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Nuclear structure investigation of even-even $^{124\text{-}138}$Ce isotopes using the geometric collective model

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Abstract:

The positive parity energy levels of the $^{124\text{-}138}$Ce isotopes were studied via the framework of the geometric collective model (GCM). The levels band structure has been investigated using the nuclear softness (NSF) and the corrected Bohr Mottelson (CBMF) formula. The energy staggering, power index, and average power law have been investigated for all isotopes. The electric quadrupole transitions probability and quadrupole moment have been calculated. The obtained results were compared with the available experimental values, and it was found that the model was able to describe the nuclear structure of these nuclei.

Keywords: Nuclear structure, Ce isotopes, Geometric Collective Model, B(E2), Quadrupole moments.

1-Introduction

The collective model appeared in 1950, which was proposed by Rainwater [1]. It was assumed that the nucleons that fall outside the closed-shell exert centrally direct pressure on the surface of the nucleus, which leads to the deformation of the nucleus from the spherical shape. Bohr and Mottelson [2] are explained that the deformed

nucleus characterized by two rotational and vibration motions. This model divided the cores into:

First, the nuclei of mass number A> 150, some of its nuclear properties are described by a model based on vibrations or vibrational motion occurring around a balanced spherical shape [3]. Second, nuclei of mass number 150 > A > 190, can be treated as representing a rotational motion of an aspherical system. The collective rotation and vibration are of great importance in understanding the higher energy levels as the statistical effects of the collective movement of nucleons. The collective model assumes that the deformation in the nucleus depends on an increase in the number of nucleons [3]. The quantitative understanding of the bands structure requires a good and consistent description of the low-lying spectrum of both energies and electromagnetic transitions. A variety of theoretical methods have been used for this purpose [4,5]

The Geometric model is one of the models that has achieved acceptable success in determining the properties of nuclei. The Geometric Collective Model (GCM) is suggested as a tool to study the nuclear structure [6]. The main term contains a set of collective nuclear features– prolate, spherical, oblate, and Triaxial [7]. The GCM succeeds in calculation the quadrupole moments, low lying energy spectra, and B(E2)values of the even-even nuclei. This model has been used in many studies as it has been applied to many isotope chains, an example is the study of isotope series $^{108\text{-}112}$Ru [8] well been studying $^{124\text{-}132}$Ba [9] and ^{182}Pt [10] and ^{152}Sm [11]. The geometry of the surface is caused by the collective behavior of nucleons, as well as the types of excitation that they produce. The shape of this surface should be roughly spherical due to the interactions between the nucleons, which are usually strong interactions. Since the physical properties of the nucleus in this model are determined by its shape, it

neglects the properties of individual particles. Therefore, this model is of great importance when studying the shape of the nucleus.

This work is principally aimed at disentangling the structure of the low-lying states of $^{124-138}$Ce isotopes. More importantly, the nuclear shape is described by intrinsic variables (β, γ). The influence of different values of the parameters on the nuclear structure is investigated. The energy of the rotational band levels was also studied in this paper using the nuclear softness formula (NSF) [12,13], in addition to calculating β and γ vibrational energy bands using the corrected Bohr Mottelson formula (CBMF) [14].

2- Theoretical formworks

2-1 Geometric Collective Model

Since its introduction, the GCM has been considered a model with a flexible approach and a wide application to describe the nuclear structure [15-17].

The description of the shape of the nucleus is possible through spherical harmonics [6]:

$$R(\theta,\Phi,t) = R_0\left(1 + \sum \alpha^*_{\lambda\mu}(t)Y_{\lambda\mu}(\theta,\Phi)\right), \quad (1)$$

R(ɵ,ϕ,t) represents the radius of the nucleus, and $\alpha_{\lambda\mu}$ are the multipole deformation parameters. The Hamiltonian is given by the relationship:

$$\hat{H} = \hat{T} + V(\beta,\gamma), \quad (2)$$

The Kinetic energy (T) is given

$$\hat{T} = \frac{1}{B_2}[\pi\times\pi]^{(0)} + \frac{P_3}{3}\left[[\pi\times\pi]^{(2)}\times\hat{\pi}\right]^{(0)}, \quad (3)$$

Where α it is the quadrupole deformation in the laboratory system and (π) is the momentum conjugate. The potential energy is given as a polynomial (6 terms) in the deformation variables β and γ

$$V(\beta,\gamma) = C_2[\alpha\times\alpha]^{(2)} + C_3\left[[\alpha\times\alpha]^{(2)}\times\alpha\right]^{(0)} + C_4\left([\alpha\times\alpha]^{(0)}\right)^2 + C_5\left[[\alpha\times\alpha]^{(2)}\times\alpha\right]^{(0)}[\alpha\times\alpha]^{(2)} + C_6\left(\left[[\alpha\times\alpha]^{(2)}\times\alpha\right]^{(0)}\right)^2 + D_6\left([\alpha\times\alpha]^{(0)}\right)^3 . \quad (4)$$

Using the two intrinsic polar coordinates, the potential operator$\hat{V}(\alpha)$ is written as [8].

$$V(\beta,\gamma) = \frac{1}{\sqrt{5}}C_2\beta^2 - \sqrt{\frac{2}{35}}C_3\beta^3\, cos\, cos\, 3\gamma + \frac{1}{5}C_4\beta^4 - \sqrt{\frac{2}{175}}C_5\beta^5\, cos\, cos\, 3\gamma + \frac{2}{35}C_6\beta^6 cos^2 3\gamma + \frac{1}{5\sqrt{5}}D_6\beta^6 , \quad (5)$$

The B_2, P_3, C_2, C_3, C_4, C_5, C_6, and $D_6$are real numbers which are determined for each isotope to fit the available experimental data. In principle, these parameters can describe a variety of shapes and structures including deformed rotor, spherical vibrator, and shape co-existence [18].

2-2 The nuclear softness formula and the corrected Bohr Mottelson formula

Bohr and Mottelson [2]were described ground band levels within the spin characteristics by using classical Hamiltonian quantization according to the properties of symmetry. For the rotational limit, the low-energy levels are proportional to the angular momentum values as (J (J + 1)). The rotational energy spectrum is written as:

$$E(J) = \left(\frac{\hbar^2}{2I}\right)J(J+1) , \quad (6)$$

Where $\Im$ is the moment of inertia. With increasing the total angular momentum, the contradiction has been found in applying this formula. Brentano *et al.* [19] have been proposed a new formula called a soft-rotor formula (SRF) to investigate the levels of the deformed and translational nuclei. In the SRF , the moment of inertia is linearly dependent on the excitation energy and total angular momentum as follows

$$I_J = I_0(1 + \sigma J + \beta E) \ , \qquad (7)$$

Then the rotational energy spectrum can be given as:

$$E(J) = \left(\frac{\hbar^2}{2I_J}\right) J(J+1) \ , \qquad (8)$$

Using the expansion of the Taylor series to the term $\Im_J$ on the ground state, one can write

$$E(J)=\frac{\hbar^2}{2}\left(\left[\frac{1}{I_0} - \left(\frac{1}{I_J^2}\frac{\partial I_J}{\partial J}\right)\right]_{J=0} + \left[\frac{2}{I_J^3}\left(\frac{\partial I_J}{\partial J}\right)^2 - \frac{1}{I_J^2}\frac{\partial^2 I_J}{\partial J^2}\right]_{J=0}\frac{J^2}{2!}\right) \ , \qquad (9)$$

Morinaga [20] was able to address the change in the moment of inertia with angular momentum J. By defining the increase in moment of inertia due to the change in angular momentum, it is the nuclear softness parameter

$$\sigma_1 = \frac{1}{\Im_0}\frac{\Delta\Im_0}{\Delta J}, \sigma_2 = \frac{1}{2!\Im_0}\frac{\partial^2\Im_0}{\partial J^2}, \sigma_3 = \frac{1}{3!\Im_0}\frac{\partial^3\Im_0}{\partial J^3} \ .$$

As this parameter is considered one of the characteristics of the rigidity nuclei. We can now express equation (9) as follows:

$$E(J)=\frac{\hbar^2 J(J+1)}{2I_0(1+\sigma_{1J})}\left(1 - \frac{\sigma_2 J^2}{(1+\sigma_1 J+\sigma_3 J^2)} - \frac{\sigma_3 J^3}{(1+\sigma_1 J+\sigma_3 J^3} + \cdots\right) \ , \qquad (10)$$

To simplify the formula, with three parameters, so it is written as follows

$$E(J)=\frac{\hbar^2 J(J+1)}{2I_0(1+\sigma_{1J})}\left(1-\frac{\sigma_2 J^2}{(1+\sigma_1 J+\sigma_3 J^2)}\right) , \quad (11)$$

The energy levels of the beta and gamma bands can also be calculated from the Correction formula for the BM-formal (CBMF)[21]:

$$E_{\beta-band}=E_\beta+\left(\frac{\frac{\hbar^2}{2I}}{\sqrt[3]{\frac{A}{N}}}\right)J(J+1)-\frac{\hbar^2}{2I}\frac{1}{\sqrt[3]{Z}E_\gamma}J^2(J+1)^2 , \quad (12)$$

$$E_{\gamma-band}=E_\gamma+\frac{1}{2}\frac{\hbar^2}{2I}J(J+1)+\frac{\hbar^2}{2I}\frac{1}{N}J^2(J+1)^2 , \quad (13)$$

Where E_γ and E_β are represent the energy of the head of γ- and β - band, respectively. A is the mass number, Z is the atomic number and N is the neutrons number respectively. According to the modified soft rotation formula (MSRF) [14] and the experimental data, the constant $(\frac{\hbar^2}{2I})$ can be found from the following

$$E(J)=\frac{\frac{\hbar^2}{2I}J(J+1)}{\left[1+\frac{D/J(J+1)}{1-C/J(J+1)}\right]} , \quad (14)$$

3-Results and discussion

To investigate the energy levels of $^{124\text{-}138}$Ce isotope, the GCM Hamiltonian parameters were adjustable so as to fit as closely the available experimental data. The used values of parameters are listed in Table (1). The obtained results are compared with experimental data[22] as shown in Figures 1-8.The sequence of energy states is well reproduced.

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Table1. The value of GCM Hamiltonian's parameters used in the present calculation. The potential parameters are given in MeV unit. The B_2 and P_3 parameters are given in (10^{-42} MeV S^2) and (10^{-42} MeV^{-1} S^{-2}), respectively.

A	C_2	C_3	C_4	C_5	C_6	D_6	B_2	P_3
124	-210	116	1461	-195	-31222	31227	54	-0.111
126	-190	150	1700	-224	-29567	28345	55	-0.11
128	-165	190	1950	-244	-27456	26312	56	-0.109
130	-125	210	2100	-274	-25689	21689	59	-0.108
132	-95	230	2250	-290	-20765	19876	63	-0.106
134	-66	300	2750	-310	-17565	17876	67	-0.104
136	-30	330	2950	-350	-13123	13645	69	-0.102
138	47	380	3350	-380	-9434	10789	75	0.099

3-1 Energy levels

The ^{124}Ce nucleus has the number of neutrons N = 66 and the energy ratio $R_{4/2}=E4_1/E2_1$equals to 3.16 according to the experimental data. Fig. 1shows the comparison between GCM, NSF, CBMF results and experimental ones [22]. It could be seen that a good agreement in ground state band levels. Experimentally, there is only scarce knowledge of energy spectra. For NSF and CBMF calculation, the energies of β and γ-band head are taken from GCM results. Experimentally result is $E(8_2^+)$ = 2.183 MeV, while in the GCM results is equal to 2.125 MeV. It also observe that the second

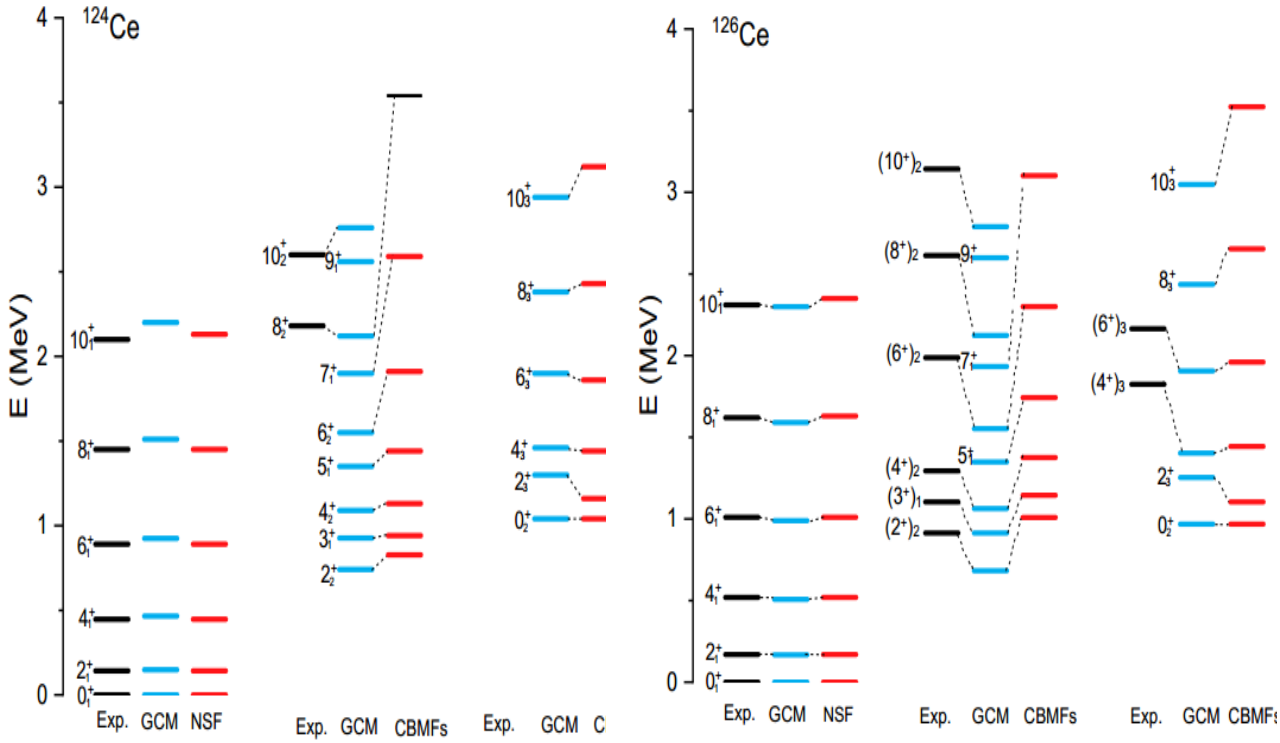

Fig. 1. Comparison between the calculated energy levels and experimental data for ^{124,126}Ce isotopes.

(10^+) state at 2.60 MeV is close to calculated J=10^+_2 at 2.762 MeV. For β- band levels, the GCM results are in good agreement with CBMF calculation up to J = 10^+. On the other hand the agreement is achieved for γ-band members up to J=5^+ level.

The CGM and NSF results are in good agreement with experimental data. For example, the energy values of the first J= (10^+) state are 2.30, 2.35, and 2.31 MeV, respectively. The calculated 0_2^+ β- band head and the 2_3^+ state nearly constant in both GCM results and CBMF calculation. The calculated energies for the 3_1^+state at 0.948 and 1.19 MeV give favorable support to it lie below 4^+_2 state, in agreement with the experimental one. However, a poor agreement was obtained for the state with J>6 in the two methods calculation as shown in Fig.1

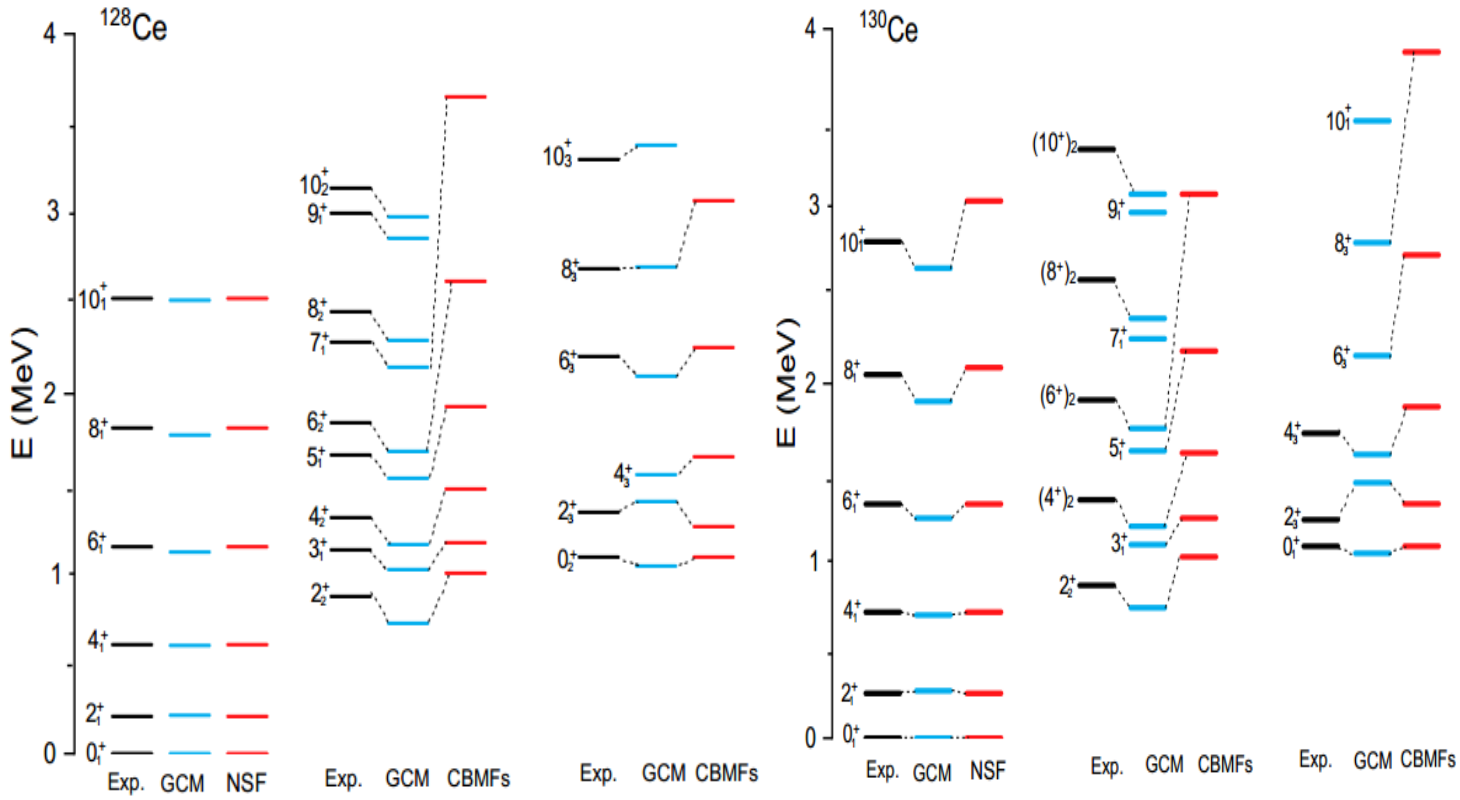

Fig. 2.Same as Fig.1, but for ^{128,130}Ce isotopes.

The calculated results of the energy states in the ^{128}Ce isotope are shown in Fig. 2. All known experimental states ($J \leq 10^{+}$) up to 3.5 MeV are included. For low-lying states, the agreement with the experiment is excellent, especially for ground-state band levels. The first 8^{+} state is predicted by GCM with energy 1.77 MeV, which is in good

agreement with the experimental one at 1.819 MeV. While it is energy equal to 1.85 MeV in the NSF calculation. This isotope has the similar behavior as previous two isotopes, i.e. the position of β and γ band states .

The GCM spectrum of the ^{130}Ce isotope is shown in Fig. 2 together with the experimental one. The calculated energy of the 2_1^+ and 4^+_1 states equal to 0.267 and 0.694 MeV in good agreement with experimental ones equal to 0.253 and 0.710 MeV, respectively. The calculated energy of the 0_2^+ state in this isotope is 0.986 MeV while it is equal to 1.02 MeV in the experimental data. Our GCM calculation predicts 2_3^+ state at 1.38 MeV and this value is close to the experimental one at 1.17 MeV, which has an uncertain spin assignment (2_3^+). For quasi-β and -γ band, it is found that the CBMF calculations have substantial agreement with available experimental data for $J \leq 4^+$ the band members.

Fig. 3 compares the energy of the low-lying states in ^{132,134}Ce nuclei. For the ^{132}Ce isotope, the calculation is successful in reproducing the experimental states. The appearance of the ground state band levels up to $J = 10^+$ at 0.355, 0.848, 1.44 , 2.13 , and 2.88 MeV are close to experimental one at 0.325, 0.858, 1.54 , 2.32 and 3.15 MeV, respectively. We should mention that the energy of the 0_2^+ and 2_2^+ states predicted is in good agreement with the experimental data. For CBMF calculation, the agreement is very good for the first three levels of the quasi- β and - γ band. There are discrepancies in reproducing the other higher energy of those bands. For example, the energy of the 10_γ^+ state equal to 6.91, 3.71, and 3.467 for the CBMF, GCM and experimental results, respectively. The first three excited $J = 2^+$ state are reproduced in the ^{134}Ce isotope as in the Fig. 3. In the GCM calculation, the first 3^+state is higher in energy. Experimentally, the 5^+_1 level is found to be at 2.05 MeV excitation energy. Our GCM and CBMF calculation places this at 2.10 and 3.29 MeV, respectively.

Fig. 4shows the energy scheme of the ^{136,138}Ce isotopes. In the case of the A=136 isotope, the GCM spectrum, a 0_2^+ state appears at 1.50 MeV which has no experimental counterpart. On the other hand, the calculated energy of 2_3^+ state at 2.150 MeV, and this very well reproduced the experimental one at 2.06 MeV. The energy of the second excited 2_2^+ state may belong to a γ-band is equal to 1.11, 1.52, and 1.09 in the GCM, CBMF, and experimental results, respectively. The GCM energy of the first and second J=6^+ state is equal to 1.96 and 2.53 MeV are close to experimental ones at 2.21 and 2.36 MeV respectively. They are calculated at 2.21 MeV (E(6_1^+) in NSF) and 5.833 (E(6_2^+) in CBMF). While the calculated 6_3^+ appears at 3.12 and 6.54 MeV in the GCM and CBMF, respectively. However, it has yet to be seen in the experiment.

From Fig. 4, it can be seen that the quality of the calculated energy spectrum of the A= 138 isotope is lower than it is in the previous isotopes. This can be understood by looking at the energy ratio E(4_1^+)/E(2_1^+) as shown in Table 2. Experimentally equal to 3.14 and 2.3 for A=124 and 138, respectively. This means the shape phase transition from deformed to spherical is observed.

Table 2. Experimental and theoretical energy ratios E(4_1^+)/E(2_1^+) values.

A	124	126	128	130	132	134	136	138
EXP	3.14	3.07	2.95	2.8	2.64	2.56	2.38	2.3
GCM	3.12	3.03	2.82	2.59	2.38	2.24	2.17	2

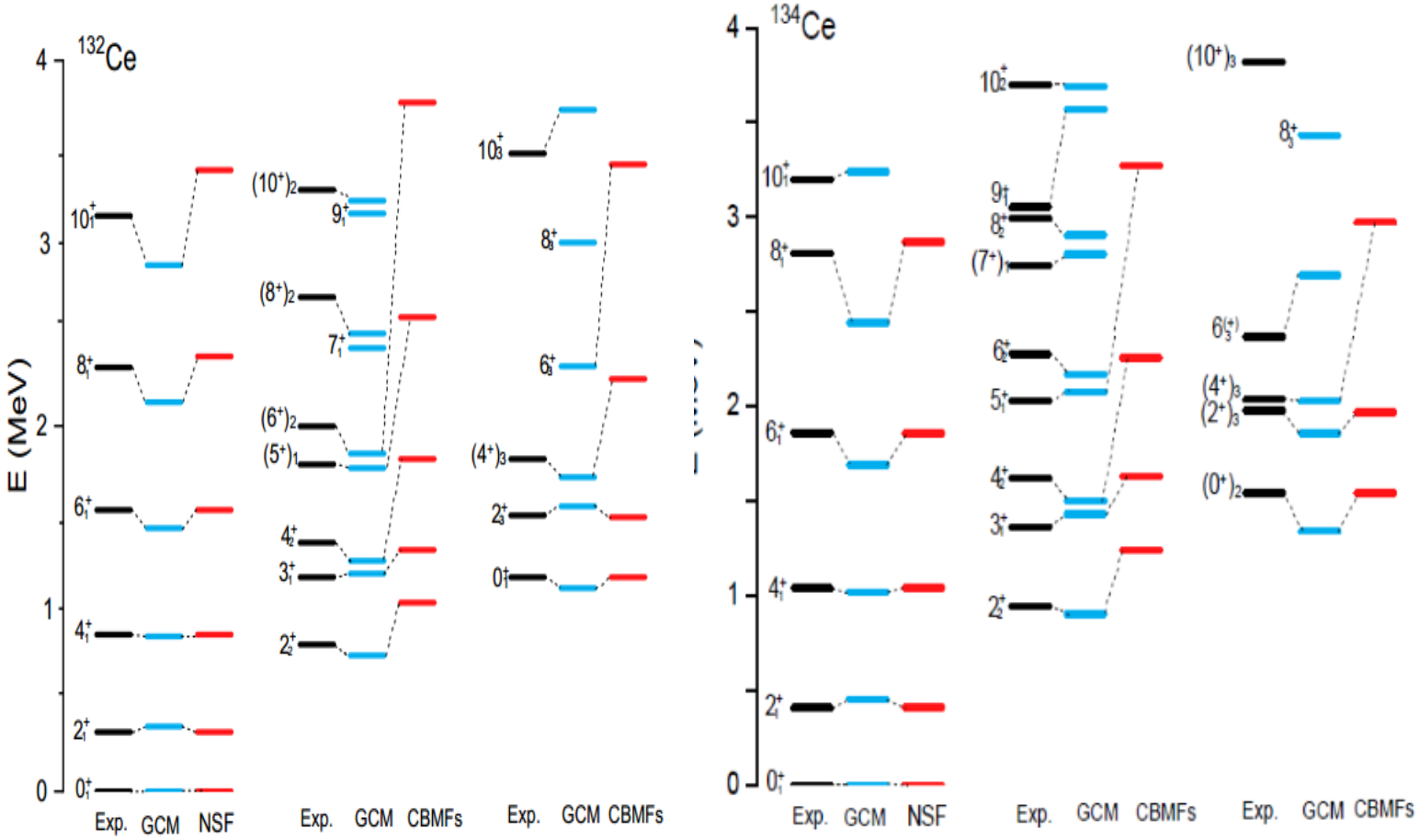

Fig. 3.Same as Fig.1, but for ^{132,134}Ce isotopes.

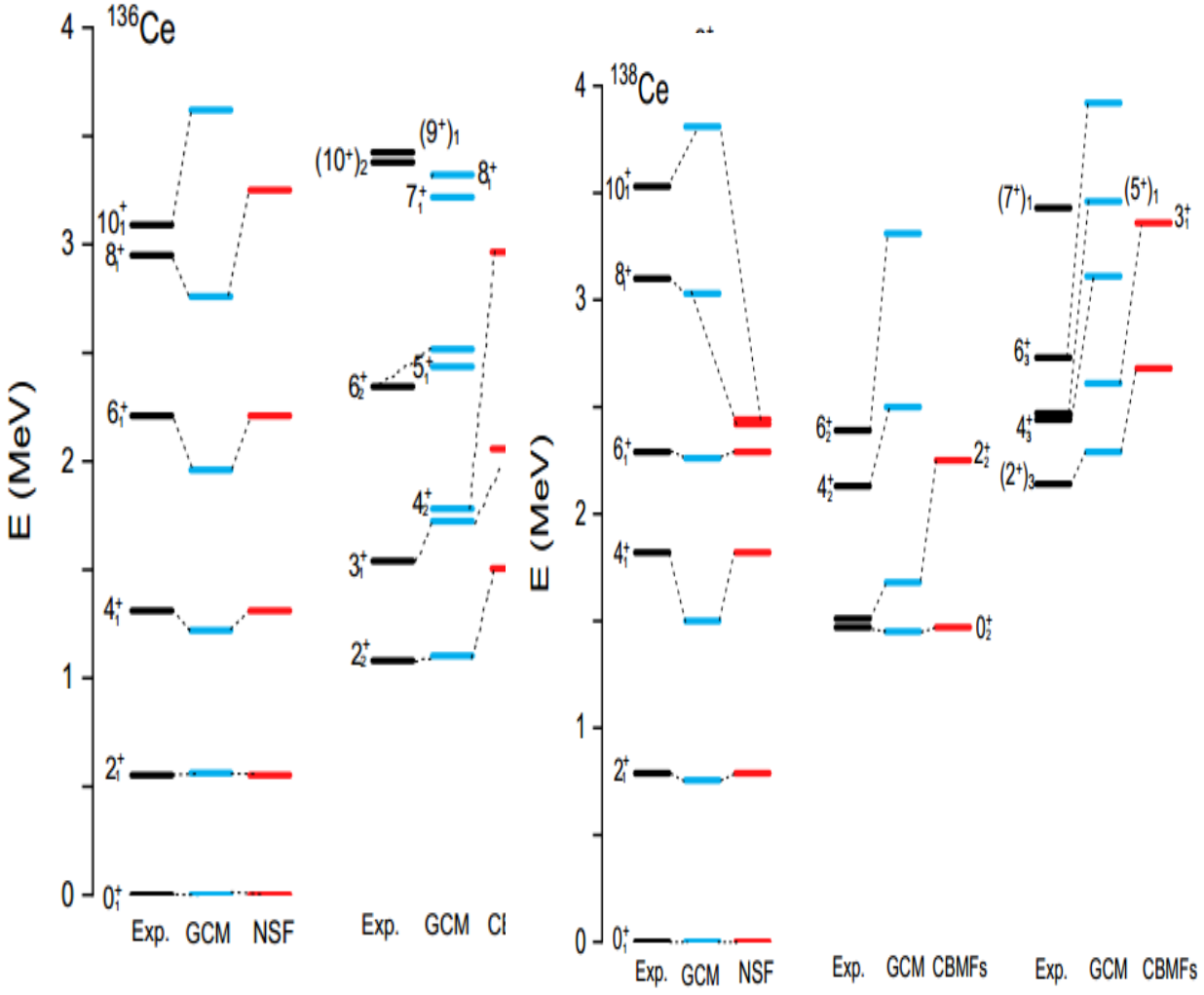

Fig. 4.Same as Fig.1 , but for ^{136,138}Ce isotopes.

3-2- Odd-even energy staggering in the γ-band

The odd even energy index (OES) can be considered as a suitable measure for determining the quality of the γ- band [23]. The odd-even energy staggering S (J) depend on both R (J) and rotor R (J) as follows [24]

$$R(J)=\frac{2(E_J-E_{J-1})}{(E_J-E_{J-2})} \quad , \qquad (15)$$

$$R(J)_{rotor}=\frac{J}{(J-\frac{1}{2})} \quad , \qquad (16)$$

$$S(J) = \left(\frac{R(J)}{R(J)_{rotor}} - 1 \right) , \qquad (17)$$

For example, the S(4) index is calculated from the energies of $(2^+, 3^+, 4^+)$ states and it is a measure to determine the type of gamma. If its value is negative, then it represents soft gamma. If it is positive, then it represents rigid gamma. For the calculated results and experimental data, the S (J) values have been calculate and plotted in the Fig. 5. Clearly, with negative value of S(4)= -1.827 in the ^{124}Ce isotope given soft gamma band. This quantity gradually decreases with increasing of the even angular momentum value. The same behavior can be observed in the ^{124,126}Ce isotopes. It can be seen a good agreement between the theoretical and experimental result for ^{128}Ce isotope. For $^{130\text{-}136}$Ce isotopes, we note the stability of the S(J) curve with the same behavior of the available experimental ones. With positive value of the S(4) and negative of S(5) for ^{138}Ce isotope accompanied by a rapid decrease as a function of angular momentum. That means the transition shape has been happened.

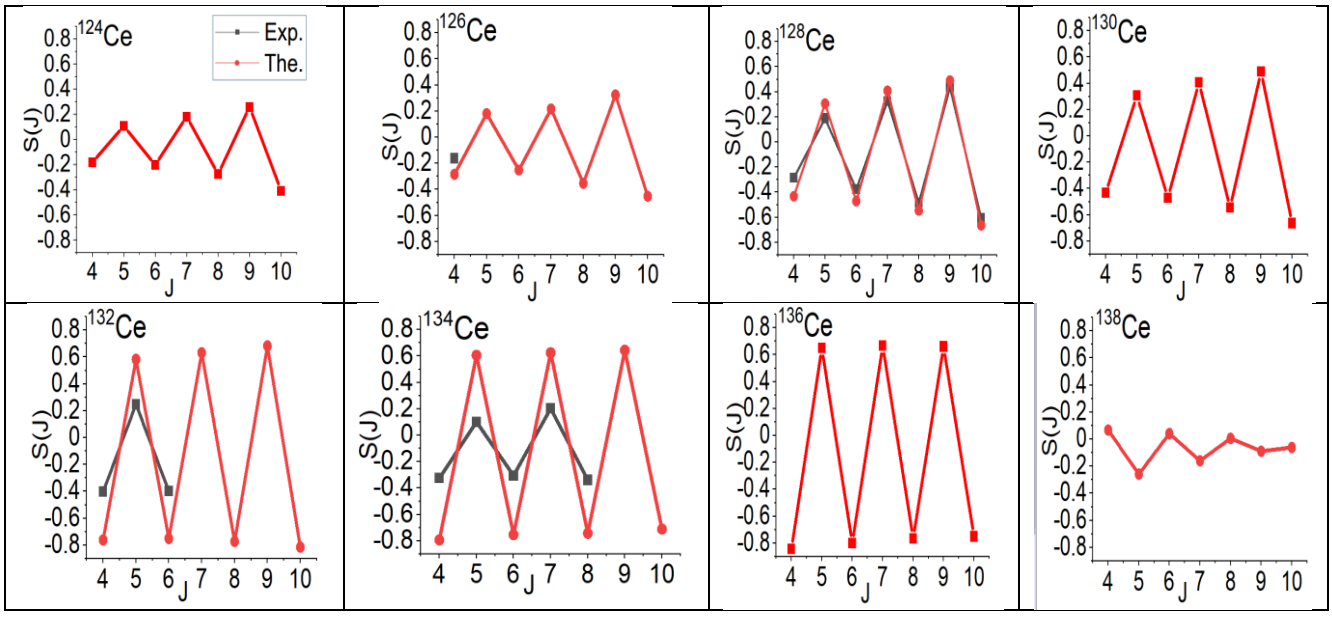

Fig.5. S (J) values as a function of the angular momentum (J) of the $^{124\text{-}138}$Ce isotopes.

3-3- Shape deformation variation with angular momentum

In order to investigate the deformation shape, the single term power law and power index *b(J)* can be used to describe the power spectrum of the ground band [25]

$$E(J) = aJ^b \ , \qquad (18)$$

Where *a* is the inverse moment of inertia. The power index *b (J)* can be correlate with the ratio of energies as

$$R(J) = \frac{E(J^+)}{E(2_1^+)} \ , \qquad (19)$$

And *b (J)* can be calculate as

$$b(J) = \frac{log\ log\left(R(J)\right)}{log\ log\left(\frac{J}{2}\right)} \ , \qquad (20)$$

If the shape of the nucleus is confirmed by increasing the angular momentum, then the stability of *b(J)* can be observed and horizontally with an increase in *J*. In order to investigate the ground state band levels, the power index b(J) values have been plotted as shown in Fig. 6 . For N=66 and 68, the behavior of the b(J) is almost horizontal, this means the stability of the nuclear structure of these two isotopes. For $^{124\text{-}132}$Ceisotopes the b(4^+) are great than 1.0, because of the unusual large gap energy between the first excited state 2_1^+ and ground state. For $^{128\text{-}132}$Ce isotopes with increasing angular momentum J, b(J) decreases specially at J=12^+ . For $^{134\text{-}138}$Ce isotope, the b(4^+) approaches the 1.0 mean these nuclei tend to spherical vibrator. It can be seen the b(J) increases sharply up to 1.0 of 6^+state in the ^{138}Ce isotope.

Fig. 7 shows the average power law index (PL) versus the number of neutrons (N) for the ground band in $^{124\text{-}138}$Ce isotopes. For ^{124}Ce (b_{av} = 1.669 (6)), which means that a large deformation occurs in the nuclear structure. The small error bar reflects the relatively stable curve , and the simple change of ground band structures . Clearly the curve

relatively steep slope with increasing neutrons number. The minimum value equal to 1.011 (22) for ^{138}Ce isotope.

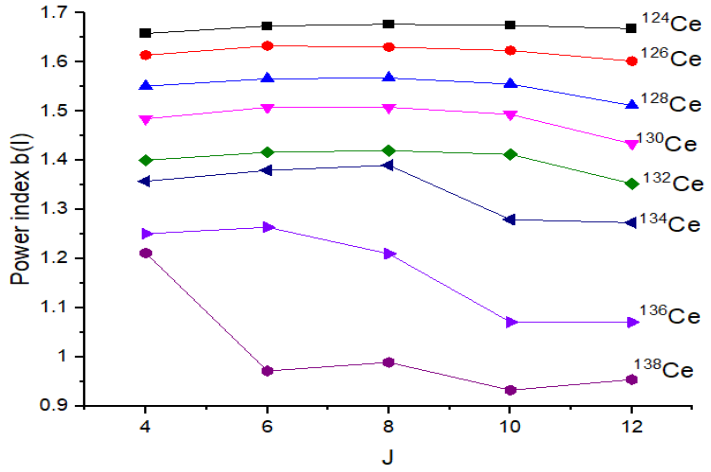

Fig.6. The change of power index b (J) with the angular momentum of the ground state band of the $^{124\text{-}138}$Ce isotopes.

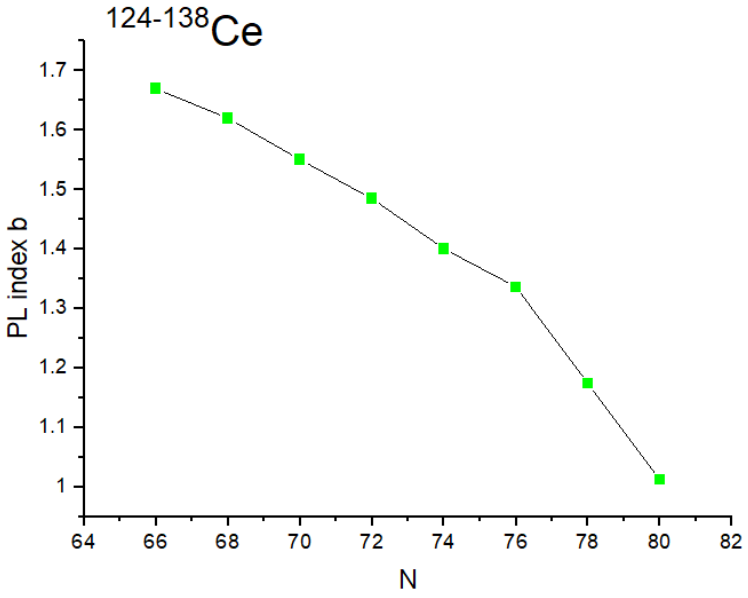

Fig. 7. The average power law index (PL) versus the number of neutrons (N) of the ground band levels in the $^{124\text{-}138}$Ce isotopes

3-4. Electromagnetic transitions

To calculate the electric quadrupole transition probabilities, the quadrupole operator has been written up to second-order in alpha as follows [6]

$$\hat{Q}_{2m} = \frac{3z}{4\pi} R_0 \left(\hat{\alpha}^*_{2m} - \frac{10}{\sqrt{70\pi}} [\ \hat{\alpha} \times \hat{\alpha}\ ]^*_{2m} \right) , \qquad (21)$$

For any collective state $|\xi, JM\rangle$, J and M are the angular momentum and its projection. The ξ represents the summarizing of all other quantum numbers. The B(E2) transition from initial state J_i to final state J_f is given by

$$B\left(E2, \delta_i J_i \rightarrow \delta_f J_f\right) = \frac{1}{2J_i + 1} \sum_{M_i, M_f, m} \left| \langle \square_i J_i M_i \rangle \right|^2$$

$$= \frac{2J_f + 1}{2J_i + 1} \left| \langle \square_f J_f \| Q_2 \| \square_i J_i \rangle \right|^2 , \qquad (22)$$

For the excited states, the quadrupole moment formula becomes:

$$Q(\delta, J) = \sqrt{\frac{16\pi}{5}} \ (J\ 0\ J\ \ -J\ 0\ J\ )\ \langle \delta, JM = J \| \hat{Q}_2 \| \delta, JM = J \rangle \ , \qquad (23)$$

To test the wave function of the states, the electric quadrupole transition probabilities have been calculated. The B(E2) values are listed in Tables 3 and 4. It can be seen from the tables that the B(E2; $2_1^+ \rightarrow 0_1^+$) values are reproduced in their qualitative structure. This probability drops strongly in value for ^{138}Ce isotopes.The investigation of the B(E2) values for the transitions from the J= 0_2^+ state on the basis of the GCM shows that the strongest transition would be expected to occur to the 2_2^+ state. To investigate the position of two phonon states i.e. 0_2^+,2_2^+, and 4_1^+, one can calculate the transition probability of the 2_2^+,4_1^+ to first excited 2_1^+ state. From the Tables , it can be seen that the B(E2; $4_1^+ \rightarrow 2_1^+$) value is larger than B(E2; $2_2^+ \rightarrow 2^+$) value for $^{124\text{-}132}$Ce

isotopes. As expected, the B(E2) of these two transitions of the $^{134-138}$Ce nuclei close together and thus reflect the characteristics of the vibrational nuclei. Such behavior occurs by decay of the 3_1^+ and 4_2^+ sates to the 2_2^+ state.

For all isotopes, the quadrupole moment of the first excited state J= 2^+ has a negative value as shown in Table 5. These values are led to the conclusion of a strongly prolate form for the ground state for all isotopes. From the Table, it can be seen that the quadrupole moment values decreased with increasing the mass number. On the other hand, we found the positive value of Q(2_2^+) state.

Table 3. Experimental and calculated B(E2) (in e^2b^2unit) for $^{124-130}$*Ce isotopes.*

	^{124}Ce		^{126}Ce		^{128}Ce		^{130}Ce	
$J_i \rightarrow J_f$	GCM	Exp.	GCM	Exp.	GCM	Exp.	GCM	Exp.
$2_1^+ \rightarrow 0_1^+$	0.4337		0.3982	0.517(90)	0.3339	0.416(45)	0.2772	0.333(15)
$2_2^+ \rightarrow 2_1^+$	0.0282		0.0412		0.0778		0.1330	
$2_3^+ \rightarrow 0_2^+$	0.2931		0.2740		0.2319		0.2062	
$0_2^+ \rightarrow 2_2^+$	0.1146		0.1561		0.2273		0.2768	
$0_3^+ \rightarrow 2_1^+$	0.0048		0.0073		0.0519		0.1143	
$3_1^+ \rightarrow 2_2^+$	0.7051		0.6284		0.5001		0.4203	
$4_1^+ \rightarrow 2_1^+$	0.6391		0.5940	0.693(63)	0.5101	0.675(15)	0.4395	0.626(75)
$4_2^+ \rightarrow 2_2^+$	0.2684		0.2551		0.2295		0.2109	
$4_3^+ \rightarrow 4_2^+$	0.0866		0.1301		0.2033		0 .230	
$5_1^+ \rightarrow 3_1^+$	0.4269		0.3981		0.3487		0.3148	
$6_1^+ \rightarrow 4_1^+$	0.7324		0.6885	0.03(11)	0.6028	0.712(18)	0.5343	0.716(60)
$6_2^+ \rightarrow 4_2^+$	0.5176		0.4773		0.3984		0.3511	
$7_1^+ \rightarrow 5_1^+$	0.6129		0.5782		0.5086		0.4686	
$8_1^+ \rightarrow 6_1^+$	0.7987		0.7567	0.033(18)	0.6703	0.937(15)	0.6035	1.087(33)

Table 4. Experimental and calculated B(E2) (in e^2b^2 unit) for $^{132\text{-}138}Ce$ isotopes.

	^{132}Ce		^{134}Ce		^{136}Ce		^{138}Ce	
$J_i \rightarrow J_f$	GCM	Exp.	GCM	Exp.	GCM	Exp.	GCM	Exp.
$2_1^+ \rightarrow 0_1^+$	0.204	0.348(26)	0.153	0.195(18)	0.1211	0.162(18)	0.0666	0.079
$2_2^+ \rightarrow 2_1^+$	0.2189		0.2171		0.2042		0.0911	
$2_3^+ \rightarrow 0_2^+$	0.1710		0.1492		0.1163		0.0639	
$0_2^+ \rightarrow 2_2^+$	0.3042		0.2363		0.5635		0.1677	
$0_3^+ \rightarrow 2_1^+$	0.1527		0.1657		0.0990		0.0053	
$3_1^+ \rightarrow 2_2^+$	0.3217		0.2607		0.2195		0.0882	
$4_1^+ \rightarrow 2_1^+$	0.3295	0.386(86)	0.2570	0.146(30)	0.2112	0.210(37)	0.1324	0.0011
$4_2^+ \rightarrow 2_2^+$	0.1818		0.1565		0.1407		0.0974	
$4_3^+ \rightarrow 4_2^+$	0.2281		0.2008		0.1780		0.0351	
$5_1^+ \rightarrow 3_1^+$	0.2496		0.2078		0.1823		0.1106	
$6_1^+ \rightarrow 4_1^+$	0.4083	0.525(300	0.3298	0.052(18)	0.2800	0.000017	0.1931	0.0003
$6_2^+ \rightarrow 4_2^+$	0.2834		0.2445		0.2213		0.1706	
$7_1^+ \rightarrow 5_1^+$	0.3644		0.3069		0.2733		0.1824	
$8_1^+ \rightarrow 6_1^+$	0.4605	0.255(52)	0.3824	0.098(15)	0.3333		0.2488	

Table 5.Quadrupole momentum (in *be* unit) of the low-lying states in $^{124\text{-}138}$Ce isotopes.

I_i^+	^{124}Ce	^{126}Ce	^{128}Ce	^{130}Ce	^{132}Ce	^{134}Ce	^{136}Ce	^{138}Ce
2^+_1	-1.31	-1.25	-1.10	-0.931	-0.580	-0.374	-0.221	-0.371
2^+_2	1.26	1.18	1.01	0 .826	0.494	0.310	0 .172	0.112
2^+_3	-0.237	-0.259	-0.151	-0.104	0 .017	-0.006	0.018	-0.405
4^+_1	-1.66	-1.57	-1.38	-1.18	-0.723	-0.477	-0.287	-0.644
6^+_1	-1.81	-1.71	-1.49	-1.27	-0.719	-0.468	-0.273	-0.863
8^+_1	-1.88	-1.77	-1.52	-1.29	-0.614	-0.384	-0.204	-1.04

3.5.**Potential energy surface**

The shape of the nucleus can be determined by the deformation coefficients (β,γ), since the value of β approaches zero for the spherical nucleus, while it increases for the deformed nucleus, and the deformation shape at $(\gamma = 0^0)$ is a prolate elliptical, and an oblate elliptical when $(\gamma = 60^0)$.The resulting potential energy surface of the $^{124-138}$Ce isotopes are given in Fig. 8. It can be seen that the minimum potential value falls at $\gamma = 0$ (prolate). It is decreases with the increase in the mass number A. the lowest value of potential V (β, γ) at $(\gamma = 30^0,60^0)$ occurs in regions $\beta<\beta_{min}$. For ^{124}Ce isotope, it was observed that the minimum potential value $\gamma = 60^0$ is less than what it is at $\gamma = 30^0$, and it is differs from what it is in other isotopes. The energy difference between (prolate − oblate)$(V_{PO}^{0-60} = [V(\beta,60)] - min[V(\beta,O)])$ is also calculated [9] as shown in Table 6.(.It was found that it is equivalent to (37%) of the potential depth of the ^{124}Ce isotope and then begins to decrease until it reaches (12%) at the ^{136}Ce isotope, while it was equal to (20%) of the potential depth at the ^{138}Ce isotope.

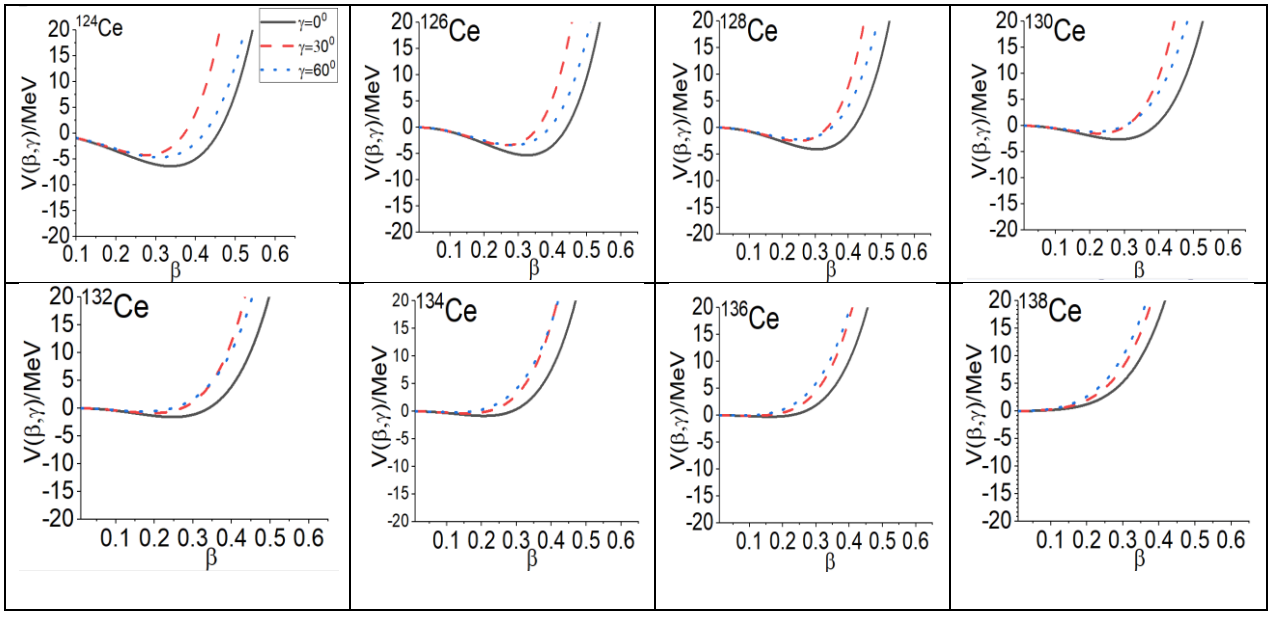

Fig. 8. Potential energy Surface as a function of the parameter β (γ= 0^0, 30^0 and 60^0)of the $^{124\text{-}138}$Ce isotopes

Table 6. The values of the V_{min} , β_{min} and V_{PO}^{0-60} for $^{124-138}$Ce isotopes.

A	124	126	128	130	132	134	136	138
V_{min}	−6.38	−5.323	−4.07	−2.639	−1.581	−0.831	−0.268	0.876
β_{min}	0.34	0.33	0.30	0.28	0.25	0.21	0.16	0.18
V_{PO}^{0-60}	1.72	1.905	1.861	1.482	1.01	0.645	0.221	0.043

4. Conclusions

We have calculated the energy levels of $^{124-138}$Ce isotopes. The obtained results confirm that the shape phase transition from deformed to spherical shape is observed. In addition, the ground state, β-and γ-band are calculated using NSF and CBMF. The investigation of odd-even staggering effect in the γ-band was done. For PES calculation, it is found that the prolate deformation is deeper than the oblate. Finally, there are still some interesting problems that need to be studied, such as the rapid decrease in the B(E2)values from the ground state band levels in the higher Ce-isotopes.

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