Heat Transfer Analysis in Air-Cooling of Spherical Food Products

Hameed B. Mahood Akram B. Attar* Shaheed Nasser Huseen**

Division of Physical Sciences and Research
Ministry of Sciences and Technology
Baghdad, Iraq
E-mail: hbmahood@yahoo.com

***,** Faculty of science**
Computer department
Thi_Qar University
***E-mail:akramattar@yahoo.com**

Abstract

The one-dimensional transient heat conduction equation, in spherical coordinates, is solved with convective surface boundary conditions during air-cooling using separation of variable analytical method. The calculation began with the transformation of the spherical coordinate to the rectangular coordinate to simplify the problem. The calculated temperature for spherical food products(apples and potatoes) are compared with measured values whom in the literature, and a good agreement was obtained. Temperature range used. And how far away from freezing.

الخلاصة

تم حل معادلة التوصيل الحراري ذات البعد الواحد والمعتمدة على الزمن وللشكل الكروي باستخدام طريقة فصل المتغيرات التحليلية وبالاستفادة من ظاهرة الانتقال الحراري من السطح الكروي بالحمل كشرط حدودي للمسألة. تم تحويل الاحداثي الكروي الى احداثي المستوي باستخدام متغيرات وسطية لغرض تبسيط المسألة بغية حلها تحليليا .تمت مقارنة درجات الحرارة التي تم الحصول عليها نظريا مع النتائج العملية والنظرية المنشورة عالميا وقد اظهرت تطابقا جيدا.

Introduction

The freezing of food products by individually quick freezing presents some difficulties due to the tenderness and high perishability of these fruits. Therefore, there is a trend to pre cool them in order to increase their firmness, to produce a high quality product. Precooling improves efficiency of the freezing operation and freezer capacity is increased due to shorter dwell times.

For designing a proper air cooling system for a particular foodstuff, prior knowledge is gathered either by performing experiments on the product or by mathematical simulation. When fruits and vegetables are precooled , heat removed by convective heat transfer from the product

surface to the cooling medium during product desiccation produces an additional cooling effect[Ansari et al, 1984].

Heat transfer characteristic during air-cooling are important for a proper design and operation of such systems. Empirical equations to predict surface heat transfers are only valid when heat transfer is by convection.

Hayash et al.(1975) studied the self-freezing of a wet substance, the problem was treated as a heat conduction problem. The solutions of the problem were derived from the analytical procedure of Neumann's solution. Cleland and Earle (1977) investigated, the one –dimensional heat transfer in infinite slabs of freezing food material cooled from both sides. Ansari et al.(1984) solved the one-dimensional heat conduction equation in spherical coordinate by a numerical technique with convective boundary condition from the surface.

Zuritz and Singh (1985) predicted temperature distribution and time- temperature history for freezing food products when subjected to fluctuating storage condition using a finite element computer model. The effects of product properties on temperature prediction were performed. Guemes et al.(1988) studied a heat transfer characteristic during air-precooling of strawberries. The effective surface heat transfer coefficients were determined and Nusselt-Reynolds correlation, which includes the effect of moisture evaporation, was developed.

In the present work a new analytical calculation is proposed using a separation of variable technique to calculate the time- temperature behavior during the air-cooling of food products. Indicates shapes and other influential parameters or restrictions introduced to simplify the problem. Temperature range and how far the values from the freezing temperature, in reference to food products type used.

Analytical approach

The one-dimensional heat conduction equation in the spherical coordinate (r,t) is only given as:

$$\frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} = \frac{1}{\alpha}\frac{\partial U}{\partial t} \qquad \textbf{(1)}$$

or

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left(rU\right) = \frac{1}{\alpha}\frac{\partial U}{\partial t} \qquad \textbf{(2)}$$

To convert the spherical coordinate to the rectangular coordinate, we define a new variable as:

$$T(r,t) = rU(r,t) \qquad \textbf{(3)}$$

Then, Eq (2) is transformed into

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial r^2} \qquad \textbf{(4)}$$

The problem is solved by using the separation of variable technique

Let:

$$T(r,t) = f(t)g(r) \tag{5}$$

Combination of Eq (4) and Eq (5) gives

$$f'(t)g(r) = \alpha f(t)g''(r) \tag{6}$$

Or

$$\frac{f'(t)}{\alpha f(t)} = \frac{g''(r)}{g(r)} = \pm\lambda \tag{7}$$

The problem under study satisfies a negative sign of λ . Then Eq (7) becomes

$$\frac{f'(t)}{\alpha f(t)} = \frac{g''(r)}{g(r)} = -\lambda \tag{8}$$

Now,

$$f'(t) = -\lambda\alpha f(t) \tag{9}$$

and

$$g''(r) = -\lambda g(r) \tag{10}$$

Equation (9) is satisfied by the exponential function as:

$$f(t) = c\exp(-\alpha\lambda^2 t) \tag{11}$$

And Eq (10) is satisfied by sine and cosine functions as:

$$g(r) = A\cos(\lambda r) + B\sin(\lambda r) \tag{12}$$

Substitution equation (11) and equation (12) into equation (5) yield

$$T(\lambda, r) = [A\cos(\lambda r) + B\sin(\lambda r)]\exp(-\alpha\lambda^2 t) \tag{13}$$

The boundary condition is

At **r=0** $\frac{\partial T}{\partial r} = 0$ **(14)**

And at r=R $\frac{\partial T}{\partial r} = -BiT$ **(15)**

Differentiation of Eq (13) and introduction of the boundary condition Eq (14) result B=0 [Carslaw and Jaeger, 1959] , hence Eq(13) becomes:

$$T(r,t) = A\cos(\lambda r)\exp(-\alpha\lambda^2 t) \qquad \textbf{(16)}$$

The boundary condition from Eq (15) [at r=R] can be written as:

$$\frac{\partial T(r,t)}{\partial r} + BiT = 0 \qquad \textbf{(17)}$$

After introduccing Eq (16) we get

$$\psi \tan\psi = BiR \qquad \textbf{(18)}$$

where

$$\psi = \lambda R \qquad \textbf{(19)}$$

The initial condition will be fulfilled if the solution can be developed in finite series [Carslaw and Jaeger, 1959] as:

$$\frac{T(r,t)}{T_o} = \sum_{n=1}^{\infty} A_n \cos(\frac{\psi_n r}{R})\exp(-\alpha\psi_n^2 t / R^2) \qquad \textbf{(20)}$$

Carslaw and Jaeger, 1959 gave the value of A_n as:

$$A_n = \frac{4\sin\psi_n}{(2\psi_n + \sin(2\psi_n))} \qquad \textbf{(21)}$$

Table IV in Carslaw and Jaeger ,1959 indicate that for BiR < 0.1 the second root ψ_2 of Eq(18) is greater than ten times the first root ψ_1 and thus, the series is rapidly converging. In this range A_1 is also very close to unity. With decreasing BiR , the coefficient A_1 tends to each unity and the higher A_n coefficients tend to each zero . According to Eq (20) at r=0 and t=0 the temperature ratio is determined by the sum of the A_n coefficient only. Since the initial temperature ratio should be equal to unity. With growing time t, convergence improves because of the second and subsequent terms are more suppressed than the first one. [Botros et al. 1989]. Therefore the calculation of the transient temperature using the first term of Eq (20) indication needs to be stressed.

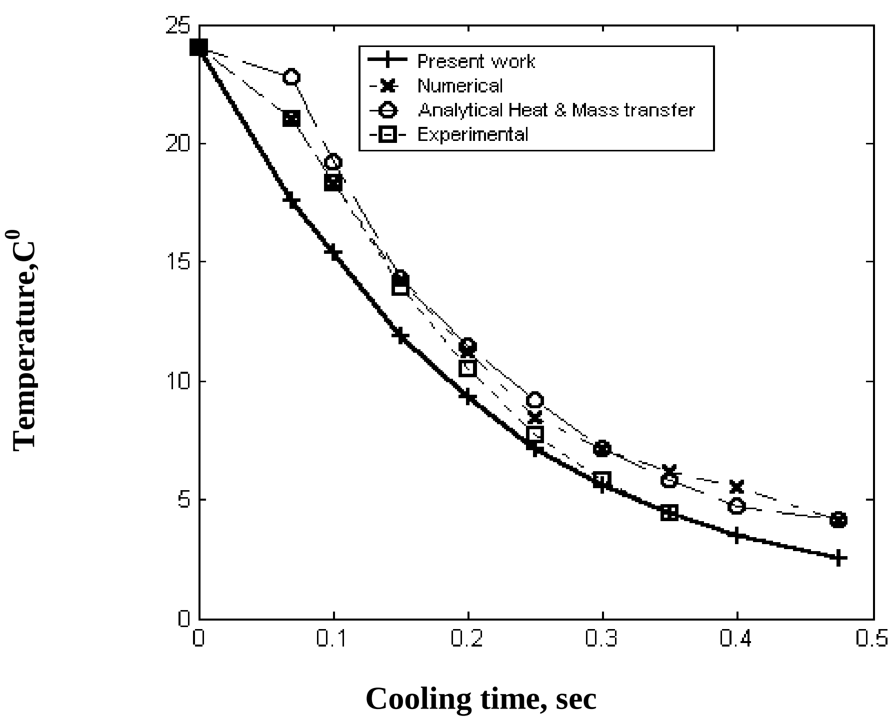
$$\frac{T(r,t)}{T_O} = \exp\left(\frac{-\alpha\psi^2 t}{R^2}\right) \qquad \textbf{(22)}$$

Results and discussion

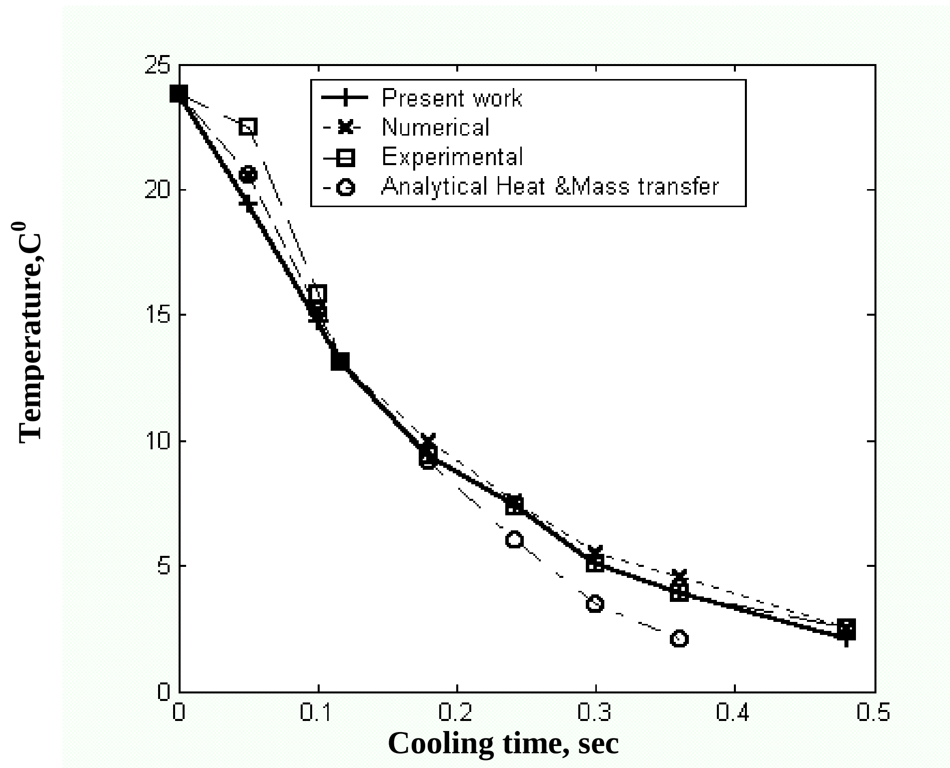
The time – temperature analytical solution for two spherical apples and two spherical potatoes was presented. It was found that the present solution with only heat transfer has lower values than the experimental and other models available in literature. The reason may be that this model assumes no mass transfer and neglect the effect of moisture evaporation phenomena which is a complex one. It depends on a number of parameters like the cooling air temperature, its humidity , water content of products , internal mass diffusion , skin effect and the temperature gradients.[Ansai et al. , 1984].

The results Figs (1,2,3 and 4) showed that at the beginning of the cooling of the spherical food products the present model predicted surface temperature which are slightly lower than the experimental values. The reason may be due to that during this period the product loses their surface moisture, and the evaporative cooling effect becomes very important. Another reason for this difference is that the true surface area of apples and potatoes might be higher than the sphere which is assumed in the analysis. The last reason of the difference may be mainly attributed to the additional cooling due to water evaporation from the food products.

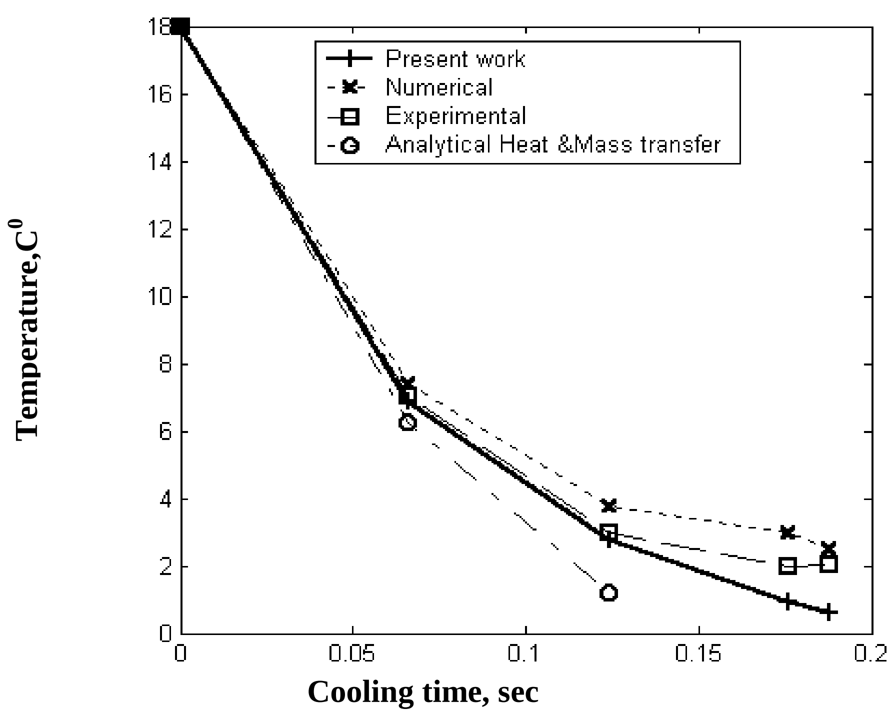
All Figures indicated a good agreement between the presented model results and the available results given by Ansai et al. (1984). Since, for Fig(1) the average divergence between the present analytical model and the experimental results is 21.6% ,for Fig(2) it equals to 5.49% , for Fig(3) it equals to 20.3% and finally for Fig(4) it equals to 30%.



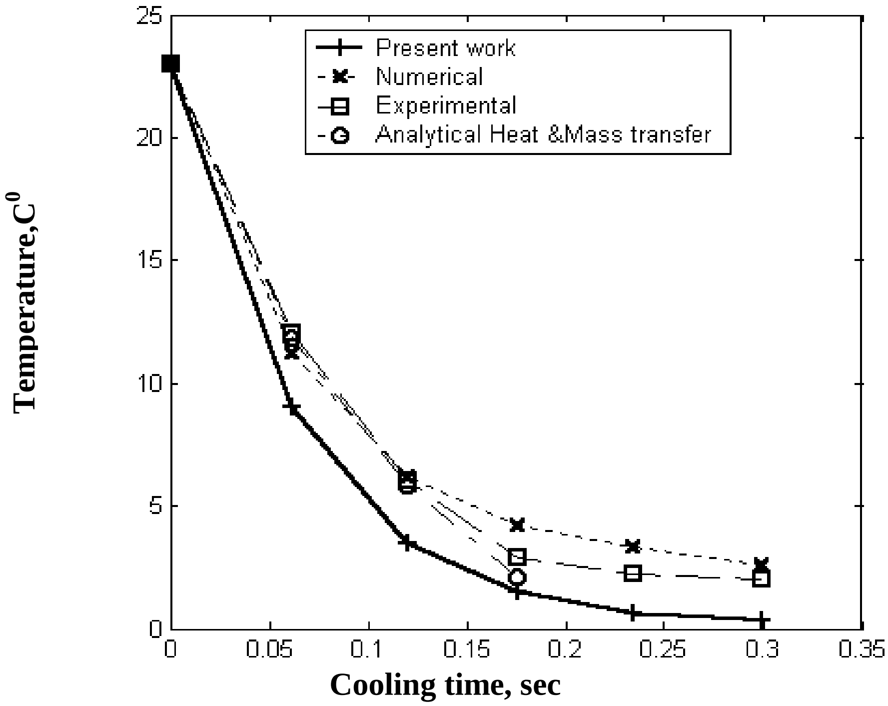
Fig(1): Time –Temperature plot of apple-1, (R=0.0388m, r=0.014m and $\rho = 807 kg/m^3$)



Fig(2): Time –Temperature plot of apple-2, (R=0.0354m, r=0.014m and $\rho = 807 kg/m^3$)



Fig(3): Time –Temperature plot of potato-1, (R=0.022m, r=0.00561m and $\rho = 1150 kg/m^3$)



Fig(4): Time –Temperature plot of potato-2, (R=0.028m, r=0.0701m and $\rho = 1136 kg/m^3$)

Conclusions

An analytical solution of heat transfer for spherical food products(apples and potatoes) like a sphere as assumed gives the following conclusions :

1- **The assumption of uniform sphere shape of apples and potatoes gives good results.**
2- **The effect of moisture appearing in the earlier stage of the cooling process and then dispersed.**
3- **The symmetrical boundary conditions are sufficient to predict the temperature-time behavior of the cooling food products.**

References

Anasri, F.A. , Charan, V. and Varma, H.K. (1984)" Heat and mass transfer analysis in air-cooling of spherical food products", Int. J. Refrigeration, Vol.7, No. 3, 194-197.

Botros, K.K., Jungowski, W.M. and Weiss, M.H. (1989)" Models and methods of simulating gas pipeline blowdown", Cand. J. Chem. Eng., Vol.67, 529-539.

Carslaw, H.S. and Jaeger, J.C. (1959)" Conduction of heat in solids", Second Eddition, Oxford Press, London.

Cleland, D.R., Pirovani, M.E. and Di Pentima, J.H. (1988)" A comparison of analytical and numerical methods of predicting the freezing time of foods", J. Food Sci., Vol. 42, No. 5, 169-173.

Hayashi, Y., Komori, T. and Katayama, K. (1975)" Analytical and experimental investigation of self-freezing", J. Heat Transfer , August, 312-325.

Zuritz, C.A. and Singh, R.P. (1985)"Modeling temperature fluctuations in stored frozen foods", Int. J. Refrig., Vol. 8, 289- 293

Nomenclature

A	**integration constant**
B	**integration constant**
c	**integration constant**
g	**variable a function of space only**
f	**variable a function of time only**
Bi	**Biot number**
R	**radius of sphere, m**
r	**space coordinate**
T	**temperature in plane coordinate**
t	**time**
U	**temperature in spherical coordinate**

Greek symbol

α	**thermal conductivity**
ρ	**density**
ψ	**arbitrary constant appearing in Eq(18)**
λ	**arbitrary constant appearing in Eq(7)**

Subscript

n	**summation index**