47, 49, 51\}$

We add $\{12, 13, 18, 32\}$ from index zero to F_1 , then
 $F_1^{-1} = \{1, 2, 3, 5, 8, 9, 10, 12, 13, 17, 18, 26, 27, 28, 29, 31, 32, 35, 36, 39, 40, 43, 46, 48, 50, 54, 55, 56\}$ is a complete $(28, 5)$ – arc, since $C_0 = 0$

Let $F_2 = E_1 \cup E_3 =$
 $\{1, 2, 3, 4, 5, 8, 9, 10, 11, 16, 17, 25, 26, 28, 29, 31, 34, 35, 36, 39, 40, 42, 47, 48, 50, 55, 56\}$
Notice that there are some lines meet F_2 in six points, hence $(k, 5)$ is un complete. So we eliminate some points from F_2 to determine a complete $(k, 5)$ – arc as follows

Let $F_2 = E_1^{-1} \cup E_3^{-1} / \{3, 16, 17\} =$
 $\{1, 2, 4, 5, 8, 9, 10, 11, 25, 26, 28, 29, 31, 34, 35, 36, 39, 40, 42, 47, 48, 50, 55, 56\}$, we notice that F_2 is incomplete, since there exist the points of index zero which are $\{12, 15, 43\}$

We add $\{15\}$ from index zero to F_2 , then
 $F_2^{-1} = \{1, 2, 4, 5, 8, 9, 10, 11, 15, 25, 26, 28, 29, 31, 34, 35, 36, 39, 40, 42, 47, 48, 50, 55, 56\}$ is a complete $(25, 5)$ – arc, since $C_0 = 0$.

10- Construction of Complete $(k, 6)$ – Arcs:-

Let $G = F_1^{-1} \cup F_2^{-1} =$
 $\{1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15, 17, 18, 25, 26, 27, 28, 29, 31, 32, 34, 35, 36, 39, 40, 42, 43, 46, 47, 48, 50, 54, 55, 56\}$, notice that there are some lines meet G in seven points, hence $(k, 6)$ – arc is incomplete arc. So we eliminate some points from G to determine a complete $(k, 5)$ – arc as follows.

Let $G = F_1^{-1} \cup F_2^{-1} / \{9, 39, 42\} =$
 $\{1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 15, 17, 18, 25, 26, 27, 28, 29, 31, 32, 34, 35, 36, 40, 43, 46, 47, 48, 50, 54, 55, 56\}$.

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Notice that G is incomplete, since there exist the points of index zero for G which are $\{21, 45, 49, 52\}$. We add $\{21, 49\}$ from index zero to G , therefore $G^1 = \{1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 15, 17, 18, 21, 25, 26, 27, 28, 29, 31, 32, 34, 35, 36, 40, 43, 46, 47, 48, 49, 50, 54, 55, 56\}$ is a complete $(34, 6)$ – arc, since $C_0 = 0$.

11- Construction of Complete $(k, 7)$ – Arcs:-

Let us take complete $(k, 6)$ – arc G^1 , G^1 is incomplete $(k, 7)$ – arc, since there exist points of index zero for G^1 which are

$\{6, 7, 9, 14, 16, 19, 20, 22, 23, 24, 30, 33, 37, 38, 39, 41, 42, 44, 45, 51, 52, 53, 57\}$

We add eight points of index zero which are $\{9, 16, 20, 33, 39, 42, 44, 57\}$ to G^1 , we denoted it by H

$H = \{1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 20, 21, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 39, 40, 42, 43, 44, 46, 47, 48, 49, 50, 54, 55, 56, 57\}$ is a complete $(42, 7)$ – arc, since $C_0 = 0$.

12- Construction of Complete $(k, 8)$ – Arcs:-

We take complete $(2, 7)$ – arc H , H is incomplete $(k, 8)$ – arc since there exist points of index zero for H which are $\{6, 7, 14, 19, 22, 23, 24, 30, 37, 38, 41, 45, 51, 52, 53\}$

We add the points of index zero to H denoted by I , then I contains all the points of the plane i.e $I = \{1, 2, 3, \dots, 55, 56, 57\}$ is a complete $(57, 8)$ – arc.

This arc is the whole plane, since each line in it contains eight points. Hence this arc is a maximal arc.

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REFERENCES

- 1- Ali, T. H., 2004 "Complete (k, n) – arc in the Projective Plane $(2, 17)$. Thesis, University of Baghdad, Iraq.
- 2- Al-Jofy , R.A.S. (1999) " Complete Arcs in a projective plane over Galois Field : , M.Sc. , Thesis , University of Baghdad Iraq.
- 3- Abdul – Hussein , M , A (1997) " Classification of $(K, 4)$ – Arc in the projective plane of order Five " , M , Sc , Thesis , University of Basrah , Iraq.
- 4- Ahmed , A , M , (1999) , " Complete Arcs in the projective plane over $GF(q)$ " . Ibn AL-Haitham , Iraq.
- 5- Ban (2001) , " The maximal (K,n) – arcs " University of Mosul , Iraq.
- 6- Hirschfeld , T . W , (1997) " Projective Geometries Over Finite Field " Oxford Press.
- 7 - Hughes , D.R and piper F . C . , (1973) , " Projective Plane Gpringer – Verlag , New York , Inc.
- 8- Mayssa G . M (2004) " Construction of (k, m) – arcs from (k, n) arcs in the Projective Plane $PG(2, 17)$, $m < n$.
- 9-Najim , A . I (2005) Complete (k, n) – arcs in the Projective Plane $(2, 13)$. Thesis , University of Baghdad , Iraq.
- 10 - Rashad (1999) " Complete arcs in a Projective Plane Over Galois Field , M .Sc. Thesis , University of Baghdad , Iraq.
- 11- Rutter , J . W , (2000) " Geometry of Curves " , Chapman and Hall / CRC.
- 12- Sawsan , J . K , (2001) " Construction of (k, n) – arcs in $PG(2, p)$, $2 \leq m < n$

بناء أقواس كاملة وأعظمية - (k, n) في المستوى الإسقاطي $PG(7, 2)$

الخلاصة

الغرض من هذا البحث هو دراسة بناء أقواس كاملة وأعظمية - (k, n) في المستوى الإسقاطي $PG(2, 7)$. قوس - (k, n) في المستوى الإسقاطي هو مجموعة من k من النقاط بحيث لا يوجد $n + 1$ نقطة منها على استقامة واحدة. قوس - (k, n) يكون كاملاً إذا لم يكن محتوياً في القوس - $(k + 1, n)$. قوس - (k, n) يكون أعظم قوس إذا وفقط إذا كان كل مستقيم في $PG(2, p)$ لا يقطع القوس أو يقطعه في n من النقاط ، وهي هنا تمثل القوسين $(k, 2)$ و $(k, 8)$.