

Stability Analysis for Fluid Flow between Two Infinite Parallel Plates II

Mahdi F. Mosa

Abdo M. Ali

Dept. of Mathematics

College of Computer sciences and Mathematics

Mosul University

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الملخص

تمت في هذا البحث دراسة الاستقرارية لنظام يمثل مائعا مثاليا بين لوحين متوازيين لا نهائيين يسخن من الأسفل واعتماد انتقال الحرارة بالتوصيل والحمل والإشعاع. وقد نوقش الموضوع تحت تأثير عدد من المعلمات ذات دلالة فيزيائية ، إذ تبين أن استقرارية النظام تعتمد على تلك المعلمات ، كما تبين أن الازعاج يزداد (ينمو) بزيادة العدد الموجي و استخدمت في إيجاد ذلك طريقة تحليلية تم من خلالها الحصول على صورة واضحة للجريان بتأثير التسخين .

ABSTRACT

A model of fluid flow with heat transfer by conduction , convection and radiation has been discussed for stability with respect to restricted parameters (k, α, r, T^*) which are proportional to: wave numbers, thermal expansion coefficient, combination of many numbers (Re, Pr, Ec, Bo, W, g) and the ratio of walls temperatures, respectively using analytical technique which illustrates that the stability of the system depends on these parameters and the disturbances with a larger wave number, grows faster than that with smaller wave number .A clear picture of the flow is shown in the velocity field $\langle u_2, v_2 \rangle$ tables and figures.

Introduction:

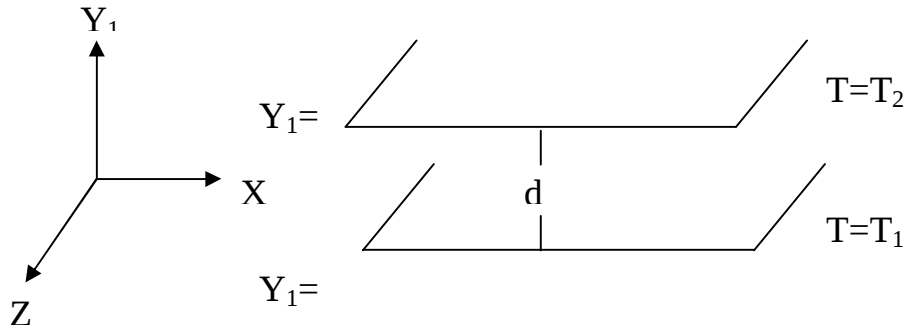
The principle of stability of fluid flow with heat transfer and its applications has been investigated by many authors such as Lorenz [1] and Yorke [4] and others.

In this paper a model of heat transfer by conduction, convection and radiation in a fluid flow between two infinite parallel flat plates has been considered.

The first like model without heating from below was investigated by Logan [2].

Model and Governing Differential Equations:

Consider, an ideal fluid confined between two infinite parallel plates $Y_1=0$, $Y_1=d$ in $X_1 Z_1 Y_1$ space separated by a distance d , and heated from below, which is under the influence of a constant gravitation field g acting in the negative Y_1 direction as shown in the following figure:



Figure(1): fluid flow between two parallel plates.

Using Boussinisque approximation and the optical thick limit and if the temperature differences between the walls and the fluid is small [3] the non- dimensional differential equations (in the new plane $x z y$, where $x=X_1/d$, $z=Z_1/d$, and $y=Y_1/d$) of the disturbance are :

$$\left. \begin{aligned} \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_2}{\partial z} &= 0 & 1 \\ \frac{Du_2}{Dt} &= -\frac{\partial p_2}{\partial x} & 2 \\ \frac{Dv_2}{Dt} &= -\frac{\partial p_2}{\partial y} + a q_2 & 3 \\ \frac{Dw_2}{Dt} &= -\frac{\partial p_2}{\partial z} & 4 \\ \frac{D\theta_2}{Dt} &= R \nabla^2 \theta_2 & 5 \end{aligned} \right\} \quad (1)$$

Where $R = \frac{g}{Pr \cdot Re} + \frac{16}{3} \frac{Ec g}{Bo \cdot w}$, $a = T_1 b$,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

(γ : Specific heat ratio, Pr: prandtl number, Re: Reynold number, Ec: Eckert number, Bo: Boltzman number, w: Bouguer number, and b : thermal expansion coefficient).

with the boundary conditions : $u_2 = v_2 = w_2 = \theta_2 = 0$, $y = 0, 1$

To investigate the system (1) (analytically) for stability or otherwise we attempt to find a solution of the form:

$$\left. \begin{aligned} u_2 &= U(y) e^{at} e^{i(k_1 x + k_2 z)} \\ v_2 &= V(y) e^{at} e^{i(k_1 x + k_2 z)} \\ - & - - \\ - & - - \\ \theta_2 &= \theta(y) e^{at} e^{i(k_1 x + k_2 z)} \end{aligned} \right\} \quad (2)$$

Where k_1, k_2 are the wave numbers and "a" is the speed number ($a = a_1 + ia_2$).

Substituting the form (2) in system (1) and using the linearized theory we have:

$$\left. \begin{aligned} ik_1 U + V' + ik_2 W &= 0 & 1 \\ aV &= -ik_1 P & 2 \\ aV &= -P' + aq & 3 \\ aW &= -ik_2 P & 4 \\ q'' - \frac{(a + Rk^2)}{R}q - \frac{T^*}{R}V &= 0 & 5 \end{aligned} \right\} \quad (3)$$

Where $T^* = (T_2 - T_1)/T_1$, (T_1 : temperature of the lower wall ,
 T_2 : temperature of the upper wall) and $k^2 = k_1^2 + k_2^2$.

From system (3) and by eliminating the pressure function in the third equation using equations (1), (2), (4) and (5) we get:

$$V^{(4)} + L_1 V'' + L_2 V = 0 \quad (4)$$

$$\text{Where: } L_1 = -\left(\frac{a}{R} + 2k^2\right) , \quad L_2 = \left(k^4 + \frac{a}{R}k^2 - \frac{aT^*}{aR}k^2\right)$$

With the boundary conditions:

$$V(0) = V(1) = V'(0) = V'(1) = 0 \quad (5)$$

In order to find a solution satisfying the boundary condition, we must investigate all cases we get from the characteristic equation.

$$m^4 + L_1 m^2 + L_2 = 0 \quad (6)$$

which is algebraic equation of degree four, it has four roots .

Thus:

$$m^2 = \frac{1}{2} \left[-L_1 \pm \sqrt{L_1^2 - 4L_2} \right] \quad (7)$$

For m^2 there are three cases (here “a” is real):

$$1. L_1^2 - 4L_2 < 0 \quad 2. L_1^2 - 4L_2 = 0 \quad 3. L_1^2 - 4L_2 > 0$$

each one has many cases.

By investigating those cases and applying the boundary conditions (5), the nontrivial solution holds:

$$\text{i) } L_1^2 - 4L_2 > 0, -L_1 < 0 \quad \text{and: } |-L_1| = \sqrt{L_1^2 - 4L_2}$$

$$\text{ii) } L_1^2 - 4L_2 > 0, -L_1 < 0 \quad \text{and: } |-L_1| > \sqrt{L_1^2 - 4L_2}$$

In the case (I), the general solution can be written as:

$$V(y) = c_1 + c_2 y + c_3 \cos \alpha_3 y + c_4 \sin \alpha_3 y, \quad (8)$$

where the roots of the characteristic equation (6) are:

$$\lambda_1 = \lambda_2 = 0, \quad \lambda_3 = i\alpha_3, \quad \text{and} \quad \lambda_4 = -i\alpha_3$$

The boundary conditions (5) give the particular solution:

$$V_n = C_n (1 - \cos(2np)y), \quad \text{where } a_3 = 2np \quad (9)$$

In this paper we illustrate the case (ii) only, where the general solution can be written as:

$$V = c_1 \cos a_1 y + c_2 \sin a_1 y + c_3 \cos a_2 y + c_4 \sin a_2 y \quad (10)$$

Solution (10) satisfies the four boundary conditions when

$$\alpha_1 = 2n\pi \quad \text{and} \quad \alpha_2 = 2m\pi \quad (\text{where } m \neq n, m=1,2,\dots$$

and $n=1,2,\dots$), and then the solution takes the form:

$$V_n = c_1 (\cos(2np)y - \cos(2mp)y) + c_2 \left(\sin(2np)y - \frac{n}{m} \sin(2mp)y \right) \quad (11)$$

where c_1 and c_2 are arbitraries such that c_2 depends on c_1 .

V_n is called the eigenfunction.

The other eigenfunctions can be found by using the relations in system (3), so the functions: v_2 , u_2 , w_2 , p_2 , and θ_2 have the following forms:

$$v_2 = V_n e^{at} e^{i(k_1 x + k_2 z)} \quad (1)$$

$$u_2 = i \frac{k_1}{k^2} [V'_n] e^{at} e^{i(k_1 x + k_2 z)} \quad (2)$$

$$w_2 = i \frac{k_2}{k^2} [V'_n] e^{at} e^{i(k_1 x + k_2 z)} \quad (3)$$

$$p_2 = -\frac{a}{k_2} [V'_n] e^{at} e^{i(k_1 x + k_2 z)} \quad (4)$$

$$\theta_2 = -\frac{a}{\alpha} \left[\frac{V''_n}{k^2} + V_n \right] e^{at} e^{i(k_1 x + k_2 z)} \quad (5)$$

(12)

We get a clear picture of the flow pattern by examining a two dimensional disturbance, and the case $n=1$, $m=2$, $k=1$, (x,y) plane with functions (12).where u_2 and v_2 are as follows:

$$u_2 = i(-2p \sin 2py + 4p \sin 4py + 2p \cos 2py - 2p \cos 4py)e^{ix}e^{at}$$

$$v_2 = \left(\cos 2py - \cos 4py + \sin 2py - \frac{1}{2} \sin 4py \right) e^{ix}e^{at}$$

To obtain the real solution, we take the real parts of the previous equations thus:

$$u_2 = -(-2p \sin 2py + 4p \sin 4py + 2p \cos 2py - 2p \cos 4py)e^{at} \sin x$$

$$v_2 = \left(\cos 2py - \cos 4py + \sin 2py - \frac{1}{2} \sin 4py \right) e^{at} \cos x$$

The following table contains the information of the signs of the functions u_2 and v_2 in the intervals $0 < x < 2\pi$ and $0 < y < 1$:

u_2	v_2
$u_2 < 0$ when: 1) $0 < x < \pi$ and $0 < y < 0.25$, $0.55 < y < 0.85$ 2) $\pi < x < 2\pi$ and : $0.25 < y < 0.55$, $0.85 < y < 1$	$v_2 < 0$ when: 1) $0 < x < \frac{p}{2}, \frac{3p}{2} < x < 2p$ and $0.3 < y < 0.7$ 2) $\frac{p}{2} < x < \frac{3p}{2}$ and: $0 < y < 0.3$, $0.7 < y < 1$
$u_2 = 0$ when : $x = 0, \pi, 2\pi$ or $y = 0.25$, $0.55, 0.85, 1$	$v_2 = 0$ when : $x = \frac{p}{2}, \frac{3p}{2}$, or $y = 0, 0.3, 0.7, 1$
$u_2 > 0$ when 1) $0 < x < \pi$ and $0.25 < y < 0.55$, $0.85 < y < 1$ 2) $\pi < x < 2\pi$ and $0 < y < 0.25$, $0.55 < y < 0.85$	$v_2 > 0$ when 1) $0 < x < \frac{p}{2}, \frac{3p}{2} < x < 2p$ and $0 < y < 0.3$, $0.7 < y < 1$ 2) $\frac{p}{2} < x < \frac{3p}{2}$ and $0.3 < y < 0.7$

Table (1): Information about the signs of the functions u_2 and v_2 in the intervals $0 < x < 2\pi$ and $0 < y < 1$.

This information and indications given in the above satiated table have been summarized by an arrow showing the direction of the velocity field $\langle u_2, v_2 \rangle$ in the following figure:

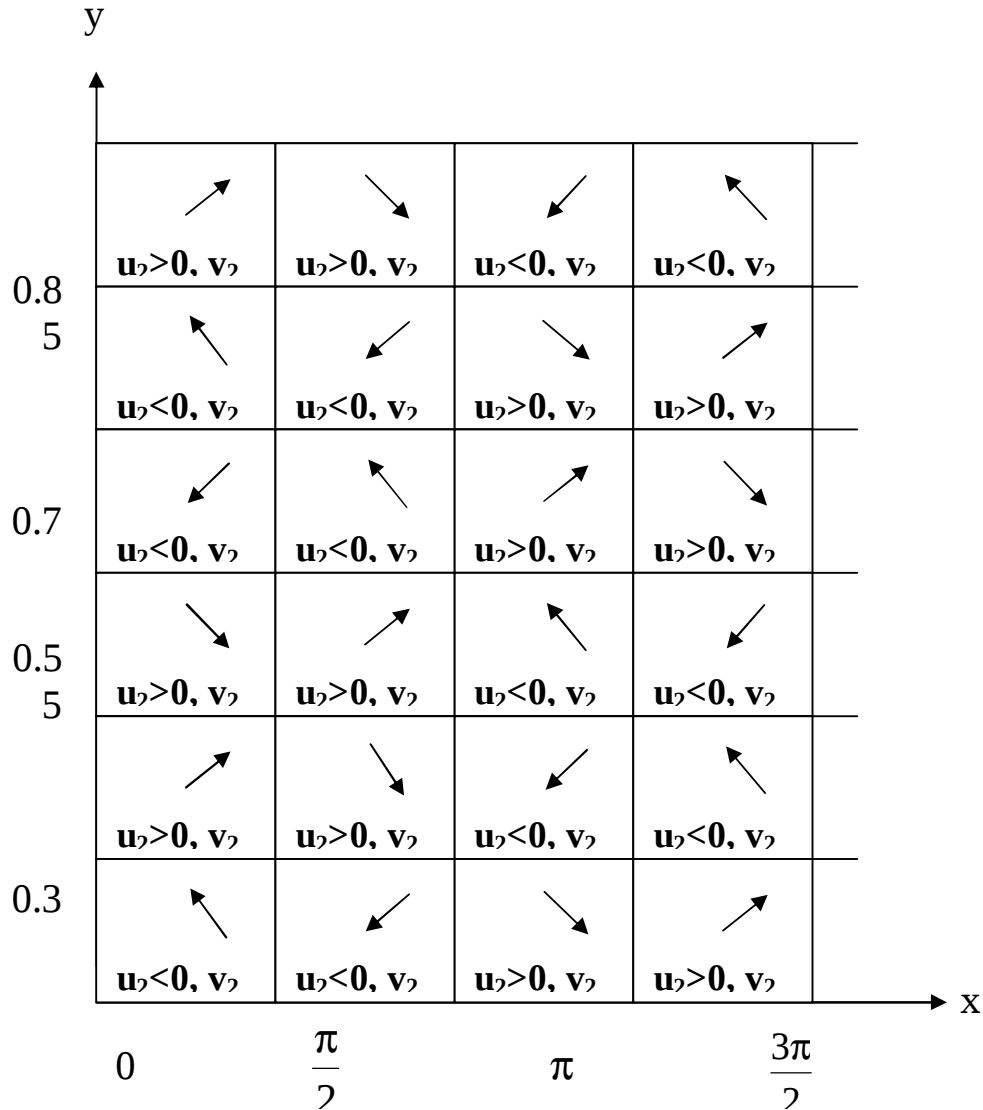


Figure (1) shows the directions of the velocity field $\langle u_2, v_2 \rangle$.

The typical streamlines are shown in the following figure, viewed from the positive z direction:

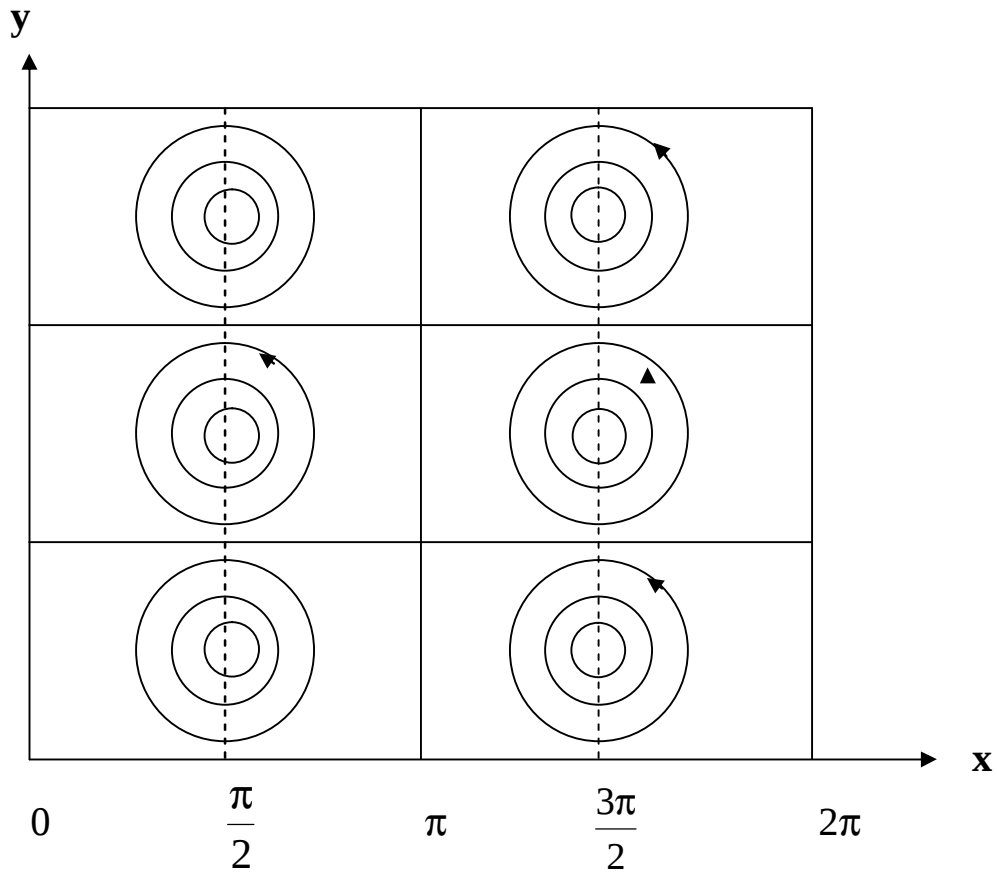


Figure (2) shows the typical streamlines

From solution (9) we obtain $a_3 = 2np$, which gives us:

$$\begin{aligned}
 a_1 &= \frac{1}{3} S_1^{1/3} + 4 S_2 \\
 a_2 &= -\frac{1}{6} S_1^{1/3} - 2 S_2 + \frac{1}{2} i \sqrt{3} \left[\frac{1}{3} S_1 - 4 \frac{n^2 \pi^2 R^2}{S_1^{1/3}} \right] \\
 a_3 &= -\frac{1}{6} S_1^{1/3} - 2 S_2 - \frac{1}{2} i \sqrt{3} \left[\frac{1}{3} S_1 - 4 \frac{n^2 p^2 R^2}{S_1} \right] \dots\dots\dots(13)
 \end{aligned}$$

Where

$$S_1 = -54rTK^2w + 6[-48n^6p^6r^6 + 81r^2T^2k^4w^2]^{1/2}$$

And
$$S_2 = \frac{n^2 p^2 r^2}{S_1}$$

The formulas in (13) give the values of the speed number where the real parts are different in signs so that it is unstable.

Conclusion:

* This analytic method gives a clear picture of the flow, as shown in figures (1) and (2) which shows that the cell pattern repeats itself every 2π units in the x-direction.

* From the eigenfunctions in (9) and (12), for arbitrary k and arbitrary ["n" in (9) or "n,m" in (12)] the cell pattern still appears but with additional cells .

* The growth rate of disturbance with large wave numbers k_1 and k_2 is faster than that with small wave numbers as formula (13) indicates .

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