

**استخدام البرمجة الضبابية لحل مشكلة النقل ثلاثي
الابعاد دراسة تطبيقية في الشركة العامة لتسويق
المنتجات البترولية**

رقية ياسر عبد الامير

كلية الامام الكاظم / العراق / ميسان

منى شاكر سلمان

الجامعة التقنية الوسطى / معهد تقني الصويرة / العراق

عذراء كامل هرموش

الجامعة التقنية الوسطى / الكلية التقنية الادارية – بغداد

Using fuzzy programming to solve the three-dimensional transportation trouble (A practical study in the General Company for Marketing Petroleum Products)

استخدام البرمجة الضبابية لحل مشكلة النقل ثلاثي الابعاد دراسة تطبيقية في الشركة العامة لتسويق المنتجات البترولية

Ruqayah Yassir Abdul Ameer *

Imam al-Kadhum college /Bagdad, Iraq

Muna Shaker Salman

Middle Technical University / Technical Institute-

Suwaira / Bagdad, Iraq

Athraa Kamel Harmoosh

Middle Technical University / Administrative

Technical College Bagdad, Iraq

رقية ياسر عبد الامير *

كلية الامام الكاظم / العراق / ميسان

منى شاكر سلمان

الجامعة التقنية الوسطى / معهد تقني الصورة / العراق

عذراء كامل هرموش

الجامعة التقنية الوسطى / الكلية التقنية الادارية – بغداد

تاريخ النشر: 2024/03/01

Received: 12/08/2023

تاريخ القبول: 2023/09/10

Accepted: 10/09/2023

تاريخ الاستلام: 2023/08/12

Published: 01/03/2024

المستخلص:

تعد مشكلة النقل احدى أهم الاساليب الرياضية التي تساعد في الاختيار المناسب لنقل البضائع من مصادر العرض إلى مراكز الطلب بأقل تكلفة ممكنة . حيث تم في هذه الدراسة حل مشكلة النقل ثلاثية الأبعاد باستخدام البرمجة الضبابية، وتضمن الضبابية في الحدود العليا والدنيا لدالة الكلفة، ولإزالة الضبابية تم استخدام دالة الانتماء الرباعية ، وقد أثبتت هذا الدراسة فعاليتها في تخفيض كلفة شحن المشتقات النفطية (النفط والكارز) من مستودعات التخزين إلى محطات الوقود ، وقد بينت النتائج أن دالة الانتماء تعطي نتائج أفضل فيما يتعلق بكلفة النقل، حيث وجد أن كلفة النقل هي (82,105,398) مليون دينار، ومن شروط ذلك ينص النموذج على وجوب حصر كلفة النقل بين الحدين الأعلى والأدنى، وقد أثبت النموذج كفاءته في تحقيق هذا الشرط، أي أن تكلفة النقل كانت في حدود 128,136,630 < كلفة < 179,411,860 .

الكلمات المفتاحية : مسائل النقل ، نموذج النقل ثلاثي الابعاد ، البرمجة الضبابية ، دالة الانتماء ، الحدود العليا والدنيا ، برنامج QM

Abstract

The transportation trouble is considered one of the most crucial mathematical methods that help in making the appropriate choice to goods transportation from supply sources to demand centers, at the lowest cost possible. In this study, the trouble of three-dimensional transportation was solved using fuzzy programming, where the fuzzy lies in the upper and lower limits, of the cost function, and to remove the fuzzy, the hyperbola belonging function has been used. This approach has demonstrated its effectiveness in lowering the cost of shipping oil derivatives (oil, kerosene) from storage facilities to gas stations. The results have shown that the hyperbola belonging function gives better

results with regard to the transportation cost, as it was found that the transportation cost is (82,105,398) million dinars, and among the conditions for this the model states that the transportation cost must be confined between the upper and lower limits, and the model has proven its efficiency in achieving this condition, meaning that the transportation cost was within the range $(128,136,630 \leq \text{Cost} \leq 179,411,860)$

Keywords: Transportation problem, Three dimensional transportation model, fuzzy programming, membership function, Upper and Lower bound, QM software.

1.Introduction:

Due to the importance of the transportation trouble as one of the mathematical methods for solving a specific type of linear programming trouble concerned with transporting a specific commodity from multiple sources to multiple demand locations at the lowest cost, many researchers have focused on studying transportation troubles and finding optimal solutions for them, whether in terms of cost or time, using methods of solving the optimum for the transportation process, and this is called the classic model (homogeneous), in which the commodity is homogeneous of one type, the means of transportation is also of one type, and the quantities offered and required are also.

Where the transportation process has become a necessary and prominent role in economic life and is considered an important activity through its contribution to providing transportation services for the import and export of various types of heterogeneous commodities, and this has increased the complexity of the mathematical model of the transportation trouble, so the three-dimensional model (Three Dimensional) indicates that a medium is the transformation is different or the type of goods transportation is different, where the approach model of the three-dimensional transformations trouble was constructing to transportation heterogeneous goods and the mathematical model of the transportation trouble was solved.

2- Research trouble:

All oil derivatives are of great importance from an economic point of view for any country, and therefore because of their direct and daily connection with the lives of citizens, and the high costs of transportation oil derivatives from storage warehouses to fuel filling stations using tank cars causes great waste from an economic point of view and imposes additional costs and burdens on the state. In this research, we reduce the cost of transportation petroleum-based goods, from the storage stores of Baghdad Governorate, to some important fuel fueling locations on both banks of Karkh and Rusafa.

3- Research objective:

The research aims to building a mathematical model to solve the trouble of three-dimensional transportation to solve the trouble of transportation heterogeneous goods or multiple different modes of transportation in order to find the

optimal decision at the lowest cost while transportation goods from supply sites to demand sites, and also to reduce the hazy transportation cost of the oil products distribution company.

4- Previous studies:

The researcher (Bakhayt Abdul-Gabbar Khaddar, 2016,p. 45) published in 2016 a research on solving the four-dimensional transportation model using the artificial intelligence algorithm (PSO), where this research included building a four-dimensional transfer model that dealt with the provider, the recipient, the type of means of transportation, and the multiplicity of the transportation material that is not homogeneous, and this model is complex, so the special methods are unable to solve the model, so the researcher resorted to using the artificial intelligence algorithm (PSO) and comparing it with the genetic algorithm (GA), where it was found that the (PSO) the algorithm is more efficient than the Genetic Algorithm (GA), as it gives a set of optimal solutions and at a good time.

The researcher (Suhad, Faisal Abboud,p. 395-410) presented a research where the mathematical model of the three-dimensional transportation trouble was built, in which the transportation of goods is heterogeneous, and the linear programming technique (Simplex) was used to solving the trouble of transformations the three food products (rice, oil, paste) from the warehouses to the requesting areas in Baghdad/Karkh, Al-Rusafa. This methodology has demonstrated its effectiveness in lowering the three items' overall transportation costs (33.747.425) million dinars compared of the company's overall cost (310,116.59\$), which is approx. (38,764,573.75) million dinars. Also, this model Achieves profits of (40137.19\$) , which is around (55.017.148.7) million dinars.

The researcher presented in 2019 (ELIODOR CONSTANTINESCU) research on the three-dimensional marine transportation model. Where it was discovered in this research that the classic transportation trouble is not a good model for marine transfer, so, suggest applying the three-dimensional transportation model in maritime activity, and this model is considered a solution to such difficult troubles, and when the daily operating costs of ships are thousands of dollars, then the scheduling process is used to achieve large profits this model is considered more realistic in real troubles, so it is necessary to develop multi-dimensional transportation models and apply them in maritime transportation to reduce costs and provide them as capital for maritime transportation companies.

The researchers (GUPTA, Kavita; ARORA, Ritu, 2021,p. 121–137) conducted a research in 2021 to solve the trouble of finite three-dimensional transportation. The finite transportation trouble is explained by defining the parallel transportation trouble. The results showed parity between the two troubles. The proposed model was applied to the Public Distribution System in North Delhi and LINGO 17.0 software was used to solve the application.

The researcher (Kadhim, 2014,P.3) presented linear models multiple objectives blurry as all of the functions of the target and transactional variables (costs , time) numbers blurry triangular used function ranks fortified to convert numbers to natural formula and then build a mathematical model and solved using the method of goal programming and method of weights parametric .

Was reached to develop models for the transportation of four multi-purpose product of two gasoline and two gas oil (kerosene) is achieved by reducing the fuzzy cost and time.

In this research got up (Jeter,2018) a multi-objective transportation problem was formulated with mixed constraints to find the optimal solution, and the fuzzy approach of the multi-objective transportation model was applied, where there are three objectives to reduce costs to a minimum, which are transportation costs, administrative costs, and lost costs (representing unintentional product damage fees). Three forms of membership functions, which are the fuzzy linear membership function, the fuzzy exponential function, and the fuzzy trigonometric function. Where the proposed model was used in the General Company for Grain Processing to reduce transportation costs to a minimum and to find the best plan for transporting the product according to the restrictions imposed on the model.

5- The concept of triple transportation: (ARORA S.R., 2004, P. 83-97)

In the classic (binary) transportation trouble, one commodity is transportation from all sources to all requesting parties, and the goal is to determine the quantities of (homogeneous) commodities that are transportation through all methods so that the total cost of transportation is minimized. But this does not exist in practice, since businessmen have resorted to trading in different types of products to increase the profits of the establishment.

The binary transportation model has been improved to become a more accurate model, which is the three-dimensional model the triple transformations trouble occurs, when transportation heterogeneous units of products from source i to destination j , in addition to the form of transportation used or the type of product, assuming it is K .

This model has become a kind of complexity due to the large number of restrictions in it, so the private methods are unable to solve this model, so we resort to the general method (Simplex method).

6- The mathematical model for the three-dimensional transportation trouble: (Faysal, Muhammad, Bashir, 2019, p.13)

The idea is based on converting the entire transportation trouble into an objective function and constraints. Following is the model:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p C_{ijk} X_{ijk} \quad \dots (1)$$

S.to

$$\sum_{i=1}^m X_{ijk} = A_{jk} \quad , j = 1, 2 \dots n, k = 1, 2 \dots p \quad \dots (2)$$

$$\sum_{j=1}^n X_{ijk} = B_{ki} \quad , i = 1, 2 \dots m, k = 1, 2 \dots p \quad \dots (3)$$

$$\sum_{k=1}^p X_{ijk} = E_{ij} \quad , i = 1, 2 \dots m, j = 1, 2 \dots n \quad \dots (4)$$

The budget constraints are as follows:

$$\sum_{k=1}^p A_{jk} = \sum_{k=1}^p B_{ki} \quad , k = 1, 2 \dots p$$

$$\sum_{i=1}^m B_{ki} = \sum_{i=1}^m E_{ij} \quad , i = 1, 2 \dots m$$

$$\sum_{j=1}^n E_{ij} = \sum_{j=1}^n A_{jk} \quad , j = 1, 2 \dots n$$

$$\sum_{j=1}^n \sum_{k=1}^p A_{jk} = \sum_{k=1}^p \sum_{i=1}^m B_{ki} = \sum_{i=1}^m \sum_{j=1}^n E_{ij}$$

Since this model includes m of Sources, n of Destination, and K of Types, in addition to that, we assume the following:

Xijk: represents K quantities of goods transportation from Source i to Location j.

Cijk: represents the variable cost per unit of K goods transformations from Source i to Location j.

Ajk: represents the total quantity of k goods demanded by site j from all sources i.

Bki: represents the total quantity of k available items that source i offers to all j locations.

Eij: represents the total quantity of all types of goods offered from source i to j locations.

7- The concept of fuzzy programming: (K.balasubramanian1 ,Dr. S. Subramanian,2018,p. 1523-1534)

The researcher Zadeh created the theory of fuzzy groups, which is very flexible in describing a wide variety of realistic cases involving a wide range of life phenomena. One of the most well-known definitions of the fuzzy group is that of Zadeh in 1965, who defines the fuzzy group as classes of elements with a continuous degree of affiliation, and that this group was distinguished by the distinctive membership function that gave each element a degr. The membership function, which represents one of the members of the arranged pair in the fuzzy group, is crucial in the construction of fuzzy groups. Knowing each member of the fuzzy group's level of affiliation and the breadth of their connections to the group is seen as a measurement. Two common functions to determine affiliation were proposed by Zadeh. The design of the membership function is influenced by both the group's and the individual's nature.

8- Hyperbola belonging function: (M. ZANGIABADI AND H. R. MALEKI, 2013, p. 61-74)

A-The Hyperbolic The following is a definition of belonging function:

$$\mu_r^H(Z_r(x)) = \begin{cases} 0 & \text{if } Z_r \leq L_r. \\ \frac{1}{2} + \frac{1}{2} \tanh \left(\frac{U_r + L_r}{2} - Z_r(x) a_r \right) & \text{if } L_r. \leq Z_r \leq U_r \\ 0 & \text{if } U_r \leq U_r. \end{cases} \dots (5)$$

whereas:

$$\alpha_r = \frac{6}{U_r - L_r} \dots (6)$$

Hyperbola belonging function $\mu_r^H[Z_r(x)]$

(Upper Limit) : U_r

(Lower Limit) : L_r

B- The fuzzy transportation trouble's mathematical model (hyperboloid belonging function)

Max = λ

S.To

$$\lambda \leq \frac{1}{2} + \frac{1}{2} \tanh \left[\alpha_r \left(\frac{U_r + L_r}{2} - Z_r \right) \right]$$

$$\sum_{i=1}^m X_{ijk} = A_{jk} \quad , j=1,2,\dots,n \quad , k=1,2,\dots,p$$

$$\sum_{j=1}^n X_{ijk} = B_{ik} \quad , i=1,2,\dots,m \quad , k=1,2,\dots,p$$

$$\sum_{k=1}^p X_{ijk} = E_{ij} \quad , i=1,2,\dots,m \quad , j=1,2,\dots,n$$

$$\sum_{k=1}^p B_{ki} \quad , k = 1,2 \dots p \quad = \sum_{k=1}^p A_{jk}$$

$$\sum_{i=1}^m E_{ij} \quad , i = 1,2 \dots m \quad = \sum_{i=1}^m B_{ki}$$

$$\sum_{j=1}^n A_{jk} \quad , j = 1,2 \dots n \quad = \sum_{j=1}^n E_{ij}$$

$$\sum_{j=1}^n \sum_{k=1}^p A_{jk} = \sum_{k=1}^p \sum_{i=1}^m B_{ki} = \sum_{i=1}^m \sum_{j=1}^n E_{ij}$$

$$X_{ij} \geq 0$$

$$0 \leq \lambda \leq 1$$

The above model can be converted into a linear model by performing mathematical steps as follows:

$$\left\{ \lambda \leq \frac{1}{2} + \frac{1}{2} \tanh \left[\alpha_r \left(\frac{U_r + L_r}{2} - Z_r \right) \right] \right\} * 2$$

$$2\lambda \leq 1 + \tanh \left[\alpha_r \left(\frac{U_r + L_r}{2} - Z_r \right) \right]$$

$$\left\{ 2\lambda - 1 \leq \tanh \left[\alpha_r \left(\frac{U_r + L_r}{2} - Z_r \right) \right] \right\} * \tanh^{-1}$$

$$\tanh^{-1} (2\lambda - 1) \leq \alpha_r \left(\frac{U_r + L_r}{2} - Z_r \right)$$

$$\tanh^{-1} (2\lambda - 1) \leq \alpha_r \left(\frac{U_r + L_r}{2} \right) - \alpha_r Z_r$$

$$\tanh^{-1} (2\lambda - 1) + \alpha_r Z_r \leq \frac{\alpha_r}{2} (U_r + L_r)$$

So the mathematical model of the hyperbolic membership function becomes as follows:

$$\text{Max} = X_{mn+1}$$

S.To

$$\left. \begin{aligned} \alpha_r Z_{r(x)} + X_{mn+1} &\leq \frac{\alpha_r}{2} (U_r + L_r) \\ \sum_{i=1}^m X_{ijk} &= A_{jk} \quad , j=1,2,\dots,n \quad , k=1,2,\dots,p \\ \sum_{j=1}^n X_{ijk} &= B_{ik} \quad , i=1,2,\dots,m \quad , k=1,2,\dots,p \\ \sum_{k=1}^p X_{ijk} &= E_{ij} \quad , i=1,2,\dots,m \quad , j=1,2,\dots,n \\ \sum_{k=1}^p A_{jk} &= \sum_{k=1}^p B_{ki} \quad , k = 1,2 \dots p \\ \sum_{i=1}^m B_{ki} &= \sum_{i=1}^m E_{ij} \quad , i = 1,2 \dots m \\ \sum_{j=1}^n E_{ij} &= \sum_{j=1}^n A_{jk} \quad , j = 1,2 \dots n \end{aligned} \right\} \quad \text{Model (2)}$$

$$\sum_{j=1}^n \sum_{k=1}^p A_{jk} = \sum_{k=1}^p \sum_{i=1}^m B_{ki} = \sum_{i=1}^m \sum_{j=1}^n E_{ij}$$

$$X_{ij}, X_{mn+1} \geq 0$$

$$0 \leq \lambda \leq 1$$

$$X_{mn+1} = \tanh^{-1}(2\lambda - 1)$$

9 - Solution algorithm:

- 1- Solving the transportation trouble (transportation cost).
- 2- Extracting Lower and Upper), i.e. solving the transportation trouble (Min-Max).
- 3- Building a mathematical model for the fuzzy transportation trouble with respect to the hyperbola belonging function.

10- Case Study:

This chapter includes the application of the presented decision-making methods represented by using linear programming ((Simplex) to solve the linear model of the three-dimensional transportation trouble (warehouse, station, type of product) and applying it to products (kerosene, oil) transported by one transportation, which is (load tanks of 36 Thousand Liters) from the three Stores (Al-Dora, Al-Karkh, Al-Rusafa) to the fuel filling stations In Baghdad. in my side of Al-Karkh and Al-Rusafa.

11- Data description:

First: The data includes three stores for storing petroleum products (kerosene, oil) for the Baghdad governorate subsidiary to the General Company for the distribution of Petroleum Products. The following table No. (1) exhibits the names of the stores and their Capacity:

Table (1) exhibits the absorptive capacity (M^3) of warehouses (B_{ki}).

Al-Rusafa	Al-Karkh	Al-Dawra	$\begin{matrix} i \\ k \end{matrix}$
13702.75	10000	14700	Kerosene
7582.25	9870	7898	Oil

Second: Through personal interviews with some individuals with expertise the Distribution Authority in Baghdad, the names of the important fueling locations on each side of Karkh and Rusafa, which numbered twelve, were obtained, which were determined according to the quantity of the monthly demand for them. Table No. (2) below displays the stations' names and its monthly demand products (kerosene, oil):

Table (2) exhibits the monthly quantity of demand (M^3) for stations (A_{jk}).

Oil	Kerosene	$\begin{matrix} K \\ J \end{matrix}$	N
-----	----------	--------------------------------------	---

2836	5932.75	Al-Qanaa	1
2716	2256	Al-Mashtal	2
3690.5	2542	Al-Jabha	3
2525	4163	Al-Bunuk	4
2044	2507	Al-Sumud	5
1759.75	2271	Al-Madayin	6
1680	2467	Al-Taaji	7
1250	4504	Al-Nursian	8
1487	2629	Al-Nuwr	9
2172	2348	Al-Dunya	10
1650	3225	Al-Shurtat2	11
1540	3558	Sector77	12

Third: General distribution of petroleum products company has a dedicated committee to measurement the kilometers distances between its warehouses and gas stations. The kilometers between warehouses and stations are displayed in table No. 3 below:

Table (3) exhibits the distances (km) between stations and warehouses.

Al-Rusafa	Al-Karkh	Al-Dawra		N
22	58	17	Al-Qanaa	1
19	59	23	Al-Mashtal	2
27	55	14	Al-Jabha	3
36	72	31	Al-Bunuk	4
41.5	65	32	Al-Sumud	5
51	77	36	Al-Madayin	6
81	78	47	Al-Taaji	7
59.5	48.5	22.5	Al-Nursian	8
78	66	41	Al-Nuwr	9
42	37	12	Al-Dunya	10
59	56	25	Al-Shurtat2	11

108	106	66	Sector77	12
-----	-----	----	----------	----

Fourth: The cost transportation oil products from storage to fuel fuel filling stations according to the distance between them, has been measured in units (JD per m³), depending on the equation used in the general a company for the distribution of petroleum Products was relied upon based on the following equation:

Transportation cost (in dinars) = load (m³) * distance (km) * transportation price in dinars

Where petroleum products (kerosene, oil) differ from each other in terms of the specific density (weight) of the products, where kerosene occupies the first place in the specific density because of its heavy weight because it is a mixture between oil (fat) and oil, and oil comes after it in the second place, and this is what we mean by it not homogeneity (Non-homogeneous) in the three-dimensional transportation, where the third variable (dimension) is the type of product, and the difference between the products is the specific weight, and each of these products has its own transportation price that differs from the other, as the transportation price of the kerosene product per cubic meter is 52.250)) the following table No. (4) shows the table of costs of transportation kerosene from warehouses to stations:

Table (4) exhibits the costs of transporting kerosene (C_{ij1})

Al-Rusafa	Al-Karkh	Al-Dawra		N
1149.5	3030.5	888.25*	Al-Qanaa	1
992.75	3082.75	1201.75	Al-Mashtal	2
1410.75	2873.75	731.5	Al-Jabha	3
1881	3762	1619.75	Al-Bunuk	4
2168.375	3396.25	1672	Al-Sumud	5
2664.75	4023.25	1881	Al-Madayin	6
4232.25	4075.5	2455.75	Al-Taaji	7
3108.875	2534.125	1175.625	Al-Nursian	8
4075.5	3448.5	2142.25	Al-Nuwr	9
2194.5	1933.25	627	Al-Dunya	10
3082.75	2926	1306.25	Al-Shurtat2	11
5643	5538.5	3448.5	Sector77	12

*/ Distance * Transportation Price $\rightarrow$ * 17 52.250 = 888.25

Regarding the cost of transportation white oil per cubic meter, it is (52), and therefore the cost is variable for all sources and stations requesting, and this is one of the most important characteristics of the heterogeneous tripartite transfer.

Table No. (5) below shows the costs of transportation white oil from warehouses to stations:

Al-Rusafa	Al-Karkh	Al-Dawra		N
1144	3016	884*	Al-Qanaa	1
988	3068	1196	Al-Mashtal	2
1404	2860	728	Al-Jabha	3
1872	3744	1612	Al-Bunuk	4
2158	3380	1664	Al-Sumud	5
2652	4004	1872	Al-Madayin	6
4212	4056	2444	Al-Taaji	7
3094	2522	1170	Al-Nursian	8
4056	3432	2132	Al-Nuwr	9
2184	1924	624	Al-Dunya	10
3068	2912	1300	Al-Shurtat2	11
5616	5512	3432	Sector77	12

*/ distance * transportation price $\rightarrow 17 * 52 = 884$

Fifth: Table No. (6) represents the total quantities of products from the warehouse to the station.

Table (6) represents the total quantities (E_{ji}) of products (kerosene, oil).

Al-Rusafa	Al-Karkh	Al-Dawra	$\begin{matrix} i \\ J \end{matrix}$	N
2334.25	3368.5	3066	Al-Qanaa	1
1693	886	2393	Al-Mashtal	2
1808	2316.5	2108	Al-Jabha	3
1664.5	2049	2974.5	Al-Bunuk	4
2437.75	1137.75	975.5	Al-Sumud	5
2306.75	1010.5	713.5	Al-Madayin	6
2336.75	1036.75	773.5	Al-Taaji	7
2738.5	1438.5	1577	Al-Nursian	8
1329	2229	558	Al-Nuwr	9
1280	860	2380	Al-Dunya	10

1500	950	2425	Al-Shurtat2	11
1274.5	2574.5	1249	Sector77	12

12- Practical application:

The first step:

Solving the transportation matrix (the cost matrix) using the approximate Vogel method and finding the lower and upper limits, where the results were as follows: (Min cost=128,136,630, Max cost=179,411,860)

The second step:

Removing blurring, i.e. applying the hyperbola belonging function to the cost matrix, as the follows:

$$\frac{1}{2} + \frac{1}{2} \tanh \left[\alpha_r \left(\frac{U_r + L_r}{2} - Z_r \right) \right] \geq \lambda$$

The above hyperbola function can be converted into a linear function for ease of solution, as follows:

$$\alpha_r = \frac{6}{U_r - L_r}$$

$$\alpha_1 = \frac{6}{179,411,860 - 128,136,630} = \frac{6}{51,275,230}$$

$$\frac{1}{2} + \frac{1}{2} \tanh \left[\frac{6}{51,275,230} * \left(\frac{179,411,860 + 128,136,630}{2} - Z_1 \right) \right] \geq \lambda$$

The third step:

Building a mathematical model for the fuzzy three-dimensional transfer trouble (hyperbola):

$$\text{Max} = X_{37}, X_{37} = \tanh^{-1}(2\lambda - 1)$$

S.to Capacity limitations of the three warehouses for kerosene product.

$$X_{(1,1),1} + X_{(1,2),1} + X_{(1,3),1} + X_{(1,4),1} + X_{(1,5),1} + X_{(1,6),1} + X_{(1,7),1} + X_{(1,8),1} + X_{(1,9),1} + X_{(1,10),1} +$$

$$X_{(1,11),1} + X_{(1,12),1} = 14700$$

$$X_{(2,1),1} + X_{(2,2),1} + X_{(2,3),1} + X_{(2,4),1} + X_{(2,5),1} + X_{(2,6),1} + X_{(2,7),1} + X_{(2,8),1} + X_{(2,9),1} + X_{(2,10),1} +$$

$$X_{(2,11),1} + X_{(2,12),1} = 10000 X_{(3,1),1} + X_{(3,2),1} + X_{(3,3),1} + X_{(3,4),1} + X_{(3,5),1} + X_{(3,6),1} + X_{(3,7),1} + X_{(3,8),1} +$$

$$X_{(3,9),1} + X_{(3,10),1} + X_{(3,11),1} + X_{(3,12),1} = 13702.75$$

Capacity restrictions of the three warehouses for the oil product.

$$X_{1,(1,2)} + X_{1,(2,2)} + X_{1,(3,2)} + X_{1,(4,2)} + X_{1,(5,2)} + X_{1,(6,2)} + X_{1,(7,2)} + X_{1,(8,2)} + X_{1,(9,2)} +$$

$$X_{1,(10,2)} + X_{1,(11,2)} + X_{1,(12,2)} = 7898$$

$$X_{2,(1,2)} + X_{2,(2,2)} + X_{2,(3,2)} + X_{2,(4,2)} + X_{2,(5,2)} + X_{2,(6,2)} + X_{2,(7,2)} + X_{2,(8,2)} + X_{2,(9,2)} +$$

$$X_{2,(10,2)} + X_{2,(11,2)} + X_{2,(12,2)} = 9870$$

$$x_{3,(1,2)} + x_{3,(2,2)} + x_{3,(3,2)} + x_{3,(4,2)} + x_{3,(5,2)} + x_{3,(6,2)} + x_{3,(7,2)} + x_{3,(8,2)} + x_{3,(9,2)} + x_{3,(10,2)} + x_{3,(11,2)} + x_{3,(12,2)} = 7582.25$$

Restrictions of the stations monthly demand for kerosene product.

$$x_{(1,1),1} + x_{(2,1),1} + x_{(3,1),1} = 5932.75$$

$$x_{(1,2),1} + x_{(2,2),1} + x_{(3,2),1} = 2256$$

$$x_{(1,3),1} + x_{(2,3),1} + x_{(3,3),1} = 2542$$

$$x_{(1,4),1} + x_{(2,4),1} + x_{(3,4),1} = 4163$$

$$x_{(1,5),1} + x_{(2,5),1} + x_{(3,5),1} = 2507$$

$$x_{(1,6),1} + x_{(2,6),1} + x_{(3,6),1} = 2271$$

$$x_{(1,7),1} + x_{(2,7),1} + x_{(3,7),1} = 2467$$

$$x_{(1,8),1} + x_{(2,8),1} + x_{(3,8),1} = 4504$$

$$x_{(1,9),1} + x_{(2,9),1} + x_{(3,9),1} = 2629$$

$$x_{(1,10),1} + x_{(2,10),1} + x_{(3,10),1} = 2348$$

$$x_{(1,11),1} + x_{(2,11),1} + x_{(3,11),1} = 3225$$

$$x_{(1,12),1} + x_{(2,12),1} + x_{(3,12),1} = 3558$$

Restrictions of the monthly demand of the stations for the oil product.

$$x_{(1,1),2} + x_{(2,1),2} + x_{(3,1),2} = 2836$$

$$x_{(1,2),2} + x_{(2,2),2} + x_{(3,2),2} = 2716$$

$$x_{(1,3),2} + x_{(2,3),2} + x_{(3,3),2} = 3690.5$$

$$x_{(1,4),2} + x_{(2,4),2} + x_{(3,4),2} = 2525$$

$$x_{(1,5),2} + x_{(2,5),2} + x_{(3,5),2} = 2044$$

$$x_{(1,6),2} + x_{(2,6),2} + x_{(3,6),2} = 1759.75$$

$$x_{(1,7),2} + x_{(2,7),2} + x_{(3,7),2} = 1680$$

$$x_{(1,8),2} + x_{(2,8),2} + x_{(3,8),2} = 1250$$

$$x_{(1,9),2} + x_{(2,9),2} + x_{(3,9),2} = 1487$$

$$x_{(1,10),2} + x_{(2,10),2} + x_{(3,10),2} = 2172$$

$$x_{(1,11),2} + x_{(2,11),2} + x_{(3,11),2} = 1650$$

$$x_{(1,12),2} + x_{(2,12),2} + x_{(3,12),2} = 1540$$

Restrictions on the total quantities of products (kerosene, oil).

$$x_{(1,1),1} + x_{(1,1),2} = 3066$$

$$x_{(2,1),1} + x_{(2,1),2} = 3368.5$$

$$x_{(3,1),1} + x_{(3,1),2} = 2334.25$$

$$x_{(1,2),1} + x_{(1,2),2} = 2393$$

$$x_{(2,2),1} + x_{(2,2),2} = 886$$

$$x_{(3,2),1} + x_{(3,2),2} = 1693$$

$$x_{(1,3),1} + x_{(1,3),2} = 2108$$

$$x_{(2,3),1} + x_{(2,3),2} = 2316.5$$

$$x_{(3,3),1} + x_{(3,3),2} = 1808$$

$$x_{(1,4),1} + x_{(1,4),2} = 2974.5$$

$$x_{(2,4),1} + x_{(2,4),2} = 2049$$

$$x_{(3,4),1} + x_{(3,4),2} = 1664.5$$

$$x_{(1,5),1} + x_{(1,5),2} = 975.5$$

$$x_{(2,5),1} + x_{(2,5),2} = 1137.75$$

$$x_{(3,5),1} + x_{(3,5),2} = 2437.75$$

$$x_{(1,6),1} + x_{(1,6),2} = 731.5$$

$$x_{(2,6),1} + x_{(2,6),2} = 1010.5$$

$$x_{(3,6),1} + x_{(3,6),2} = 2306.75$$

$$x_{(1,7),1} + x_{(1,7),2} = 773.5$$

$$x_{(2,7),1} + x_{(2,7),2} = 1036.75$$

$$x_{(3,7),1} + x_{(3,7),2} = 2336.75$$

$$x_{(1,8),1} + x_{(1,8),2} = 1577$$

$$x_{(2,8),1} + x_{(2,8),2} = 1438.5$$

$$x_{(3,8),1} + x_{(3,8),2} = 2738.5$$

$$x_{(1,9),1} + x_{(1,9),2} = 558$$

$$x_{(2,9),1} + x_{(2,9),2} = 2229$$

$$\begin{aligned}
x_{(3,9),1} + x_{(3,9),2} &= 1329 \\
x_{(1,10),1} + x_{(1,10),2} &= 2380 \\
x_{(2,10),1} + x_{(2,10),2} &= 860 \\
x_{(3,10),1} + x_{(3,10),2} &= 1280 \\
x_{(1,11),1} + x_{(1,11),2} &= 2425 \\
x_{(2,11),1} + x_{(2,11),2} &= 950 \\
x_{(3,11),1} + x_{(3,11),2} &= 1500 \\
x_{(1,12),1} + x_{(1,12),2} &= 1249 \\
x_{(2,12),1} + x_{(2,12),2} &= 2574.5 \\
x_{(3,12),1} + x_{(3,12),2} &= 1274.5
\end{aligned}$$

Budget constraints.

$$\begin{aligned}
\sum_{k=1}^p Bki &= \sum_{k=1}^p Ajk \\
[25350.25, 38402.75] &= [25350.25, 38402.75] \\
\sum_{i=1}^m Eij &= \sum_{i=1}^m Bki \\
[21193, 19857, 22703] &= [21193, 19857, 22703]
\end{aligned}$$

$$\sum_{j=1}^n Ajk = \sum_{j=1}^n Eij$$

$$\begin{bmatrix} 8768.75 & 4972 & 6232.5 \\ 6688 & 4551 & 4030.75 \\ 4147 & 5754 & 4116 \\ 4520 & 4875 & 5098 \end{bmatrix} = \begin{bmatrix} 8768.75 & 4972 & 6232.5 \\ 6688 & 4551 & 4030.75 \\ 4147 & 5754 & 4116 \\ 4520 & 4875 & 5098 \end{bmatrix}$$

$$\sum_{j=1}^n \sum_{k=1}^p Ajk = \sum_{k=1}^p \sum_{i=1}^m Bki = \sum_{i=1}^m \sum_{j=1}^n Eij$$

$$63753 = 63753 = 63753$$

Hyperbolic belonging function constraint (cost constraint)

$$\left\{ \frac{1}{2} + \frac{1}{2} \tanh \left[\frac{6}{51,275,230} * \left(\frac{179,411,860 + 128,136,630}{2} - Z_1 \right) \right] \geq \lambda \right\} * 2$$

$$1 + \tanh \left[\frac{6}{51,275,230} * \left(\frac{307,548,490}{2} - Z_1 \right) \right] \geq 2\lambda$$

$$\left\{ \tanh \left[\frac{6}{51,275,230} * (153,774,245 - Z_1) \right] \geq 2\lambda - 1 \right\} * \tanh^{-1}$$

$$\frac{6}{51,275,230} * (153,774,245 - Z_1) \geq \tanh^{-1}(2\lambda - 1)$$

According to the following entry in the

$$\alpha_r Z_{r(x)} + X_{mn+1} \leq \frac{\alpha_r}{2} (U_r + L_r)$$

$$\begin{aligned} & \frac{6}{51,275,230} Z_{1(x)} + X_{37} \\ & \leq \frac{3}{51,275,230} (179,411,860 + 128,136,630) * (51,275,230) \end{aligned}$$

$$6Z_1(x) + 51,275,230X_{37} \leq 922,645,470$$

$$\begin{aligned} & 6(888.25X_{1,(1,1)} + 1201.75X_{1,(2,1)} + 731.5X_{1,(3,1)} + 1619.75X_{1,(4,1)} + 1672X_{1,(5,1)} + 1881X_{1,(6,1)} + 1306.25 \\ & X_{1,(7,1)} + 2455.75X_{1,(8,1)} + 1175.625X_{1,(9,1)} + 2142.25X_{1,(10,1)} + 627X_{1,(11,1)} + 3448.5X_{1,(12,1)} + 3030. X_{2,(1,1)} + \\ & 3082.75 X_{2,(2,1)} + 2873.75 X_{2,(3,1)} + 3762 X_{2,(4,1)} + 3396.25 X_{2,(5,1)} + 4023.25 X_{2,(6,1)} + 2926 X_{2,(7,1)} + 4075.5 \\ & X_{2,(8,1)} + 2534.125 X_{2,(9,1)} + 3448.5 X_{2,(10,1)} + 1933.25 X_{2,(11,1)} + 5538.5 X_{2,(12,1)} + 1149.5 X_{3,(1,1)} \\ & + 992.75 X_{3,(2,1)} + 1410.75 X_{3,(3,1)} + 1881 X_{3,(4,1)} + 2168.375 X_{3,(5,1)} + 2664.75 X_{3,(6,1)} + 3082.75 X_{3,(7,1)} + 4232. \\ & 25 X_{3,(8,1)} + 3108.875 X_{3,(9,1)} + 4075.5 X_{3,(10,1)} + 2194.5 X_{3,(11,1)} + 5643 X_{3,(12,1)} + 884 X_{1,(1,2)} + 1196 \\ & X_{1,(2,2)} + 728 X_{1,(3,2)} + 1612 X_{1,(4,2)} + 1664 X_{1,(5,2)} + 1872X_{1,(6,2)} + 1300 X_{1,(7,2)} + 2444 X_{1,(8,2)} + 1170 \\ & X_{1,(9,2)} + 2132 X_{1,(10,2)} + 624 X_{1,(11,2)} + 3432 X_{1,(12,2)} + 3016 X_{2,(1,2)} + 3068 X_{2,(2,2)} + 2860 X_{2,(3,2)} + 3744 \\ & X_{2,(4,2)} + 3380 X_{2,(5,2)} + 4004 X_{2,(6,2)} + 2912 X_{2,(7,2)} + 4056 X_{2,(8,2)} + 2522 X_{2,(9,2)} + 3432X_{2,(10,2)} + 1924 \\ & X_{2,(11,2)} + 5512 X_{2,(12,2)} + 1144 X_{3,(1,2)} + 988 X_{3,(2,2)} + 1404 X_{3,(3,2)} + 1872 X_{3,(4,2)} + 2158 X_{3,(5,2)} + 2652 \\ & X_{3,(6,2)} + 3068 X_{3,(7,2)} + 4212 X_{3,(8,2)} + 3094 X_{3,(9,2)} + 4056 X_{3,(10,2)} + 2184 X_{3,(11,2)} + 5616 X_{3,(12,2)} + 51,27 \\ & 5 \end{aligned}$$

The fourth step: After solving the model of the fuzzy three-dimensional transportation trouble (hyperbola) in the QM program, the results were as follows:

$$\begin{aligned} & x_{1,(3,2)} = 4246, x_{1,(5,1)} = 738, x_{1,(6,1)} = 2268, x_{1,(7,1)} = 1492, x_{1,(8,1)} = 2033, \\ & x_{1,(9,1)} = 755, x_{1,(11,1)} = 971, x_{1,(12,1)} = 3302, x_{2,(5,1)} = 97, x_{2,(7,1)} = 975, \\ & x_{2,(8,1)} = 2471, x_{2,(9,1)} = 1674, x_{2,(10,1)} = 2248, x_{2,(11,1)} = 2279, x_{2,(12,1)} = 256, \\ & x_{3,(1,1)} = 6501, x_{3,(2,1)} = 2256, x_{3,(3,1)} = 297, x_{3,(4,1)} = 4183, x_{3,(5,1)} = 1672, x_{3,(6,1)} = 3, x_{1,(1,2)} = 1337, \\ & x_{1,(2,2)} = 96, x_{1,(4,2)} = 1808, x_{1,(8,2)} = 391, x_{1,(9,2)} = 1287, x_{1,(10,2)} = 729, x_{1,(11,2)} = 1250, x_{2,(2,2)} = 322, \\ & x_{2,(3,2)} = 313, x_{2,(5,2)} = 2044, x_{2,(6,2)} = 1756, x_{2,(7,2)} = 1680, x_{2,(8,2)} = 859, x_{2,(10,2)} = 1343, \\ & x_{2,(12,2)} = 1540, x_{3,(1,2)} = 1499, x_{3,(2,2)} = 1698, x_{3,(3,2)} = 2377, x_{3,(4,2)} = 707, Z_1(x) = 176.35, 82,105,398, 1 \\ & , X_{45} = 1, X_{58} = 1, X_{6,10} = 1, X_{77} = 1, X_{83} = 1, X_{99} = 1 \end{aligned}$$

$$X_{37} = \tanh^{-1}(2\lambda_{-1})$$

$$\{1.51 = \tanh^{-1}(2\lambda_{-1})\} * \tanh$$

$$\tanh(1.51) = (2\lambda_{-1})$$

$$0.026 = 2\lambda_{-1}$$

$$2\lambda_{-1} = 1.026$$

$$\lambda_{-1} = \frac{1.026}{2}$$

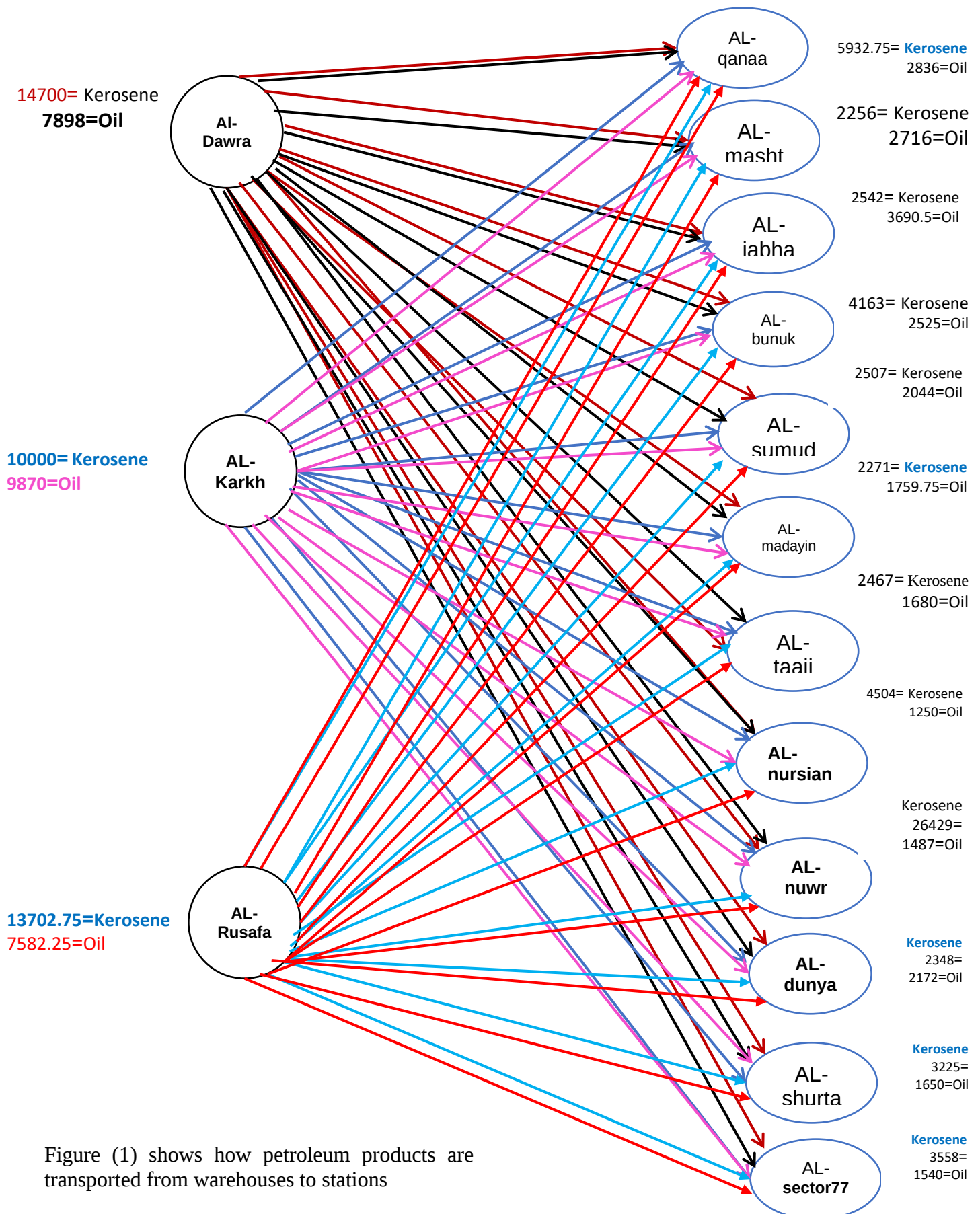


Figure (1) shows how petroleum products are transported from warehouses to stations

1. By solving the three-dimensional transportation model for petroleum products from warehouses to fuel gas stations on both parties of Karkh & Rusafa, using fuzzy programming, it was found that the total cost of transportation is (82,105,398) dinars.
2. Full utilization of the production capacity of the three warehouses of products (kerosene, oil).
3. investigation all demand and supply constraints, as well as budget constraints
4. When comparing the three-dimensional transportation model with the bilateral transportation model and its solution by traditional methods, we find that the three-dimensional transportation model is better than the approximate Vogel method, because the total cost of transporting products at once and by one means of transportation is less than transporting the product separately, and this process costs the company costs Great transfer.
5. Through the use of the hyperbola belonging function as constraints in the mathematical model, the results showed that the transportation cost is (82,105,398) million dinars.

Recommendations:

- 1- Generalizing the fuzzy three-dimensional transportation model in all oil companies and in other governorates.
- 2- Using this model to reduce time and it can be used to increase profits as well.
- 3- Providing a database in governmental and private institutions to facilitate the task of researchers in developing research.

Funding

None

Acknowledgement

None

Conflicts of Interest

The author declares no conflict of interest.

English References:

- Arora S.R.,(2004): " Three dimensional fixed charge bi criterion indefinite quadratic transportation trouble", yugoslav journal of operations research, no.1,p.p: 83-97.
- Bakhayt Abdul-Gabbar Khaddar,(2016): "Solving bi-objective 4-dimensional transportation trouble by using pso" , department of statistics, college of economic & administration, university of baghdad, baghdad, iraq.
- Eliodor Constantinescu: "Three dimensional maritime transportation models",international conference on maritime and naval science and engineering.

- Faysal ,Muhammad, Bashir,(2019) ; Muhsin, Al-Zawbai, Ubayd Mahmud ; Ahmad, Muhammad, Al-Tayyib Umar ,2019, " Building a three-dimensional maritime transportation model to find the best solution by using the heuristic algorithm" vol. 7, issue 2, pp.87-99, p.13
- Gupta, Kavita; Arora, Ritu,(2021): "Three dimensional bounded transportation trouble" 31 , number 1, 121–137.
- jeter, alaa shnaishel, (2018), "multi-objective constrained fuzzy transportation with mixed constraints using different membership functions," journal of economic and administrative sciences, issue 107, volume 24, pp. 614-629.
- K.Balasubramanian¹ , Dr. S. Subramanian,(2018): " An approach for solving fuzzy transformation trouble" international journal of pure and applied mathematics, volume 119 no. 17 2018, 1523-1534.
- Kadhim, Ihsan Jawad, (2014) "Transportation problem with fuzzy multiple objective functions with practical application on some petroleum products," university of baghdad, college of administration and economics, master thesis.
- M. Zangiabadi and H. R. Maleki,(2013): " Fuzzy goal programming technique to solve multiobjective transportation troubles with some non-linear membership functions" iranian journal of fuzzy systems vol. 10, no. 1, pp. 61-74.
- Suhad, Faisal Abboud: "Solving the trouble of three-dimensional transportation using linear programming," research extracted from a master's thesis, journal of economic and administrative sciences, issue 103, volume 24, pages 395-410.