Modified SEE Integral Transform and its Applications

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Abstract

In this paper , we propose a new definition of the modified SEE (Sadiq – Emad – Eman) integral transform Sa {f(t)} for a piece-wise continuous function of exponential order further reduces to simple SEE transform for a = e, where a≠1 and a > 0 . Also we prove basic results of this modified SEE integral transform. Also, we introduce simple examples illustrate the solution of ordinary differential equations by modified SEE integral transform.

Keywords: SEE integral transform , convolution , iverse SEE integral transform , integral transformation , modified integral transform.

1. Introduction

Basic concept of SEE transform the SEE (Sadiq – Emad – Eman) integral transform defined for function of exponential order we consider function in the set a defined by :

$$A=[f(t): \exists M, L1, L2 > 0\ |f(t)| < Me\ Lj|t|\ , \text{if } t \in (-1)x[0,\infty)].$$

For a given function in the set A , the m must be finite number , L1 and L2 may be finite or infinite.

SEE integral transformation denoted by the operator S , defined by the integral equation:

$$S\ \{f(t)\} = \frac{1}{v} n \int_{t=0}^{\infty} e\text{-}vt\ f(t)\ dt = T(v). \qquad n \in z\ , L1 \le V \le L2\ , t \ge 0$$

The variable v in this integral transform is used to factor the vanable t in the argument of the function f , [1 , 2].

Table (1) , [1 , 2].

S.N	Name of property	Mathematical form
1	Linearity	S{af1(t)+bf2(t)}=aS{f1(t)}+bS{f2(t)} Where a,b are constants.
2	Change of scale	If S {f(t)}=T(v) 0, then S{f(at)}= $\frac{1}{a}$n+1 T($\frac{v}{a}$).
3	Shifting	S{eatf(t)}= $\frac{(v-a)}{v}$ T (v-a)
4	convolution	S {f1(t)*f2(t)}= vn S{f1(t)} . S{f2(t)}
5	First derivative	S{y′(t)}= $\frac{-1}{v}$n y(0)+vT(v).
6	Second derivative	S{y″(t)}= $\frac{-y'(0)}{v}$n - $\frac{y(0)}{v}$n-1 + v2 T(v).
7	Mth derivative	S{y(m)(t)}= $\frac{-y(0)}{v}$n - $\frac{y(0)}{v}$n-1 - ,,, - $\frac{y(0)}{v}$n-m+1 +vm T(v)

Table (2) , [1 , 2].

S.N	F(t)	S{f(t)}= T(v)
1	1	$\frac{1}{v}$n+1
2	T	$\frac{1}{v}$n+2
3	t2	$\frac{2}{v}$n+3
4	tm , mϵN	$\frac{m!}{v}$n+m+1
5	tm , m>-1	$\frac{}{v}$n+m+1
6	eat ,a≡constant	$\frac{1}{v}$n(v-a)
7	Sin(at)	$\frac{a}{v}$n(v2+a2)
8	Cos(at)	$\frac{v}{v}$n(v2+a2)
9	Sin h(at)	$\frac{a}{v}$n(v2-a2)
10	Cos h(at)	$\frac{v}{v}$n(v2-a2)

11	t f(t)	$\frac{-n}{v}$ T(v) - $\frac{d}{dv}$ T(v)

2. Definition of modified SEE transformation:

The modified SEE transform for a function f(t) which is a piece – wise continuous and of exponential order is defined by the integral equation :

Sa {f(t)}= $\frac{1}{v}$n $\int_{t=0}^{\infty} a$-vt f(t) dt = Ta(v) …. (3,1)

a϶(o ,∞) ┃{1} , n϶Ƶ , t≥0 , L1< v < L2 where L1 and L2 >0 .

modified SEE integral transform denoted by the operator Sa{.} .

Provided that the integral equation in eq.(3,1) exists.

For a=e ,the modified SEE transform become simple SEE integral transform.

2.1. linear property of modified SEE transform:

If g(t) and h(t) are two function of t whose modified SEE transform exists ,then for any contant α and ß are contants :

Sa{α g(t)+ ß h(t) }= α Sa{g(t)}+ ß Sa{h(t)}=αT1a(v)+ßT2a(v).

Proof :

Sa{α g(t)+ß h(t)}

= $\frac{1}{v}$n$\int_{t=0}^{\infty} a$-vt(αg(t)+ßh(t))dt

=$\frac{α}{v}$n $\int_{t=0}^{\infty} a$-vt g(t)dt+ $\frac{ß}{v}$n$\int_{t=0}^{\infty} a$-vt h(t)dt

= α Sa{g(t)}+ ß Sa{h(t)}

which, on using eq (3.1), we get desired

Proposition(3.1): The following properties are valid for modified SEE transform :

If f(t)=1 , then Sα{1}=$\frac{1}{v}$n+1 , (v>0)

If f(t)=t , then Sα{t}=$\frac{1}{v}$n+2 , (v>0) Log(

If f(t)=tm , then Sα{tm}=$\frac{\Gamma(m+1)}{v}$m+n+1 (Log(a))^2 , (v>0)

If f(t)=ebt , then Sα{ebt}=$\frac{1}{v}$n $\int_{t=0}^{\infty} a$-vt ebt dt =$\frac{1\,(\text{Log(a)})^m}{v^n\,(vlog(a)-b)}$

where (v log (a)> ┃b┃).

If f(t) = sin(bt) ,

then Sα{sin(bt)}=$\frac{1}{v}$n$\int_{t=0}^{\infty} a$-vt sin(bt)dt

=$\frac{1}{v}$n$\int_{0}^{\infty} e$-vt log(a) . $\frac{e^{bt}-e^{-bt}}{2i}$ dt =$\frac{1}{2iv}$n $\int_{0}^{\infty}$[e-t(vlog(a)-b)- e-t(vlog(a)+b)]dt

$\frac{b}{v^{n[v^2(\log(a))^2+b^2]}}$, (v log(a)>0)

If f(t)= cos(bt), then

Sα{cos (bt)}=$\frac{v}{v^{n}[v^2(\log(a))^2+b^2]}$, (v log(a)>0)

If f(t)= sinh(at), then

Sα{ sinh(at)}=$\frac{1}{v}$n $\int_0^\infty a$-vt [$\frac{e^{bt}-e^{-bt}}{2}$] dt

= $\frac{b}{v^{n}[v^2(\log(a))^2-b^2]}$ where (v log(a)>│b│)

If f(t)= cosh(bt), then

Sα{ cosh(bt)}=$\frac{v}{v^{n}[v^2(\log(a))^2-b^2]}$ (v log(a)>│b│) .

2.2. Unit step function

Now, we present a method to find the solution of certain initial value problem with discontinuous forcing function. For example, an extreme force acting on a mechanical system of equations or a voltage applied to an electrical circuit can be turned off after a certain period of time. Therefore we discuss the case of function having jump type discontinuity at t=k.

U(t-k)= $\begin{cases} 0 & x < k \\ 1 & x \geq k \end{cases}$ (3.2)

Which, on taking the modified SEE transform

Sa{u(t-k)}= $\frac{1}{v}$n $\int_{t=0}^\infty a$-vt u(t-k)dt

Now, in view of (3.3), we get :

Sa{u(t-k)}= $\frac{1}{v}$n $\int_{t=0}^\infty a$-vt(0) dt + $\frac{1}{v}$n $\int_{t=0}^\infty a$-vt(1)dt = $\frac{1}{v}$n $\int_{t=0}^\infty a$-vtdt ,

{u(t-k)}= $\frac{1}{v}$n $\int_{t=0}^\infty e$-vt log(a) dt , {u(t-k)}= $\frac{1}{v}$n [$\frac{-1}{v\log(a)}$ (e-vt log(a)) $\Big|_0^\infty$,

= $\frac{a^{-vk}}{v^{n+1}\log(a)}$.

2.3. First shifting property:

If Sα{f(t)}= Ta (v;a) , then a>0
(a≠1), we have

Sα{ebt f(t)}= $\frac{(v\log(a)-b)^n}{v^n}$ T (v log(a)).

Proof :

By using eq (3.1) , we write

Sα{ebt f(t)}= $\frac{1}{v^n}\int_{t=0}^\infty a$-vt ebt f(t)dt ,

= $\frac{1}{v^n}\int_{t=0}^\infty e$-vt log(a)+bt f(t)dt ,

=$\frac{1}{v^n}\int_{t=0}^\infty e$-t(v log(a)-b) f(t)dt,

$=\frac{1}{v^n(v\log(a)-b)}$n (v log(a)-b)n $\int_{t=0}^{\infty} e$-t(v log(a)-b) f(t)dt,
$= \frac{(v\log(a)-b)^n}{v^n}$.

2.4. Second shifting property:

Sα{f(t-b)}= a-vb Sα{f(t)}
Proof: with the definition of modified SEE transform in eq .(3.1), we can write:
Sα{f(t-b)}=$\frac{1}{v^n}\int_{t=0}^{\infty} e$-vt log(a) f(t-b)dt

By putting Ʈ=t-b , dƮ=dt
T=Ʈ+b , therefore:
Sα{f(t-b)}=$\frac{1}{v^n}\int_{0}^{\infty} e$-vƮ log(a) f(Ʈ)dt ,
= a-vb Sα{f(t)}.
2.5. Convolution property:

Sa { f2(t) * f2(t) } = Vn Sa { f2(t)} Sa {f2(t)}
Proof. Clear

3. Simple applications of modified SEE transform of ordinary differential equations:

The following examples illustrated the use of the modified SEE transform in solving certain initial value problems described ordinary differential equations.
Example (4.1) Consider the differential equation
y'+y=0, y(0)=1.
Solution: Taking modified SEE transform to above equation, we get:

4. Results and conclusion:

In this paper, our motive is to introduce a modified form of SEE integral transform that is more general and works on a large domain ($\boldsymbol{a} \in (\mathbf{0}, \infty \setminus \{\mathbf{1}\})$. The result obtained by the modified SEE transformation in potential problems of applied mathematics will be more precise and accurate.

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