

Original Research**CALCULATING THE NATURAL FREQUENCY OF PRE-TWISTED BEAM**

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Abstract: Beams in two configurations; uniform and non-uniform, are common structural components utilized for several engineering applications. Thus, the studies dealing with their behavior under dynamic and vibrations have been increased. In this research, the transverse vibration phenomena of pre-twisted beams were experimentally and theoretically studied by investigating the effect of twisting angles on the first three transverse natural frequencies. In the present experimental part, the pre-twisted beams are manufactured using a 3D printer, and the fundamental frequencies of manufactured pre-twisted beams are measured by a suitable rig. In the theoretical work, the finite element method is used to simulate the transverse vibration behavior of the pre-twisted beams. The accuracy of the simulation process is checked by comparison of the first natural frequencies calculated by the finite element method (using ANSYS Workbench Software) with those measured experimentally. The results show that there is an excellent agreement between the experimental results and finite element outputs. For the clamped-free pre-twisted beam, there is no critical twisting angle. The critical twisting angle is equal to the mode number for simply – simply and clamped simple pre-twisted beam. While there is more than one value of critical twisting angle for the clamped-clamped pre-twisted beam.

Keywords: ANSYS workbench; finite element method; non-uniform beam; transverse vibration; twisting angle

1. Introduction

Beams are key structural components and are classified according to their geometry into: (i) uniform or tapered, (ii) slender or thick, and (iii) twisted or curved beams [1]. Several engineering applications (aeronautics, robotics, architecture, and other innovative engineering applications) use uniform isotropic beams. Besides, considerably available research dealt with the transverse vibration of this type of beam [1]. Experimentally, the research found that better or more suitable distribution of strength and mass may be provided by non-uniform beams in comparison to uniform beams [2-4]. Generally, the partial differential equations of vibration behavior for uniform beams are solved analytically and the analytical solution is more complicated when the non-uniformity effects are considered [1]. Therefore, several approximate solutions are used to study the vibration behavior of non-uniform beams. Each approximate method makes several assumptions to reduce the effect of the nonlinear parts in partial differential equations due to the

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non-uniformity effect. Some of these approximate methods are the Frobenius method (FM) [5, 6], Adomian decomposition method (ADM) [7], Galerkin method (GM) [8, 9], finite element method (FEM) [10–14] and Rayleigh-Ritz method (RRM) [15, 16].

Prevalent previous research dealt with the non-uniform cross-section area which leads to variation in material (i.e. mass) and second moment of area. , the pre-existing beams are non-uniform beams that have uniform material distribution but with varying second moments of area. Many studies analyzed the dynamic and vibration behavior of rotating pre-twisted beams using different techniques based on the Euler beam theory [17] or Timoshenko beam theory [18].

Many studies investigated the vibration or dynamic behavior of pre-twisted beams or blades. Abrate [19] and Dawson [20, 21] used the Rayleigh-Ritz method to study the vibration behavior of a pre-twisted blade. Gupa and Rao [22] used FEM to calculate the natural frequencies of doubly tapered and twisted beams based on Timoshenko's theory. Hodges et al. [23] estimated the first natural frequencies of non-uniform rotating beams using the transfer matrices method. Also, Lin et al. [24] based on Timoshenko theory and used modified the transfer matrices method to study the dynamic behavior of a blade considered a non-uniform pre-twisted beam in Kuang and Hsu [25, 26]. With the same consideration, Swaminathan and Rao [27] calculated the natural frequency of the pre-twisted beam. Subrahmanyam et al. [28] studied the effect of pre-twisting in the blade on the shear deflection, rotary inertia, and vibration using the Reissner method. Also, papers dealing with the behavior of vibration for the rotating composite laminated blade or composite laminated pre-twisted beam were made [1,29].

In the current paper, the fundamental frequencies of pre-twisted beams are determined experimentally and theoretically. Experimentally, pre-twisted beams with different twisted angles are manufactured using 3D printers, and then the natural frequencies of these pre-twisted beams are measured using a suitable rig. The finite element method is applied to simulate the free vibration behavior of pre-twisted beams using ANSYS software (Workbench).

2. Methodology

2.1 Manufacturing of pre-twisted beams

The experiments were conducted using a Tronxy 3D printer shown in Fig. 1, and PETG (Polyethylene Terephthalate Glycol-Modified) filament. A pre-twisted beam design was implemented with specific dimensions, width, thickness, length, (10, 15, and 350) mm and five twisting angles (0° , 90° , 180° , 270° , and 360°). The printing process involved carefully controlling the printer settings, including temperature, layer height, and print speed, to ensure accurate and consistent results. The twisting angle variations were achieved by precise measuring and controlling the angle using additional fixtures and equipment.

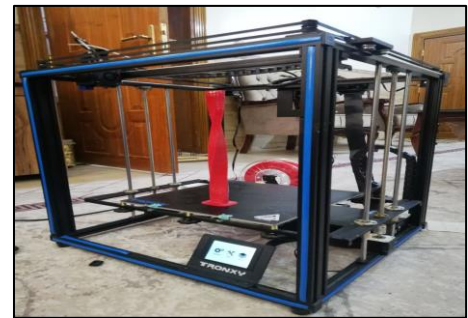


Figure 1. Tronxy x5sa pro 3D printer

2.1.1 3D printer technology.

Tronxy is a brand that specializes in manufacturing and providing 3D printers for

various user levels, from beginners to experienced enthusiasts. Here are some key points about Tronxy 3D printers:

1. Studying build quality and robust construction. The printers are durable and provide a firm platform for accurate 3D printing.
2. Wide range of 3D models to cater to different needs and budgets.
3. Fused Deposition Modeling (FDM) technology, which involves melting and depositing filament layer by layer to create the desired object.
4. It is a user-friendly interface and accessible to beginners, with intuitive controls, and clear instructions.
5. Good printing accuracy and resolution that allows users to create detailed and precise 3D models. The printers often provide adjustable settings for layer height and printing speed, enabling users to achieve the desired level of detail.
6. It offers various connectivity options, including USB and SD card interfaces, allowing users to transfer their 3D models conveniently. They are also compatible with popular slicing software, such as Cura and Simplify3D, providing flexibility in choosing the software that best suits their needs.

2.1.2 Filament

PETG (Polyethylene Terephthalate Glycol-Modified) filament exhibits excellent mechanical properties, which are shown in Table. 1. which makes it suitable for this experiment. It possesses high tensile strength, offering resistance to external forces and

impacts. Additionally, PETG filament displays flexibility, allowing for torsion without compromising structural integrity. It is also heat-resistant, with a higher melting temperature compared to other filaments, ensuring stability at elevated temperatures. Furthermore, PETG filament demonstrates chemical resistance, making it durable in various environmental conditions. PETG is a versatile material with numerous applications across various industries.

Table 1. PETG mechanical properties

Impact Strength	Heat Deflection	Flexural Strength	Tensile Strength
8.1 kJ/m ²	70°C	69 MPa	50 MPa

2.2. Physical and Mechanical Properties of PETG

The tensile test was carried out on the PETG to obtain the tensile properties of PETG according to the (ASTM D638) [30] using five specimens with dimensions as given in Fig. 2. Also, the density of PETG was measured experimentally. The required physical and mechanical properties are listed in Table. 2, and it's approximately equal to that in the available literature.

Table 2. Mechanical and Physical Properties of PETG.

Property	Symbol	Unit	Value
Modulus of Elasticity	E	N/m ²	2.1*10 ⁹
Density	ρ	kg/m ³	1270
Poisson Ratio	ν	---	0.425

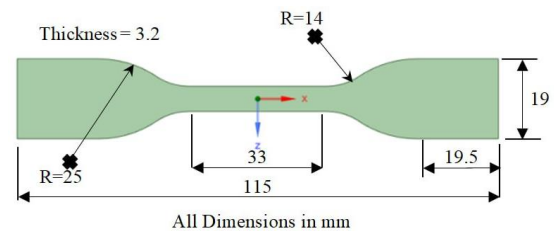


Figure 2. ASTM D638 Standard Dimensions [30].

3. Free Vibration Test of Pre-Twisted Beam

The vibration test performs the first natural frequencies of the pre-twisted beam. The apparatus of this test consists of a base, support

arm, beam sample, accelerometer, hammer, amplifier, and an oscilloscope as shown in Fig. 3.

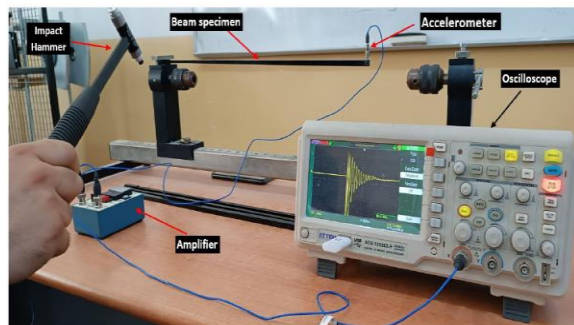


Figure 3. Free Vibration Test Apparatus.

The free vibration test on pre-twisted beams is carried out using the following step-by-step procedure given in [2–4]:

1. Set up the pre-twisted beam in a vibration test rig. The beam should be securely clamped at one end to prevent any unwanted movement during the test.
2. Mount the accelerometer on the end of the beam sample
3. Excite the beam by the impact of the hammer towards the acceleration reader.
4. Store the data through the digital storage oscilloscope and then save this data as CSV by Flash Ram.
5. The signal obtained from the digital storage oscilloscope represents the acceleration data. This signal was very noisy, so it was treated by the Sigview program for noise removal to get the final signal as shown in Fig. 4.
6. The final signal was transformed by Fast Fourier Transformation (FFT) to get the fundamental natural frequency as shown in Fig. 5.
7. Repeat the test for various twisting angles (0° , 90° , 180° , 270° , and 360°), to investigate the

natural frequencies of the beam as shown in Fig. 6.

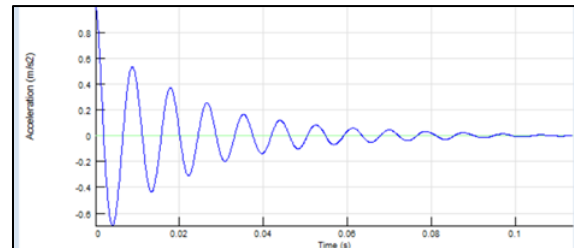


Figure 4. Acceleration Vs. Time.

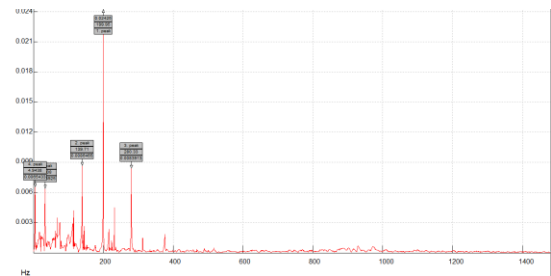


Figure 5. Fast Fourier Transformation(FFT).



Figure 6. Beam Samples with Different Twisting Angles (θ).

4. Finite Element Simulation

ANSYS Workbench software has been used in the present work to simulate the free vibration of a pre-twisted beam. The beam has (10mm) thickness and (15mm) width and it was twisted with different twisting angles (from 0° to 620° with step 45°). Several length values for the pre-twisted beam are considered (350, 437.5, 525, 612.5, and 700) mm. Fig. 7, illustrates the geometry and mesh of three pre-twisted beams with twisting angles (90° , 360° and 450°) when the length of the beam is (350mm). The element

SOLID186 with 20-Node is used in this model and convergent criteria are applied for selecting the suitable size of the element [31–39]. The number of elements and nodes are (450) and (2939) respectively when the length of the beam is 350 mm and the twisting angle is zero (i.e. uniform beam). When the length of the beam is 700 mm and the twisting angle is (720°), the number of elements and nodes are (852) and (5851) respectively.

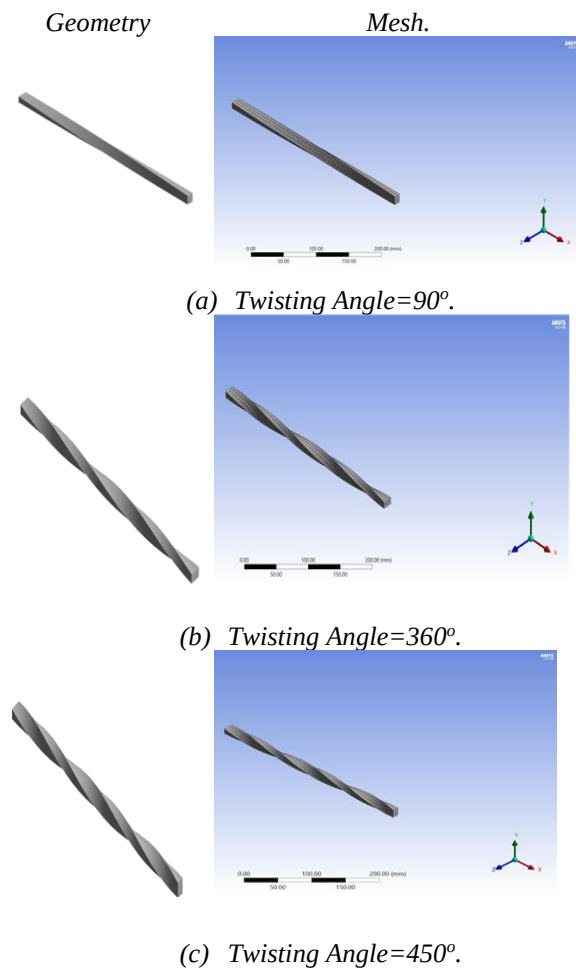


Figure 7. Geometry and Mesh of Pre-Twisted Beams with Twisting Angles (a) 90° , (b) 360° , and (c) 450° .

The boundary conditions applied in this model are:

- (a) Simply-Supported Beam: The linear displacements in x,y, and z directions (U_x , U_y , and U_z) of the left cross-section area of

the beam are zero. The linear displacements in y and z directions (U_y and U_z) of the right cross-section area of the beam are zero too.

- (b) Clamped-Clamped Beam: All displacements (linear and rotational) of the left and right cross-section area of the beam are zero.
- (c) Clamped-Simply Supported Beam: All displacements (linear and rotational) of the left cross-section area of the beam are zero. The linear displacements in the y and z direction (U_y and U_z) of the right cross-section area of the beam are zero.
- (d) Clamped-Free Beam: All displacements (linear and rotational) of the left cross-section area of the beam are zero only.

In all cases used in theoretical work, the convergent criteria are considered.

5. Results and Discussion

5.1 Comparison between the Experimental and Theoretical Results

The experimental results of the first natural frequency of the pre-twisted beam that has dimensions (10, 15, and 350) mm with clamped-free support (CFB) and with five twisting angles (0° , 90° , 180° , 270° , and 360°). The comparison between the natural frequency measured experimentally and that calculated by ANSYS software is made as illustrated in Fig. 8. The maximum absolute error percentage between the experimental and theoretical results is (5.22%) at a twisting angle of (270°).

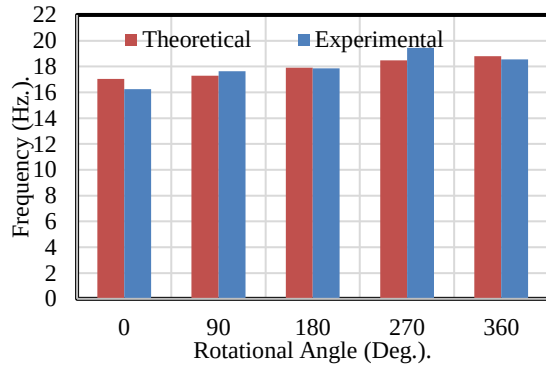


Figure 8. The Comparison Between the Experimental and Theoretical Natural Frequency.

5.2 Theoretical Results

In theoretical work, the first three natural frequencies are calculated to study the free vibration of the pre-twisted beam. The width and thickness of the beam are (15 and 10) mm respectively, while the length of the beam is varied as (350, 437.5, 525, 612.5, and 700) mm. The pre-twisted beams are supported as Clamped- Clamped support (C-C), Simply-Simply support (S-S), Clamped-Free support (C-F), and Clamped-Simply support (C-S).

5.2.1 Clamped-Free Pre-Twisted Beam

The effect of the twisting angle on the first, second, and third transverse natural frequencies for the clamped-free pre-twisted beam are illustrated in Fig. 9. The first transverse natural frequency increases slightly with the increase in the twisting angle shown in Fig. 9-a. Also, the first transverse natural frequency at any twisting angle decreases with increasing the length of the clamped-free pre-twisted beam as illustrated in Fig. 10-a. This happens because (i) the increasing of pre-twisted beam length leads to a reduced Experimental Study of using the effect of supports as illustrated in Equation 1, for uniform beam [37].

$$\omega_n = \left(\frac{(2n-1)\pi}{2} \right)^2 \left(\frac{EI}{\rho AL^4} \right)^{0.5} \quad (1)$$

Where:

ω_n Natural Frequency (rad/sec).

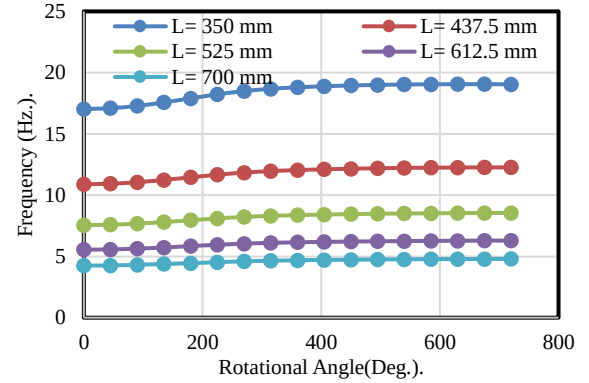
E Modulus of Elasticity (N/m²).

L Length of Beam (m).

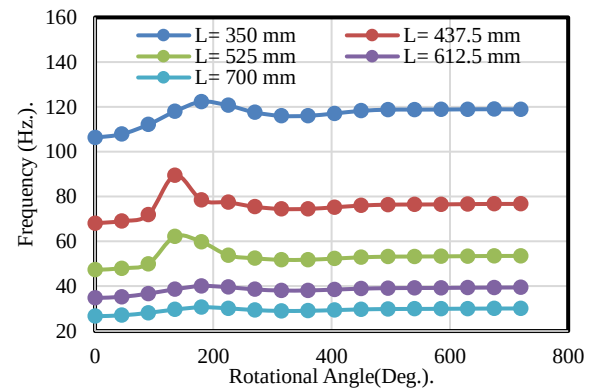
A Cross Section Area of Beam (m²).

I Second Moment of Area (m⁴).

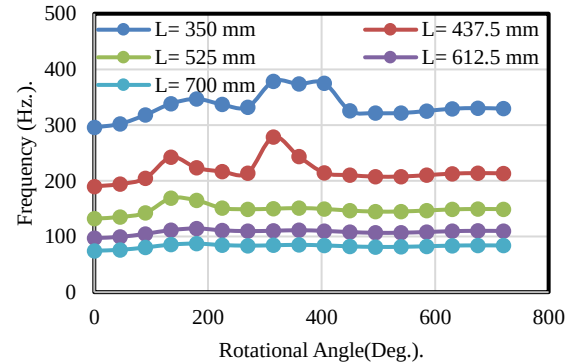
ρ Density (kg/m³).



(a) 1st Natural Frequency.



(b) 2nd Natural Frequency



(c) 3rd Natural Frequency

Figure 9. The Effect of Twisting Angle on the Natural Frequency of Clamped-Free Pre-Twisted Beam.

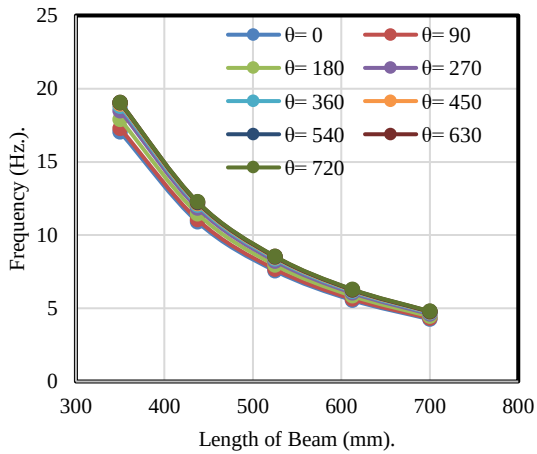
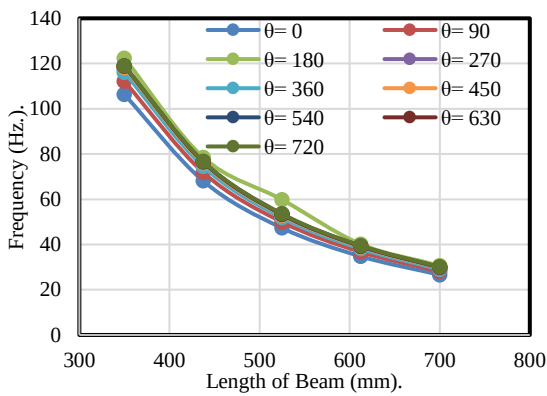
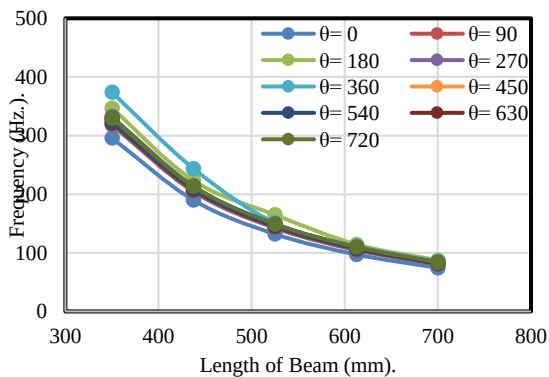
(a) 1st Natural Frequency.(b) 2nd Natural Frequency(c) 3rd Natural Frequency

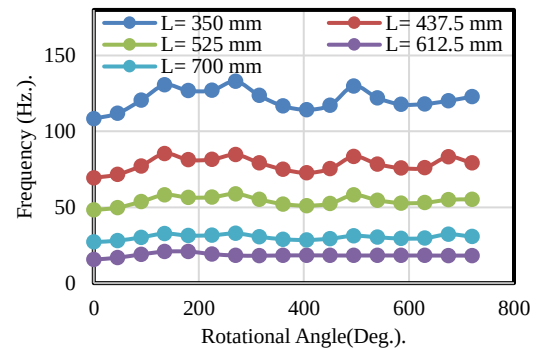
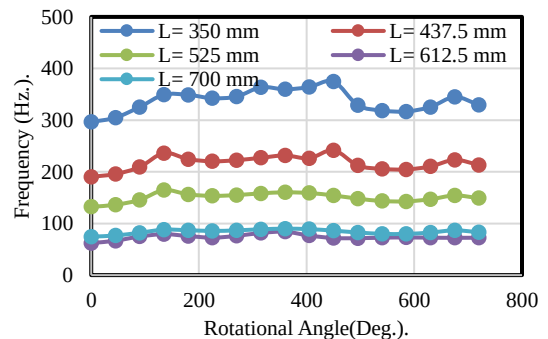
Figure 10. The Effect of Length on the Natural Frequency of Clamped-Free Pre-Twisted Beam.

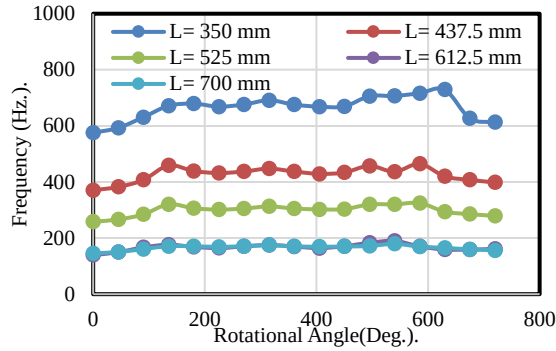
(ii) the increasing twisting angle leads to an increase in the equivalent moment of area (I) and this causes a natural frequency increase.

For second and third transverse natural frequencies, it can be noted that there is a critical value of twisting angle which gives a maximum natural frequency. For example, the maximum second transverse natural frequency occurs at twisting angles (180o, 135o, 135o, 180o and 180o) when the length of the pre-twisted beam is (350, 437.5, 525, 612.5, and 700) mm respectively shown in Fig. 9-b. On the other side, the second and third transverse natural frequencies at any twisting angle decrease with increasing the length of the clamped-free pre-twisted beam as shown in Fig. 10 -b & c.

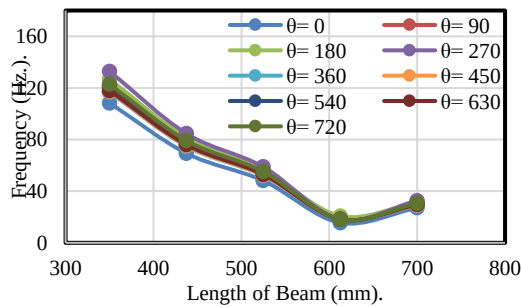
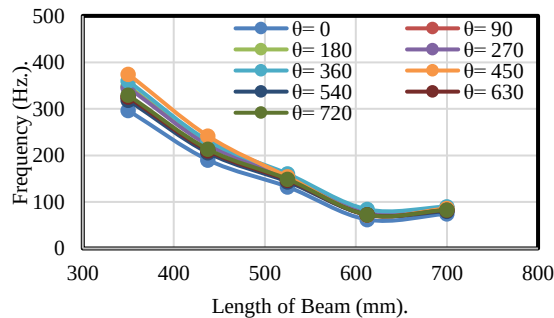
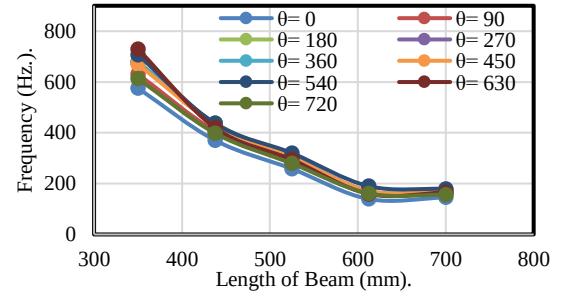
5.2.2 Clamped-Clamped Pre-Twisted Beam

Fig. 11 shows the effect of the twisting angle on the first three natural frequencies of the clamped-clamped pre-twisted beam. It can be seen that there is more than one critical twisting angle.

(a) 1st Natural Frequency.(b) 2nd Natural Frequency

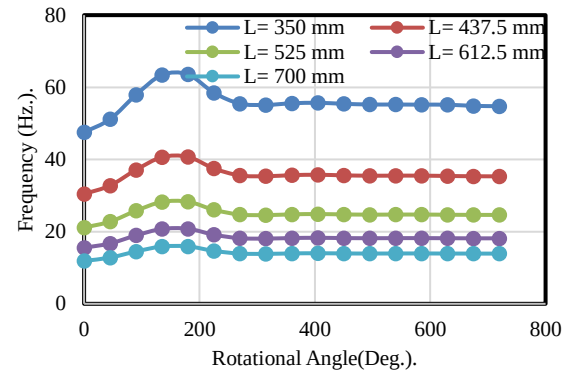
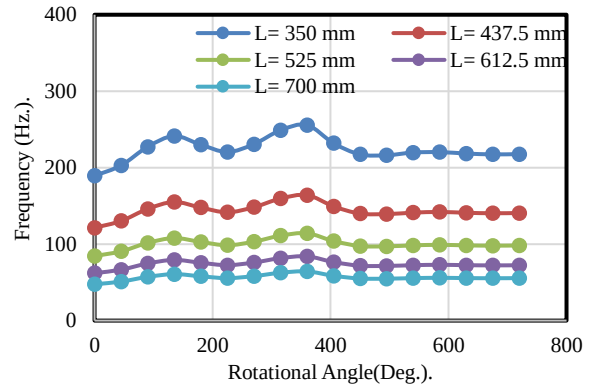
(c) 3rd Natural Frequency**Figure 11.** The Effect of Twisting Angle on the Natural Frequency of Clamped-Clamped.

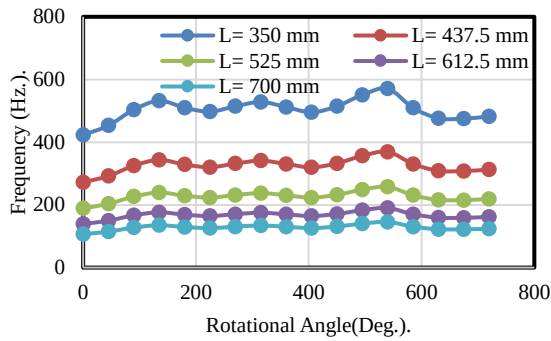
Also, the value of these critical twisting angles and their corresponding natural frequencies depend on the length of the pre-twisted beam and mode number. The effect of beam length on the first three natural frequencies of the clamped-clamped pre-twisted beam is illustrated in Fig. 12. It can be noted that there is a critical length of the beam which gives the minimum natural frequencies and this value is (612.5mm) in this work.

(a) 1st Natural Frequency.(b) 2nd Natural Frequency(c) 3rd Natural Frequency**Figure 12.** The Effect of Length on the Natural Frequency of Clamped-Clamped Pre-Twisted Beam.

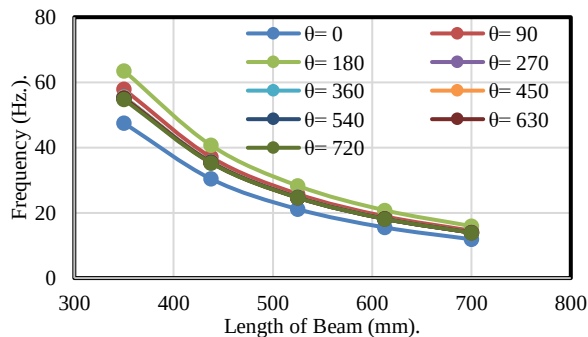
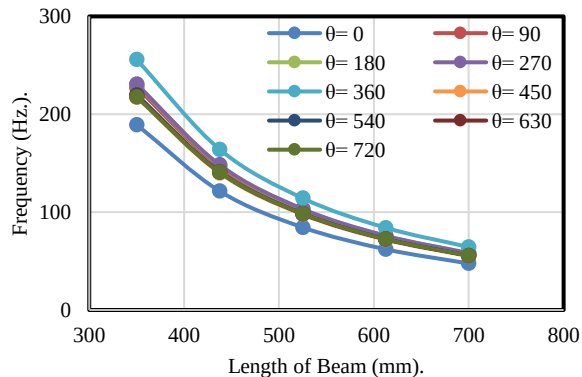
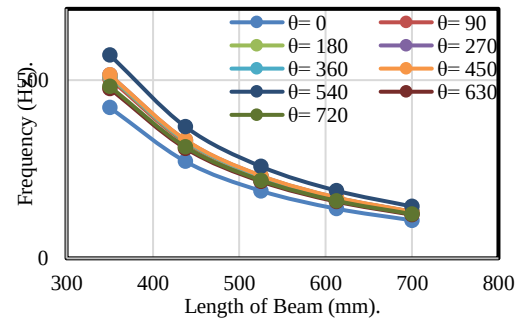
5.2.3 Simply-Simply Pre-Twisted Beam

For a simply pre-twisted beam, the effect of twisting angle on the first three natural frequencies shows that the number of critical twisting angles is equal to the number of modes as shown in Fig. 13.

(a) 1st Natural Frequency.(b) 2nd Natural Frequency

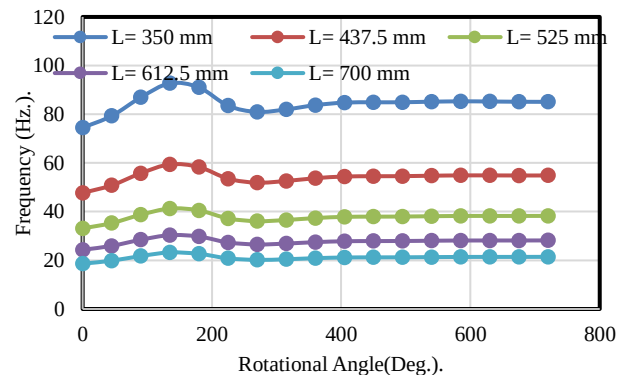
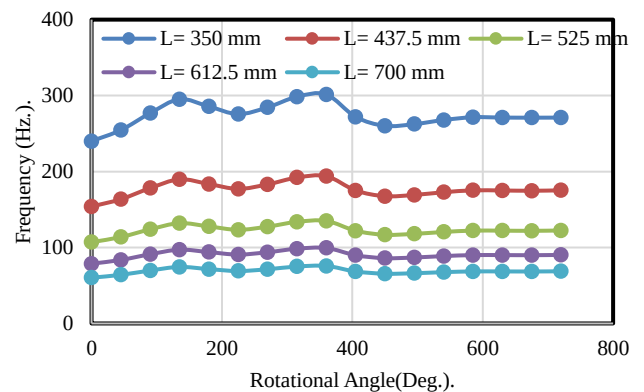
(c) 3rd Natural Frequency**Figure 13.** The Effect of Twisting Angle on the Natural Frequency of Simply-Simply Pre-Twisted Beam.

In other words, there is one critical twisting angle for the first natural frequency, and for the second natural frequency, there are two critical twisting angles while there are three critical twisting angles for the third natural frequency. In Fig. 14, the first three natural frequencies decrease with increasing the length of the pre-twisted beam.

(a) 1st Natural Frequency.(b) 2nd Natural Frequency(c) 3rd Natural Frequency**Figure 14.** The Effect of Length on the Natural Frequency of Simply-Simply Pre-Twisted Beam

5.2.4 Clamped-Simply Pre-Twisted Beam

For the clamped-simply pre-twisted beam, Fig. 15 shows the variation of first, second, and third natural frequencies due to twisting angles for different lengths of the pre-twisted beam. Similar to a simply-simply pre-twisted beam, the number of critical twisting angles is equal to the mode number.

(a) 1st Natural Frequency.(b) 2nd Natural Frequency

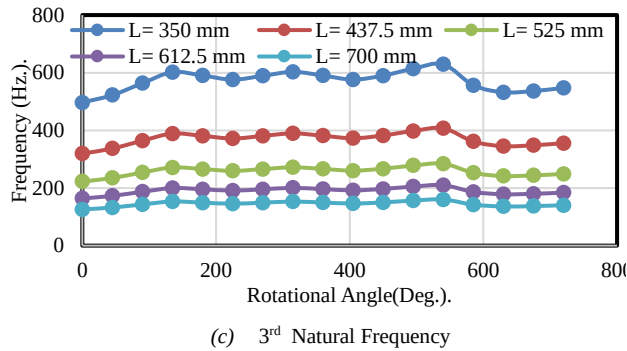


Figure 15. The Effect of Twisting Angle on the Natural Frequency of Clamped- Simply Pre-Twisted Beam.

Also, the values of the critical twisting angle and the corresponding natural frequency depend on the length of the pre-twisted beam and mode number. On the other side, the first three natural frequencies decrease with increasing the length of the pre-twisted beam as shown in Fig. 16.

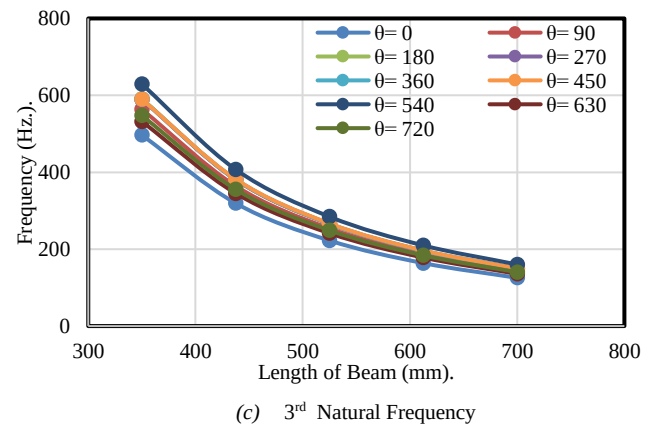
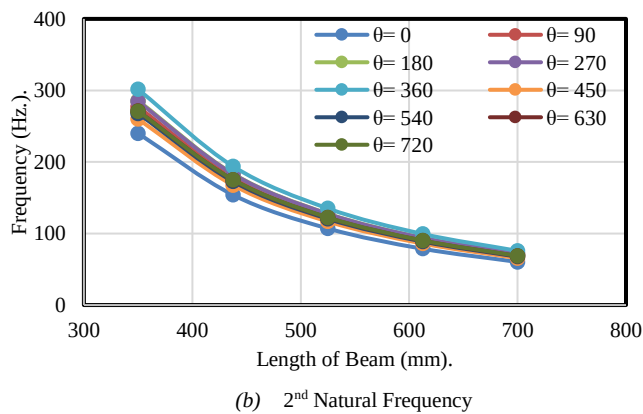
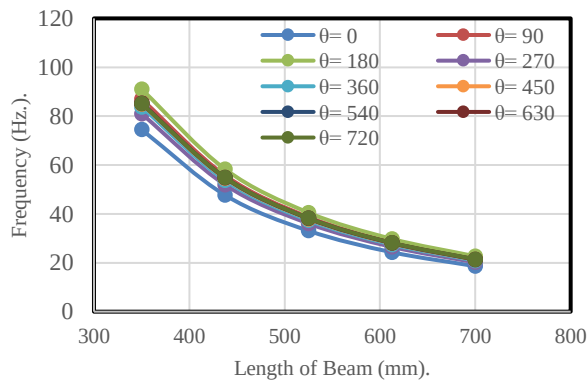
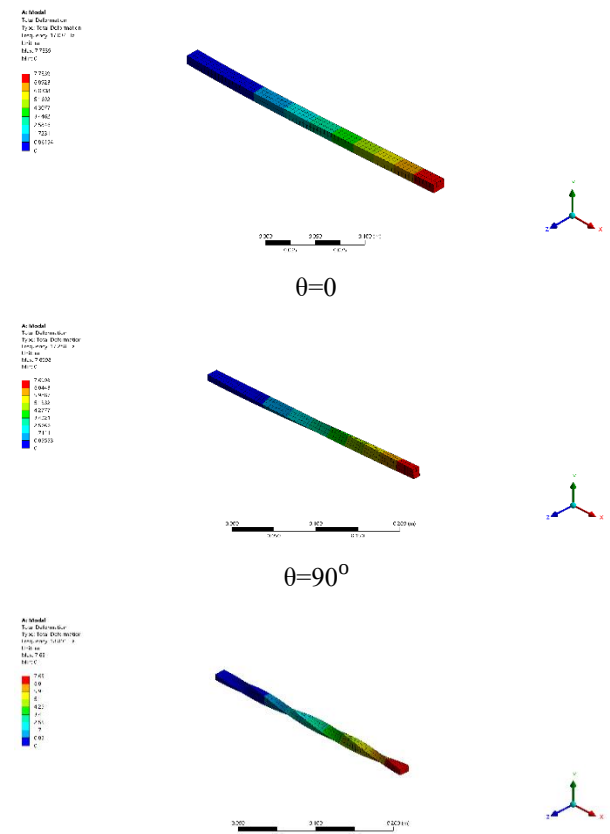


Figure 16. The Effect of Length on the Natural Frequency of Clamped- Simply Pre-Twisted Beam.

5.3 Finite Elements Results:

In this section, the first mode vibration of the different pre-twisted beams with different supporting types is illustrated in Fig.17, 18, 19, and 20. Four twisting Angles (0° , 90° , 360° , and 540°) are considered. Generally, the mode shapes of the first mode are similar for each supported type.



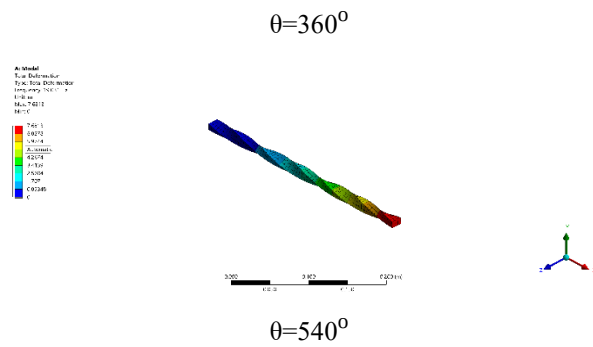


Figure 17. First Mode Vibration of Clamped-Free Pre-Twisted Beam

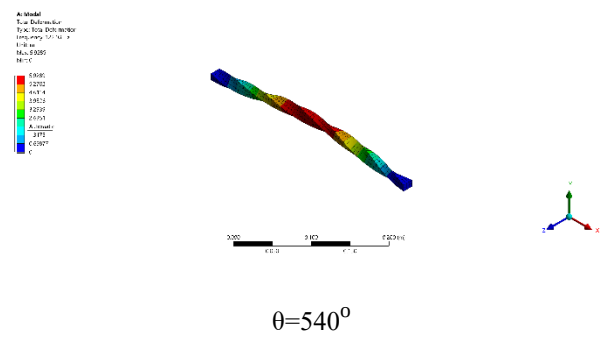
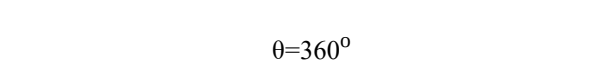
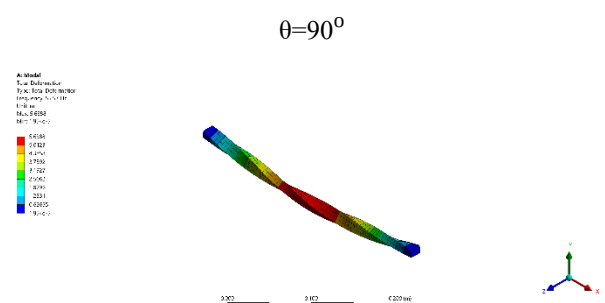
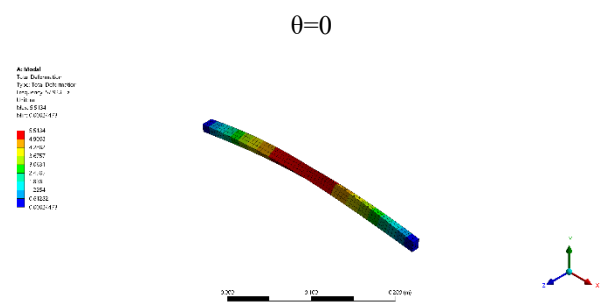
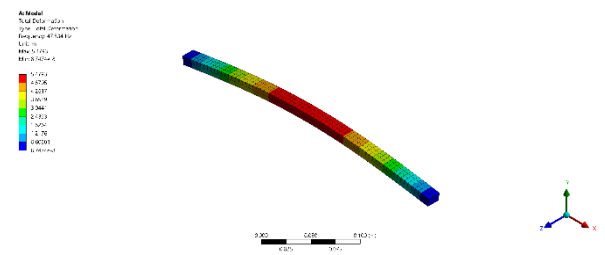
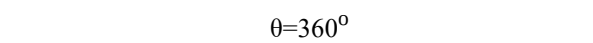
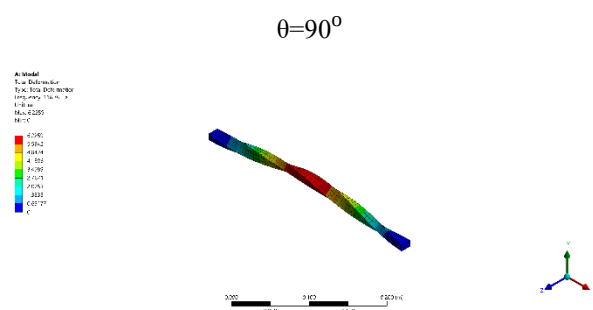
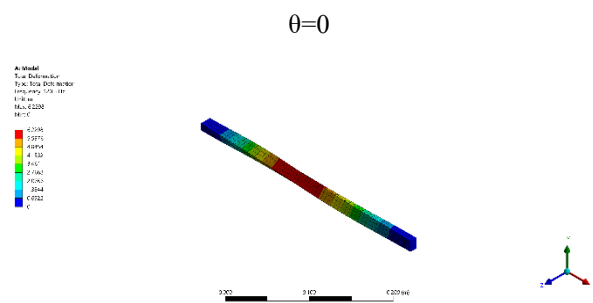
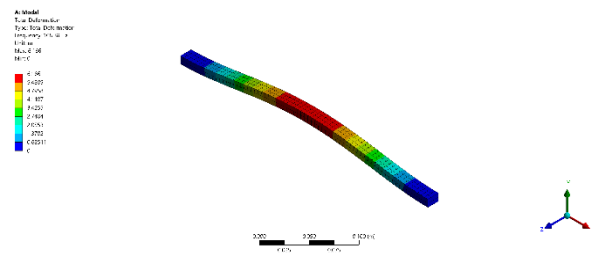


Figure 18. First Mode Vibration of Clamped-Clamped Pre-Twisted Beam



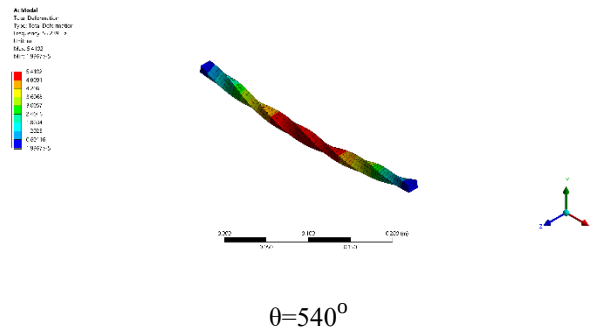


Figure 19. First Mode Vibration of Simply Supported Pre-Twisted Beam

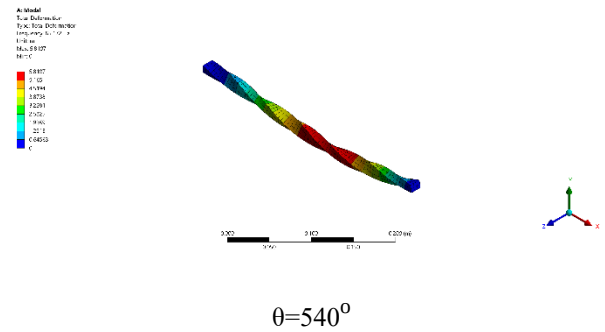


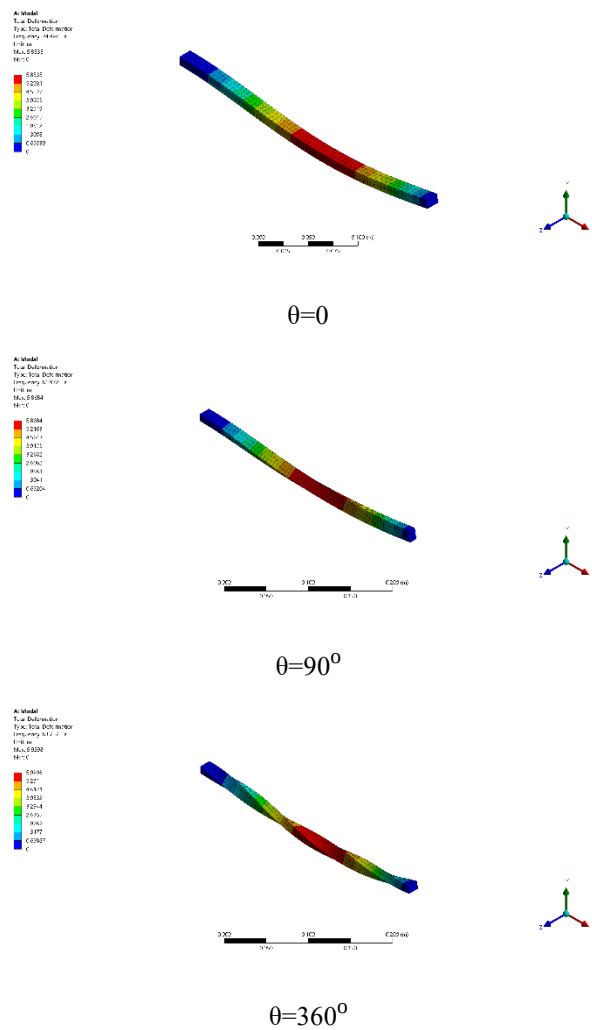
Figure 20. First Mode Vibration of Clamped-Simply Pre-Twisted Beam

6. Conclusions and Future Works

In this work, the first three natural frequencies of the transverse pre-twisted beams are calculated theoretically using the ANSYS Workbench Software. The accuracy of the simulation process is checked by comparing the simulation results with the experimental one. In the present experimental work, the 3D printer was used to manufacture the pre-twisted beam, and the fundamental frequency was then measured.

From previous sections, the following points can be concluded: The 3D printer is a suitable process to manufacture a beam with complex geometry like the pre-twisted beam. There is an excellent agreement between the experimental and finite element-free vibration results of pre-twisted beams. For a clamped-free pre-twisted beam, there is no critical twisting angle. While the values of the critical twisting angle are equal to the mode number for simply – simply and clamped simple pre-twisted beam. While there is more than one value of critical twisting angle for the clamped-clamped pre-twisted beam.

For the clamped-clamped pre-twisted beam, there is a critical length that produces the minimum natural frequencies and its value is (612.5mm) and this fact is not noted in other support types.



For future works, the effect of (width/thickness) ratio on the natural frequencies of pre-twisted beams will be investigated experimentally and theoretically using finite element and Rayleigh-Ritz methods.

Conflict of interest

The authors confirm that the publication of this article causes no conflict of interest

Authors' contributions:

Zainab M. Shukur: (Corresponding author) developed the Methodology of the research, made the formal analysis and investigation wrote the original draft, and was responsible for funding acquisition.

Raghad Azeez Neamah developed the methodology of the research, made the formal analysis and investigation wrote the original draft, and was responsible for funding acquisition.

Husam Jawad Abdulsamad made the formal analysis and investigation, participated in writing the article, reviewed previous literature, and participated in editing and project administration.

Luay S. Al-Ansari and Sutartip Wittayapiyanon developed the conceptualization, made the formal analysis and investigation, participated in writing the article, reviewed previous literature, and participated in editing and project administration.

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